Combining these with the new measurements of $|\eta_{00}|$, we obtain the further entries in Table I.

It is clear from these discussions that the largest errors reside in those of the angles of η_{+-} and η_{00}. More accurate measurements of $\hat{\eta}_{+-}$, and measurements of $\hat{\eta}_{00}$ or $\hat{\eta}_{00}-\hat{\eta}_{+-}$ or $\mathrm{Re}\epsilon$, would serve to narrow the degree of uncertainty.

It is interesting to notice that

$$\left|\mathrm{Im}\frac{A_2}{A_0}\right| \leq 3.2\times10^{-3}. \qquad (16)$$

Comparing this with the corresponding K^+ decay rate

$$\left|\frac{A_2^+}{A_0}\right| = 0.055, \qquad (17)$$

one sees that it is likely that $\mathrm{Im}A_2/\mathrm{Re}A_2 \sim 10^{-1}$. In other words[11] the $|\Delta I|>\frac{1}{2}$ amplitude seems still largely CP conserving.

We would like to thank Professor C. N. Yang for suggesting this investigation and for his continuous guidance.

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TESTS FOR CP NONINVARIANCE WITH $|\Delta T|>\frac{1}{2}$ IN THE PARTIAL RATES FOR $K^\pm\to\pi^0+\pi^\pm$ AND $K^\pm\to\gamma+\pi^0+\pi^\pm$

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CP noninvariance with $|\Delta T|>\frac{1}{2}$ and CPT invariance implies that the partial rates, $r_\pm$, for $K^\pm\to\pi^0+\pi^\pm$, and also the partial rates, $r_\pm{}^1$ for $K^\pm\to\gamma+\pi^0+\pi^\pm$, will be unequal. An approximate phenomenological analysis is formulated, and suggests the possibility of $|1-(r_-/r_+)|\cong10^{-3}$ and $(r_-{}^1/r_+{}^1)\cong 3$.

The recent discovery[1,2] that the rate for $K_L{}^0\to2\pi^0$ is significantly larger than one-half of the rate for $K_L{}^0\to\pi^++\pi^-$ implies that CP-nonconserving nonleptonic decay amplitudes occur which violate the nonleptonic $|\Delta T|=\frac{1}{2}$ rule.[3,4] The origin of such relatively small amplitudes may be instrinsic to the weak interaction,[3,4] or they may arise from an electromagnetic correction to the weak interaction,[5] if the electromagnetic interaction is not CP-invariant.[6,7] Of course, ordinary electromagnetic interactions must give rise to small corrections to weak interactions which violate the nonleptonic $|\Delta T|=\frac{1}{2}$ rule.[8]

The decays $K^\pm\to\pi^0+\pi^\pm$ must, of course, proceed into a pure $T=2$ state with $|\Delta T|=\frac{3}{2},\ \frac{5}{2}$.

With CP noninvariance these partial rates for the K^+ and K^- need not be equal. However, from CPT invariance and to lowest order in the weak interaction, it is known[9,10] that

$$r_+(K^+\to\pi^0+\pi^+)+\sum_n r_+^n(K^+\to n+\gamma+\pi^0+\pi^+)$$
$$=r_-(K^-\to\pi^0+\pi^-)$$
$$+\sum_n r_-^n(K^-\to n+\gamma+\pi^0+\pi^-), \quad (1)$$

where $r_\pm$ and $r_\pm^n$ denote the respective partial rates for K^+ and K^-, and the sum extends over any number of photons. Limiting the sum to a single photon gives

$$r_+-r_-\cong r_-^1-r_+^1. \quad (2)$$

Equation (2) limits the difference in partial rates for the two-pion decays of $K^\pm$ to be of order α, the fine-structure constant, assuming CPT invariance. Since essentially nothing is known experimentally about r_-^1, and the errors on the present determination of r_- are large,[11,12] it remains an important experimental problem to verify the implication of Eq. (2).

The purpose of this note is to give a simple, but approximate, phenomenological analysis of the origin of the possible differences in $r_\pm$ and in $r_\pm^1$, in order to delineate the circumstances under which these differences would be anomalously small, and to note that, at present, there is no reason to believe that the quantity $|1-(r_-/r_+)|$ may not be of order of the CP-nonconserving effects that experimentalists have become accustomed to search for in many experiments, namely of order of a tenth of a percent, while a gross difference between r_+^1 and r_-^1 may exist.

