[20]In order to satisfy Eq. (10) and the other requirements without getting an $I=2$ piece, it is necessary to add an additional vector field Ψ_μ which is a scalar with respect to the entire SU(3)⊗SU(3) group. A suitable current, which contains only $I=0$, 1, would then be $\partial_\nu[\Psi_\nu(\psi_\mu{}^3+3^{-1/2}\psi_\mu{}^8-6^{-1/2}\psi_\mu{}^0)-(\mu\leftrightarrow\nu)]$.

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KAON DECAY AND PION PHASE SHIFTS

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We consider decays of neutral kaons under the following hypotheses:

(a) We assume that the short-lived and long-lived neutral kaons are each characterized by a unique complex mass; and that the conventional formalism,[1] with TCP invariance, applies.

(b) $\Gamma(K_S\to3\pi)\ll\Gamma(K_L\to3\pi)$. This follows[2] from the approximate validity of the $\Delta I=\frac{1}{2}$ rule, and the known rates for $K_L\to3\pi$ and $K^+\to3\pi$.

(c) There are no $\Delta S=-\Delta Q$ currents. (The weaker statement that the $\Delta S=\pm\Delta Q$ form factors are relatively real suffices.)

(d) $K_S\to2\pi$ satisfies an approximate $\Delta I=\frac{1}{2}$ rule. Thus, we exclude the possibility of a mixture of $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ giving the $\Delta I=\frac{1}{2}$ branching ratio, but the wrong relative phase.

(e) We must exclude the possibility that $\cot(\delta_0-\delta_2)=2\Delta m/\Gamma_S$. We discuss separately what happens when this relation is nearly satisfied. δ_I denotes the pion phase shift at the kaon mass for $I=0, 2$; $\Delta m=m_L-m_S$, and henceforth we use the notation $\tan\varphi_w=2\Delta m/\Gamma_S\approx\tan40°$.

Under these hypotheses, we obtain a new relation among the four measurable quantities φ_w, φ, δ, and R. Here φ denotes the phase of the amplitude for $K_L\to\pi^++\pi^-$ relative to the amplitude for $K_S\to\pi^++\pi^-$; δ denotes $\delta_0-\delta_2$; R is the branching ratio for $K_L\to2\pi$, $R=\Gamma(K_L\to\pi^0+\pi^0)/\Gamma(K_L\to\pi^++\pi^-)$. The relation is

$$2R-1=3\sin^2(\varphi-\varphi_w)[1+3\tan^2(\varphi_w+\delta)]-3\sin2(\varphi-\varphi_w)\tan(\varphi_w+\delta). \quad (1)$$

In principle, the experimental evaluation of R, φ, and φ_w determines the difference of the pion phase shifts, without the necessity of measuring the phase φ_0 of the $K_L\to2\pi^0$ amplitude relative to the $K_S\to2\pi^0$ amplitude. Of course, an analogous relation holds for φ_0,

$$\tfrac{1}{2}R^{-1}-1=3\sin^2(\varphi_0-\varphi_w)[\tfrac{3}{4}\tan^2(\varphi_w+\delta)-\tfrac{1}{4}]-\tfrac{3}{2}\sin2(\varphi_0-\varphi_w)\tan(\varphi_w+\delta). \quad (2)$$

Note that two values of φ give the same value of R. For $R=\frac{1}{2}$, $\varphi=\varphi_w$ corresponds to the $\Delta I=\frac{1}{2}$ rule for K_L, while the other solution different from φ_w corresponds to a mixture of $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ giving the $\Delta I=\frac{1}{2}$ branching ratio but the wrong phase. Another interesting case is $\varphi=\pm\frac{1}{2}\pi-\delta$, which occurs in Bowen's model of CP-invariance violation.[3] It is clear from Eq. (1) that this gives $R=2$, the pure $\Delta I=\frac{3}{2}$ result.

Derivation of Eq. (1).—Denote the matrix element for the decay of K^0 into two pions with $I=0$ and 2 by $A\exp i(\delta_0+\alpha)$ and $B\exp i(\delta_2+\beta)$, respectively, where A and B are real and α and β are CP-nonconservation parameters with $-\frac{1}{2}\pi\le\alpha,\beta\le\frac{1}{2}\pi$. From TCP invariance, the corresponding matrix elements for $\bar{K}^0$ decay are obtained from these by replacing α by $-\alpha$ and β by $-\beta$. The physical particles are given by

$$K_L=\{K^0-\bar{K}^0+\epsilon(K^0+\bar{K}^0)\}[2(1+\epsilon^2)]^{-1/2},$$
$$K_S=\{K^0+\bar{K}^0+\epsilon(K^0-\bar{K}^0)\}[2(1+\epsilon^2)]^{-1/2}, \quad (3)$$

