average plasma characteristics that takes place in a length scale $L=U_1\Delta t$, where $U_1$ is the velocity of the front and $\Delta t$ is its duration on the oscilloscope trace. Typically the front arrives in a time 0.8 $\mu$sec. Thus $U_1 \cong 8\times10^6$ cm/sec and $L\cong 0.35$ cm after allowing for the 1.6-mm probe diameter. (Note that this is an upper limit for $L$ since the shock may have slowed down before reaching the probes, and the response time of the circuit is about $10^{-8}$ sec.)

The length $L=0.35$ cm contrasts with collision mean free paths in air at 25 $\mu$ which are of order 10 cm. If we tentatively identify the sharp front as due to an ion wave shock, we see from Eq. (2) that an ion density $N_1\cong 10^{12}$-$10^{13}$ cm$^{-3}$ would give a shock thickness $L_s \cong 0.3$-$0.1$ cm, in reasonable agreement with what is observed. A density of $10^{13}$/cm$^3$ corresponds to a degree of ionization in the glow discharge at 25 $\mu$ of $\sim 1\,\%$.

Using the relation $I=AeN_e(KT_e/2\pi m_e)^{1/2}$ for the current to the positive probe of area $A$, the observed 1-A signals are obtained if we assume that behind the shock front (subscripts 2) $N_{e2}\cong 4N_1 \cong 4\times10^{12}$/cm$^3$ and $T_{e2}\cong 5\times10^5$ °K. The high electron temperature $T_{e2}$ should drop rapidly further behind the shock because of thermal conduction into the colder driver plasma of LiD, but this effect on the probe current is overcome by an increase in electron density as the LiD plasma arrives at the probe.

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# EXCITATION OF ELECTRON-PLASMA WAVES BY THE INTERACTION OF AN ION BEAM WITH A PLASMA*

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This paper reports the interaction of an energetic beam of ions with a plasma that is produced by the ion beam.

The experimental arrangement is given in Fig. 1. From a duoplasmatron ion source surrounded by an insulator, a beam of hydrogen ions is extracted by an accelerating electrode. The beam is focused and divided into mass components by a magnetic lens. Through the entrance aperture, 2 cm in diameter, practically only particles of one component enter the plasma chamber. Negative-potential electrodes in front of and behind the plasma chamber prevent the electrons from leaving the plasma chamber along the beam. The neutral gas pressure in the plasma chamber can be varied by a gas leak. In all experiments hydrogen was used. The pressure in the drift space depended a little on the gas pressure in the plasma chamber. Because the beam current, entering the plasma chamber, changed smoothly with the pressure in the drift space, an automatically driven gas leak in the drift space was provided which always adjusted the pressure to a given value (about $6\times10^{-5}$ Torr). The beam current, entering the plasma chamber, was measured by a movable calorimeter.

The plasma chamber had an outer diameter of about 20 cm, was 125 cm in length, and was made of stainless steel. The end plates were of iron. The whole plasma chamber could be used as a cavity. Two loops were used to excite a TM mode. The quality $Q$ of this cavity for these modes was about 500. In addition there was a Langmuir probe and, at the end of the chamber, a pin probe for detecting excited frequencies.

Normally the experiment was performed with a $H_2^+$ beam, because the highest currents at that time were achieved with $H_2^+$. Some exper-