In order to discuss possible differences in $r_\pm$ and in $r_\pm^1$ it is necessary to consider the coupling between the $T=2$, S-wave π^0-$\pi^\pm$ state with total energy equal to m_K, the K-meson mass, and the states of γ-π^0-$\pi^\pm$. We do this in the following model. We consider the simplest allowed γ-π^0-$\pi^\pm$ configuration, that with two pions in a relative P state and an electric-dipole photon. We assume that, at this energy, all of the matrix elements of the T matrix for the coupled system are small, and we approximate the T matrix by its leading term, the real, symmetric (assuming time-reversal invariance for this part of the problem[13]) reactance matrix, K,[14] given by

$$K=\begin{pmatrix}\delta & b\\ b & a\end{pmatrix}$$

with

$$\delta=\langle\pi^0\pi^\pm|K|\pi^0\pi^\pm\rangle,$$
$$b=\langle\gamma\pi^0\pi^\pm|K|\pi^0\pi^\pm\rangle,$$
$$a=\langle\gamma\pi^0\pi^\pm|K|\gamma\pi^0\pi^\pm\rangle. \quad (3)$$

We simplify the treatment further by (1) neglecting the three-particle scattering matrix element a, which is proportional to the π^0-$\pi^\pm$ P-wave scattering amplitude at total energies below m_K; and (2) by approximating the production matrix element b by an "average" matrix element $\bar{b}$, as follows:

$$b\to\bar{b},$$

where

$$\frac{4\pi|\bar{b}|^2}{q^2}=\sigma(\pi^0+\pi^\pm\to\gamma+\pi^0+\pi^\pm). \quad (4)$$

In Eq. (4), q is the center-of-mass momentun of the π^0-$\pi^\pm$ system at total energy m_K, and σ denotes the total production cross section at m_K. With the normalization of matrix elements implied by Eq. (4), we have simply

$$\frac{|\delta|^2}{q^2}=\frac{d\sigma}{d\Omega}(\pi^0+\pi^\pm\to\pi^0+\pi^\pm), \quad (5)$$

i.e., δ is the (assumed small) $T=2$, S-wave π-π phase shift at m_K. The approximate K matrix can be diagonalized by the orthogonal transformation U,

$$U=\begin{pmatrix}\cos\lambda & -\sin\lambda\\ \sin\lambda & \cos\lambda\end{pmatrix} \quad (6a)$$

$$UKU^{-1}=\begin{pmatrix}\tan\delta_1 & 0\\ 0 & \tan\delta_2\end{pmatrix}\cong\begin{pmatrix}\delta_1 & 0\\ 0 & \delta_2\end{pmatrix} \quad (6b)$$

with

$$\tan2\lambda=-2\bar{b}/\delta\cong2\lambda \text{ for small } \lambda. \quad (6c)$$

The (small) real phases, δ_1 and δ_2, are the scattering eigenphases corresponding to the two eigenstates of the coupled system, which are given by

$$|1\rangle=\cos\lambda|\pi^0\pi^\pm\rangle-\sin\lambda|\gamma\pi^0\pi^\pm\rangle$$
$$|2\rangle=\sin\lambda|\pi^0\pi^\pm\rangle+\cos\lambda|\gamma\pi^0\pi^\pm\rangle. \quad (7)$$

Consider now the matrix elements for the decays of $K^\pm$ into the (incoming-wave) eigenstates, with possible CP noninvariance in the effective Hamiltonian, H (and with $e^{i\delta_{1,2}} \cong 1 + i\delta_{1,2}$):

$$\langle(-)1|H|K^{\pm}\rangle \cong |A_1|(1+i\delta_1)e^{\pm i\Delta_1}$$
$$\langle(-)2|H|K^{\pm}\rangle \cong |A_2|(1+i\delta_2)e^{\pm i\Delta_2}. \qquad (8)$$