where ϵ is taken to be real without loss of generality, and is known to be small in magnitude ($|\epsilon| \leq 10^{-2}$) with this choice of convention. The matrix elements for the decays of physical particles are

$$M(K_S \to \pi^+ + \pi^-) = \sqrt{2}A(\cos\alpha + i\epsilon\sin\alpha)e^{i\delta_0} + B(\cos\beta + i\epsilon\sin\beta)e^{i\delta_2},$$

$$M(K_S \to \pi^0 + \pi^0) = A(\cos\alpha + i\epsilon\sin\alpha)e^{i\delta_0} - \sqrt{2}B(\cos\beta + i\epsilon\sin\beta)e^{i\delta_2},$$

$$M(K_L \to \pi^+ + \pi^-) = \sqrt{2}A(i\sin\alpha + \epsilon\cos\alpha)e^{i\delta_0} + B(i\sin\beta + \epsilon\cos\beta)e^{i\delta_2}.$$

$$M(K_L \to \pi^0 + \pi^0) = A(i\sin\alpha + \epsilon\cos\alpha)e^{i\delta_0} - \sqrt{2}B(i\sin\beta + \epsilon\cos\beta)e^{i\delta_2}, \quad (4)$$

in which we have dropped a common factor of $(1+\epsilon^2)^{1/2}$. We approximate these expressions by making use of hypothesis (d), which requires that $|B\cos\beta| \ll |A|$, and of the fact that $\Gamma(K_L \to 2\pi) \ll \Gamma(K_S \to 2\pi)$, which requires that $|\epsilon| \ll 1$ and $|\alpha| \ll 1$. We obtain

$$M(K_S \to \pi^+ + \pi^-) \approx \sqrt{2}Ae^{i\delta_0},$$

$$M(K_S \to \pi^0 + \pi^0) \approx Ae^{i\delta_0},$$

$$M(K_L \to \pi^+ + \pi^-) \approx \sqrt{2}A(\epsilon + i\alpha)e^{i\delta_0} + iB\sin\beta e^{i\delta_2},$$

$$M(K_L \to \pi^0 + \pi^0) \approx A(\epsilon + i\alpha)e^{i\delta_0} - \sqrt{2}iB\sin\beta e^{i\delta_2}. \quad (5)$$

From unitarity, the following sum rule is easily deduced,[4]

$$\epsilon(\Gamma_S + i2\Delta m) = \sum_n M(K_L \to n)M^*(K_S \to n), \quad (6)$$

where $M(K_{L,S} \to n)$ is the matrix element for the decay into channel n, and the sum extends over all open channels and includes phase space. Provided that $\epsilon \neq 0$, we may use use Eq. (6) to determine the phase of the right-hand side. (Bowen[3] has discussed the case of $\epsilon = 0$. He obtains, with the above hypotheses, $R = 2$ and $\varphi = \pm\frac{1}{2}\pi - \delta$.) Because of our hypotheses (b) and (c), only the $I = 0, 2$ two-pion channels contribute significantly to the right-hand side of Eq. (6). We obtain

$$\tan\varphi_w = \alpha/\epsilon + \lambda B\sin\beta/A\epsilon, \quad (7)$$

where $\lambda = B\cos\beta/A$ is known to be small from hypothesis (d). It must be included in Eq. (7) because of the possibility that $B\sin\beta \gg A\epsilon$. Defining $a = \alpha/\epsilon$, and $b = 2^{-1/2}B\sin\beta/A\epsilon$, we may write for the relative phase φ and the branching ratio R, from Eq. (5),

$$2R = \left|\frac{(1+ia)e^{i\delta} - 2ib}{(1+ia)e^{i\delta} + ib}\right|^2, \quad (8)$$

$$\varphi = \mathrm{Arg}[(1+ia) + ibe^{-i\delta}], \quad (9)$$

$$\varphi_0 = \mathrm{Arg}[(1+ia) - 2ibe^{-i\delta}]. \quad (10)$$

Evidently, $\varphi = \varphi_0 = \varphi_w$ and $R = \frac{1}{2}$ if the $\Delta I = \frac{1}{2}$ rule describes both K_S and K_L. Equations (7) and (9) may be solved for a and b, so that Eq. (8) becomes an expression for R in terms of φ, φ_w, δ, and λ,

$$2R = 1 - M\sin2(\varphi - \varphi_w)\tan(\varphi_w + \delta) + \sin^2(\varphi - \varphi_w)\{M^2 - 2M + M^2\tan^2(\varphi_w + \delta)\}, \quad (11)$$

where

$$M = 3\{1 + \lambda(\cos\delta - \tan\varphi_w \sin\delta - \lambda)^{-1}\}. \quad (12)$$