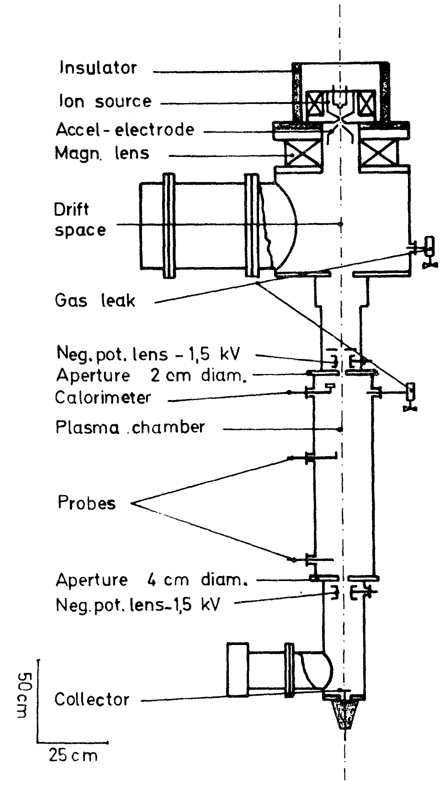

FIG. 1. Experimental arrangement setup.

iments have been done with $H^+$. The maximum currents through the 2-cm aperture varied for $H_2^+$ between 15 and 25 mA for 30 to 40 keV. Beam densities were about $10^8$ $cm^{-3}$.

Two types of experiments were performed. In the first one the variation of rf signals, picked up by the pin probe and detected with a Hewlett-Packard wide-band spectrum analyzer, was studied, under varying pressure conditions. In the second one the plasma density was measured as a function of the neutral gas pressure by the phase shift of the cavity, when it was excited in the $TM_{010}$ mode.

The results of the experiments with the $H_2^+$ beam are given in Fig. 2. The points are the frequencies of the waves excited at pressure $p$. The scale for the frequencies is found on the right-hand side of the ordinate axis. The crosses give the square root of the frequency shift of the cavity, excited in the lowest TM mode $TM_{010}$. The scale for this is given on the left-hand side of the ordinate axis of Fig. 2. This frequency shift is proportional to the plasma density. The square root of the shift is, then, proportional to the plasma frequency.

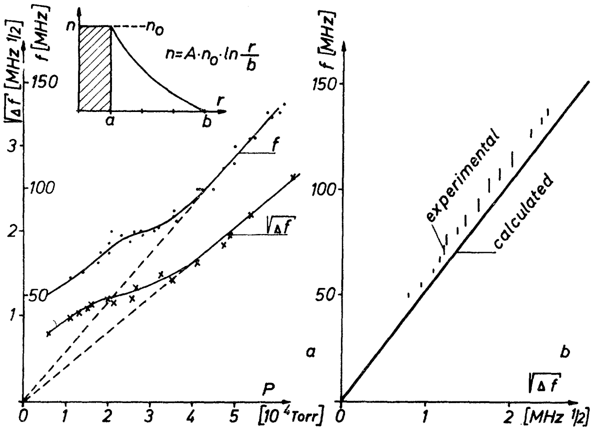

FIG. 2. (a) Square root of the frequency shift of the cavity (crosses) and detected frequencies (dots) as a function of pressure. Insert: assumed density profile; $a$, beam radius and $b$, plasma radius. (b) The experimentally obtained frequencies and the calculated plasma frequency as a function of the square root of the frequency shift.

These experimental observations can be explained by the following ideas. The ion beam is injected into the discharge region from the duoplasmatron source, whereupon it produces a plasma in the discharge region by colliding with the background gas. The beam then interacts with the plasma by the double-stream interaction mechanism. The familiar one-dimensional dispersion relation for the beam moving with a velocity $v_0$ through a plasma is

$$1-\frac{\omega_{pb}^2}{(\omega-kv_0)^2}-\frac{\omega_e^2+\omega_{pi}^2}{\omega^2}=0,$$

where $\omega_{pb}$ is the ion-plasma frequency of the beam, $\omega_e$ is the electron-plasma frequency of the plasma, $\omega_{pi}$ is the ion-plasma frequency of the plasma, and $\omega$ and $k$ are the frequency and wave number of the interaction. To maintain charge neutrality we have

$$n_e=n_i+n_{pb},$$

where $n_e$ is the electron density, $n_i$ is the plasma-ion density, and $n_{pb}$ is the beam-ion density. Temperature effects have been neglected because measurements with Langmuir probes have shown that the temperature of the plasma electrons is smaller than 1 eV.