In Eq. (8), the positive sign in the exponent goes with K^+, the negative sign with K^-, and CP noninvariance is contained in the real phases, Δ_1 and/or Δ_2, different from 0 or π. The physical decay matrix elements can be explicitly constructed from Eqs. (7) and (8):

$$\langle(-)\pi\pi^0\pi^{\pm}|H|K^{\pm}\rangle = \alpha_+ = \alpha\exp(\pm i\Delta_\pi) \cong \exp(\pm i\Delta_1)(1+i\delta_1)|A_1|\{\cos\lambda + \rho\sin\lambda(1+i\bar{\delta})e^{\pm i\bar{\Delta}}\}, \qquad (9a)$$

$$\langle(-)\gamma\pi\pi^0\pi^{\pm}|H|K^{\pm}\rangle = \beta_\pm = \beta\exp(\pm i\Delta_\gamma) \cong \exp(\pm i\Delta_1)(1+i\delta_1)|A_1|\{-\sin\lambda + \rho\cos\lambda(1+i\bar{\delta})e^{\pm i\bar{\Delta}}\}, \qquad (9b)$$

with

$$\bar{\delta} = \delta_2 - \delta_1 \approx -\delta \text{ for small } \lambda,$$
$$\bar{\Delta} = \Delta_2 - \Delta_1,$$
$$\rho = |A_2|/|A_1| \approx \big||\beta_+/\alpha_+|\exp(i\Delta_\gamma) + \lambda\exp(i\Delta_\pi)\big| \quad \text{for small } \lambda \text{ and } |\beta_+/\alpha_+| \ll 1. \qquad (9c)$$

For small λ, the CP-nonconserving eigenphases, Δ_1 and Δ_2, are given approximately in terms of the CP-nonconserving phases of the physical amplitudes, Δ_π and Δ_γ, by

$$\tan\Delta_1 \cong \frac{\sin\Delta_\pi - \lambda|\beta_+/\alpha_+|\sin\Delta_\gamma}{\cos\Delta_\pi - \lambda|\beta_+/\alpha_+|\cos\Delta_\gamma}, \qquad (10a)$$

$$\tan\Delta_2 \cong \frac{\lambda\sin\Delta_\pi + |\beta_+/\alpha_+|\sin\Delta_\gamma}{\lambda\cos\Delta_\pi + |\beta_+/\alpha_+|\cos\Delta_\gamma}, \qquad (10b)$$

$$|\sin\bar{\Delta}| \cong |\bar{\Delta}| \approx |\beta_+/\alpha_+||\Delta_\pi - \Delta_\gamma|/\rho \quad \text{for small } \Delta_\pi \text{ and } \Delta_\gamma. \qquad (10c)$$

The matrix element β_+ again represents an "average" over the three-particle phase space; we have $4\pi|\beta_+|^2 = \text{Rate}(K^+ \to \gamma + \pi^0 + \pi^+,\ E1) \cong r_+{}^1$ and also $|\alpha_+|^2 = dr_+/d\Omega$ ($K^+ \to \pi^0 + \pi^+$). From Eqs. (9a)-(9b) we can compute the CP-nonconserving observables

$$|D_\pi| = \frac{2\big||\alpha_+|^2 - |\alpha_-|^2\big|}{|\alpha_+|^2 + |\alpha_-|^2} \cong \left|1 - \frac{r_-}{r_+}\right| \cong \frac{|2\rho\bar{\delta}\sin2\lambda\sin\bar{\Delta}|}{\{(\cos\lambda + \rho\sin\lambda\cos\bar{\Delta})^2 + (\rho\sin\lambda\sin\bar{\Delta})^2 + (\rho\bar{\delta}\sin\lambda\cos\bar{\Delta})^2\}}, \qquad (11a)$$

$$|D_\gamma| = \frac{2\big||\beta_+|^2 - |\beta_-|^2\big|}{|\beta_+|^2 + |\beta_-|^2} = \frac{2|1 - (r_-{}^1/r_+{}^1)|}{1 + (r_-{}^1/r_+{}^1)} \cong \frac{|2\rho\bar{\delta}\sin2\lambda\sin\bar{\Delta}|}{\{(-\sin\lambda + \rho\cos\lambda\cos\bar{\Delta})^2 + (\rho\cos\lambda\sin\bar{\Delta})^2 + (\rho\bar{\delta}\cos\lambda\cos\Delta)^2\}}. \qquad (11b)$$