We may set $M \approx 3$, thus obtaining Eq. (1), provided that

$$|\cos(\varphi_w + \delta)| \gg 1/30 \geq \lambda\cos\varphi_w.$$

Equation (1) is approximately valid if $\varphi_w + \delta$ differs from $\pm\frac{1}{2}\pi$ by at least several degrees. If, on the other hand, $\cos(\varphi_w + \delta)$ is small compared with 1, Eq. (11) takes the approximate form

$$2R = 1 - 2\chi + \chi^2, \quad (13)$$

where $\chi \approx 3(\varphi - \varphi_w)[\cos(\varphi_w + \delta) - \lambda\cos\varphi_w]^{-1}$. Evidently, any non-negative value of R may be attained; and R is an exceedingly sensitive function of $\varphi - \varphi_w$. The case of small $\cos(\varphi_w + \delta)$ is not without interest in view of a recent model of Truong.[5] He notes that if $K \to 2\pi$ and the kaon-mass splitting are both dominated by an s-wave, $I=0$, pion-pion resonance, somewhat higher than the kaon mass to ensure $m_L > m_S$, then the relation $\cos(\varphi_w+\delta)=0$ must be satisfied. For small δ_2, this is precisely the exceptional limit we have just examined.

Comparison with experiment. – Presently available experimental data permit us to determine $\delta_0-\delta_2$ approximately. Inserting into Eq. (1) the values of $R=2.9\pm0.6$ [6] and $\varphi=0.60\pm0.23$,[7] we obtain the inequality

$$10^\circ \leq \delta_0-\delta_2 \leq 60^\circ.$$

This result is compatible with other determinations of the pion phase shifts at 500 MeV.[8]

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ERRATUM

SECONDARY PARTICLE YIELDS AT 0° FROM THE NEW STANFORD ELECTRON ACCELERATOR. A. Barna, J. Cox, F. Martin, M. L. Perl, T. H. Tan, W. T. Toner, T. F. Zipf, and E. H. Bellamy [Phys. Rev. Letters 18, 360 (1967)].

The following typographical errors appear: (1) In the last two paragraphs of the left-hand column of page 360, "state" is printed consistently for "stage." (2) An omission has been made on page 360. Beginning at 20 lines from the bottom of the right-hand column it should read "The $K/(\pi+\mu+e)$ and $p/(\pi+\mu+e)$ ratios were measured by varying the gas pressure in the Cherenkov counter. The proportion of muons in the beam was measured by the number of particles which penetrated the 1.6-m-thick iron absorber." (3) In Table I on page 361, and also in Ref. 6, "±3.8 mr" has been printed instead of "±3.9 mr." (4) In Ref. 7, "turn off" has been printed for "turned off."

MINIMUM ENERGY OF POSITRONS IN METALS. S. M. Kim, A. T. Stewart, and J. P. Carbotte [Phys. Rev. Letters 18, 385 (1967)].

The figures in this paper have been interchanged. The captions, however, are correct; only the drawings are transposed.

POSSIBLE CP-NONINVARIANT EFFECTS IN $\pi\pi\gamma$ DECAY OF CHARGED K MESONS. G. Costa and P. K. Kabir [Phys. Rev. Letters 18, 429 (1967)].

The function $I_{\text{int}}(\omega)$ defined in Eqs. (9) and (10) is only one-half as large as I_{int} defined by Good; consequently, the correct value of C_1 in Eq. (11) should be $C_1 \simeq 0.65$.

The s-wave $\pi^+\pi^0$ phase shift δ_0 should be taken at the center-of-mass energy equal to the kaon mass M. The statement that δ_1 is the p-wave $\pi^+\pi^0$ phase shift becomes exact in the limit when rescattering corrections to the $\pi\pi\gamma$ final state may be neglected. The corrections in the $\pi\pi\gamma$ state are of relative order α and may be ignored; the corrections leading from $\pi\pi$ states to $\pi\pi\gamma$ states have been estimated by H. Chew, Nuovo Cimento 26, 1109 (1962), who finds that they are negligible for values of $|x_e|$ as large as those we are considering. One would be surprised if corrections from $3\pi\to\pi+\pi+\gamma$, which only affect the phase of x_m, greatly exceeded those from $2\pi\to\pi+\pi+\gamma$.

In Eq. (11), the denominator on the left-hand side should be $B^+ + B^-$.

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