A solution of the dispersion equation for real $k$ with $\omega_{pi}^2$ neglected compared with $\omega_e^2$ is shown in Fig. 3 (see also Imshennik and Morozov[1]).

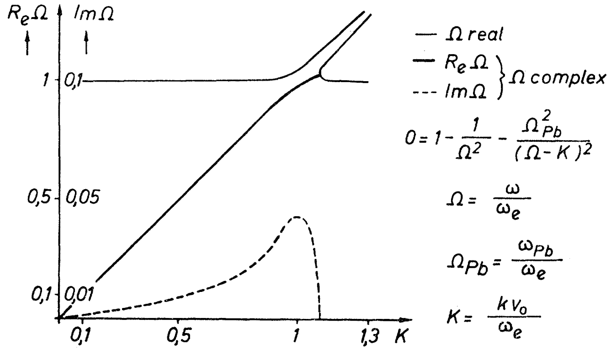

FIG. 3. Dispersion equation for cold beam and plasma.

The normalized frequency $\Omega = \omega/\omega_e$ is given as a function of the normalized wave number $K = kv_0/\omega_e$. It can be seen that $\Omega$ is complex for $K \leqslant 1$. The imaginary part of $\Omega$ of the complex solution has a maximum for $K = 1$. For this value of $K$ we find $\Omega \approx 1$, too. It follows that $\Omega \approx K$ or $\omega \approx kv_0$, which is the Cherenkov condition for the excitation of waves. The growth rate results from the interaction of the space-charge waves of the beam with the electron plasma oscillations. Because of this interaction of the ion beam with a plasma we expect to find waves in the experiment with the electron plasma frequency. As can be seen from Fig. 2(b), where we have plotted the excited frequencies against the square root of the frequency shift, these frequencies are proportional to the square root of the frequency shift. From this it follows that the frequency is a function of the square root of the density, that is, of the plasma frequency. In order to verify that the excited frequency is the electron-plasma frequency itself, we had to determine the density in the beam region quantitatively. For this purpose we had to assume a density profile. For higher pressures it seemed realistic to assume a profile where the density depends only on the radius and obeys the following diffusion equation:

$$D\nabla^2 n + \nu_i n_{pb} = 0$$

($D$ = diffusion coefficient, $\nu_i$ = ionization frequency). This diffusion law is fulfilled as long as electric fields are unimportant. In this equation $D$ is assumed as inversely proportional to the pressure and $\nu_i$ as directly proportional to the pressure. From this it follows that the density should vary proportionally with the square of the pressure. The plasma frequency or the square root of the frequency shift should then be proportional to the pressure. This is found experimentally [Fig. 2(a)] for the higher pressure region. The excited frequencies as well as the square root of the frequency shift lie for higher pressure on straight lines through the origin. For lower pressures the values lie above the straight lines. In the intermediate pressure range the plasma density (as found by the frequency-shift measurements) is comparable with the beam density. In this situation space-charge forces become important and the above diffusion equation is no longer satisfied.

The solution of the diffusion equation shows that the density outside the beam varies as

$$n = n_0 \frac{\ln r - \ln b}{\ln a - \ln b}.$$

This logarithmic variation was also found experimentally, when we determined the dependence of the ion saturation current to a Langmuir probe on the radius.

If we now assume a constant density in the beam region (see insert in Fig. 2), plasma frequencies can be calculated which very closely approximate the measured frequencies. This can be seen from Fig. 2(b). There the plasma frequencies, calculated from the frequency shift of the resonator with the described density profile, lie on the thick line. From this we deduce that the ion beam really has excited waves with the electron-plasma frequency, as was expected from theory.

The amplitude of the oscillations was highest for the pressure between 2 and $3 \times 10^{-4}$ Torr. For higher pressures the amplitude decreased to zero. This may be the effect of collisions. We have made some measurements with a proton beam, too. Similar results were obtained, but in this case we also detected the excited frequencies at higher pressures and densities.

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