From Eqs. (11a)-(11b) we note the essential results of this analysis that are independent of the model. The CP-nonconserving observables will vanish in any one of the following circumstances: (I) CP invariance, implying $\bar{\Delta} = 0$ or π; (II) CP noninvariance, but "accidentally" $\bar{\Delta} = 0$ or π (for example $\Delta_\pi = \Delta_\gamma \neq 0$ or π); (III) CP noninvariance, but vanishing coupling between the S-wave π^0-$\pi^\pm$ state and the γ-π^0-$\pi^\pm$ states, i.e., $\lambda \to 0$; and (IV) CP noninvariance, but vanishing strong-interaction phase shift in the S-wave π^0-$\pi^\pm$ state at total energy m_K, i.e., $\delta \to 0$, or, more generally, δ_1 and $\delta_2 \to 0$ (or accidentally, $\delta_1 = \delta_2 \neq 0$).

It remains to make a rough numerical estimate of $|D_\pi|$ and $|D_\gamma|$. We have one important relevant piece of experimental data[15,12]

$$y^2=\left|\frac{\beta_+}{\alpha_+}\right|^2\cong\frac{r_+^1}{4\pi(dr_+/d\Omega)}=\frac{r_+^1}{r_+}\cong 5\times10^{-4}. \quad (12)$$

If we now assume[16] $|\gamma|\gg y$ and therefore $\rho\approx\lambda$ (but still $|\lambda|<1$), we obtain, using Eqs. (11a)-(11b), (10c), (6c), and (12),

$$|D_\pi|\cong 4y|\lambda\bar{\delta}|\,|\Delta_\pi-\Delta_\gamma|$$

$$\cong(9\times10^{-2})|\bar{b}|\,|\Delta_\pi-\Delta_\gamma|, \quad (13a)$$

$$|D_\gamma|\cong\frac{4y|\Delta_\pi-\Delta_\gamma|}{(|\lambda\bar{\delta}|+y^2/|\lambda\bar{\delta}|)}\cong\frac{(9\times10^{-2})|\Delta_\pi-\Delta_\gamma|}{(|\bar{b}|+y^2/|\bar{b}|)}. \quad (13b)$$

Noting again the definition of $\bar{b}$ in Eqs. (3) and (4), and remembering the suppression of both α_+ and the inner bremsstrahlung contribution to β_+ by the nonleptonic $|\Delta T|=\frac{1}{2}$ rule, a reasonable estimate[16] for $|\bar{b}|$ is

$$|\bar{b}|\approx|\beta_+/\alpha_+|=y\cong 2.24\times10^{-2}. \quad (14)$$

From Eqs. (11a)-(11b), (13a)-(13b), and (14), we have the following maximal estimate for the CP-nonconserving observables, with[17] $|\Delta_\pi-\Delta_\gamma|\sim\frac{1}{2}$:

$$|1-(r_-/r_+)|\cong10^{-3}, \quad (15a)$$

$$(r_-^1/r_+^1)\cong 3. \quad (15b)$$

Equations (15a)-(15b) are the basis for the estimates stated in the last sentence of the third paragraph above; together with Eq. (10) they are clearly consistent with the approximate restriction imposed by CPT invariance, namely Eq. (2). On the other hand, if $(r_-^1/r_+^1)\cong 0$, Eq. (2) gives

$$|1-(r_-/r_+)|\cong r_+^1/r_+\cong 0.5\times10^{-3}. \quad (16)$$

We are well aware that it will be experimentally difficult to achieve a measurement of the very small difference in Eq. (15a), and even the gross difference in Eq. (15b) is a difficult measurement because of the rare process involved. Nevertheless, in the light of the very significant new experimental results on CP noninvariance,[1,2] it is to be strongly expected that these CP-nonconserving effects exist, and only in the circumstances (II)-(IV) enumerated above will $|1-(r_-/r_+)|$ perhaps be depressed more than an order of magnitude below the estimate of Eq. (15a). On the other hand, a much larger experimental value for $|1-(r_-/r_+)|$ would cause concern for CPT invariance in a nonleptonic weak decay.

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m_K, we have $|\lambda| \cong |\bar{b}/\delta| > 0.06$, whereas $y \cong 0.02$; thus $|\lambda| \gg y$ may be a first rough approximation in estimating $|D_\pi|$ and $|D_\gamma|$. Clearly, we are trying to estimate essentially one number which is a function of the physical quantities b and δ, which are presently not well known, but for which at least informed estimates can be made. It may be that $\bar{b} \approx \delta y$ is a more reasonable estimate. However, we have already tended to underestimate y (Ref. 15).

[17]If inner bremsstrahlung dominates the radiative decays Δ_γ would differ from Δ_π only because of the radiating pion being off mass shell; however, a direct emission amplitude could cause Δ_γ to differ significantly from Δ_π, especially if this amplitude is CP noninvariant. The estimate (15b), in particular, is a maximal estimate for the total radiative rates; the ratio may be as large only over a portion of the spectrum where the $E1$ amplitude may be dominant.

η DECAY, CURRENT ALGEBRA, AND THE C-NONCONSERVING ELECTROMAGNETIC CURRENT

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A striking success of the hypotheses of current algebra[1] and partially conserved axial-vector current[2] has been the correct prediction of the $K_2^0 \to \pi^+ + \pi^- + \pi^0$ matrix element,[3] under the assumption that the linear matrix element observed inside the Dalitz plot can be extrapolated to the points where the pion four-momenta vanish. Since the $\eta \to \pi^+ + \pi^- + \pi^0$ matrix element has the same linear form[4] as that in K_2^0 decay, one might expect that the methods used to treat K_2^0 decay will work also for the η. But as Sutherland[5] and Itzykson, Jacob, and Mahoux[6] have pointed out, in the usual picture of η decay, current algebra, combined with the assumption of a linear matrix element out to the points where pion four-momenta vanish, implies that $\eta \to 3\pi$ is forbidden! The argument is a simple one. Let $J_\mu = J_\mu^{(0)} + J_\mu^{(1)}$ denote the electromagnetic current, which consists of isoscalar (0) and isovector (1) pieces. The decay $\eta \to \pi^a + \pi^b + \pi^c$ proceeds via a second order virtual electromagnetic interaction,

$$A(\eta \to \pi^a + \pi^b + \pi^c) \propto \langle \pi^a \pi^b \pi^c | e^2 \int d^4y\, D_{\mu\nu}(y) T[J_\mu(y) J_\nu(0)] | \eta \rangle$$
$$= 2\langle \pi^a \pi^b \pi^c | e^2 \int d^4y\, D_{\mu\nu}(y) T[J_\mu^{(0)}(y) J_\nu^{(1)}(0)] | \eta \rangle, \tag{1}$$

where $D_{\mu\nu}$ is the photon propagator. The $J_\mu^{(0)} J_\nu^{(0)}$ and $J_\mu^{(1)} J_\nu^{(1)}$ terms do not contribute on account of G parity. According to the usual current-algebra methods, the amplitude for η decay, in the limit as $q^c \to 0$, is the matrix element of the equal-time commutators arising from expanding $(\partial/\partial x_\xi) \times T[\mathfrak{F}_{c\xi}^5(x) J_\mu^{(0)}(y) J_\nu^{(1)}(0)]$; that is,

$$A(\eta \to \pi^a + \pi^b + \pi^c)|_{q^c=0}$$
$$\propto \langle \pi^a \pi^b | e^2 \int d^4y\, D_{\mu\nu}(y) T\{[F_c^5(y_0), J_\mu^{(0)}(y)] J_\nu^{(1)}(0) + J_\mu^{(0)}(y)[F_c^5(0), J_\nu^{(1)}(0)]\} | \eta \rangle. \tag{2}$$

(As usual, $\mathfrak{F}_{c\xi}^5$ and F_c^5 denote, respectively, the isospin-c axial-vector current and charge.) According to the current algebra,

$$[F_c^5(y_0), J_\mu^{(0)}(y)] = 0, \quad [F_c^5(0), J_\nu^{(1)}(0)] = i\epsilon_{c3d} \mathfrak{F}_{d\nu}^5(0) = \text{isovector}. \tag{3}$$

Hence Eq. (2) is of the form $\langle \pi^a \pi^b | \int d^4y$ isovector $|\eta\rangle$, which vanishes since an s-wave two-pion state can only have $I = 0$ or 2. Thus we conclude that $A(\eta \to \pi^a + \pi^b + \pi^c)$ vanishes when any of the three pion four-momenta is zero. Assuming a linear matrix element implies that $\eta \to 3\pi$ is forbidden. This means that, in the usual picture of η decay, the similarity of the K_2^0 and η decays within their Dalitz plots must be regarded as an accident.