PARAMETERS η_{+-}, η_{00}, ϵ FOR K^0-$\bar{K}^0$ DECAY

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(Received 10 January 1967)

In two recent experiments[1,2] the parameter $|\eta_{00}|$ was measured to be $(4.9\pm0.5)\times10^{-3}$. In this short note we want to discuss the implications of these experiments on some parameters involved in K^0-$\bar{K}^0$ decay. Especially, we estimate the errors involved in our present knowledge on these decay parameters.

We use the following formulas and notation of Wu and Yang[3]:

$$\eta_{+-}=\tfrac{1}{2}[\epsilon+2^{1/2}iF\,\mathrm{Im}A_2/A_0], \quad (1)$$

$$\eta_{00}=\tfrac{1}{2}[\epsilon-2^{1/2}2iF\,\mathrm{Im}A_2/A_0], \quad (2)$$

where

$$\epsilon=\frac{-M_i+i(y_{\mathrm{lep}}+y_{3\pi})}{A_0^{\,2}+i(m_S-m_L)}$$

and

$$F=\exp[i(\delta_2-\delta_0)]. \quad (3)$$

Therefore,

$$\eta_{00}=\tfrac{3}{2}\epsilon-2\eta_{+-}. \quad (4)$$

The parameter $A_0^{\,2}$ is to a great accuracy $\frac{1}{2}$ the decay rate of K_S and is taken from Trilling.[4] The parameter (m_S-m_L) is the mass difference between the two K's and is taken from Alff-Steinberger et al.[5] The value of $|\eta_{+-}|$ is taken from Christenson et al.,[6] $\hat{\eta}_{+-}$ from Rubbia and Steinberger.[7] $|\eta_{00}|$ is taken from Ref. 1, as the more accurate of the two recent experiments:

$$A_0^{\,2}=(0.578\pm0.010)\times10^{10}\ \mathrm{sec}^{-1},$$

$$m_S-m_L=-(0.541\pm0.025)\times10^{10}\ \mathrm{sec}^{-1},$$

$$|\eta_{+-}|=(1.94\pm0.09)\times10^{-3},$$

$$\hat{\eta}_{+-}=34°\pm13°,$$

and

$$|\eta_{00}|=(4.9\pm0.5)\times10^{-3}. \quad (5)$$

We divide the discussion into two parts. In the first part we assume $y_{\mathrm{lep}}=y_{3\pi}=0$. In the second part we do not make such an assumption.

(1) We assume in this part $y_{\mathrm{lep}}=y_{3\pi}=0$. This assumption is prompted by the possibility raised in Ref. 3 and by Truong[8] that CP nonconservation is absent in the strong, electromagnetic, and that part of the weak interaction which satisfies $|\Delta I|=\frac{1}{2}$. The recent experiments[1,2] are consistent with this assumption. Furthermore,[3] if $\Delta Q=\Delta S$, then $y_{\mathrm{lep}}=0$. Also the lack of any appreciable perpendicular polarization of the μ in $K_{\mu3}$ decay[9] is consistent with $y_{\mathrm{lep}}=0$. It is also likely that $y_{3\pi}$ is very small (i.e., $y_{3\pi}/A_0^{\,2}\ll|\eta_{+-}|$): The rates $R[K_S^{\,0}\to3\pi]$ have been estimated[10] to be $<10^4$ sec^{-1}. In Ref. 4, $R[K_L^{\,0}\to3\pi]$ is given to be 7×10^6 sec^{-1}. Using[3]

$$y_{3\pi}^{\,2}\leq R[K_S^{\,0}\to3\pi]R[K_L^{\,0}\to3\pi], \quad (6)$$

one obtains

$$y_{3\pi}\leq2.65\times10^5\ \mathrm{sec}^{-1}. \quad (7)$$

In this case it is straightforward to evaluate average values and errors of various parameters. They are listed as version A in Table I.

(2) If we do not assume $y_{\mathrm{lep}}=y_{3\pi}=0$, our knowledge of these parameters y is limited only to the positivity requirement of the decay matrices. Namely, for any mode C,

$$y_C^{\,2}\leq R[K_S^{\,0}\to C]R[K_L^{\,0}\to C]. \quad (8)$$

To utilize Eq. (8), we assume

$$R[K_S^{\,0}\to3\pi]\leq R[K_L^{\,0}\to3\pi]$$

$$R[K_S^{\,0}\to\mathrm{lep}]\leq R[K_L^{\,0}\to\mathrm{lep}]. \quad (9)$$

Together with the data tabulated in Ref. 4, these positivity conditions imply

$$|y_{\mathrm{lep}}|<13\times10^6\ \mathrm{sec}^{-1},$$

$$|y_{3\pi}|<7\times10^6\ \mathrm{sec}^{-1}, \quad (10)$$

Not having any good measurement on y, we assume

$$y_{\mathrm{lep}}=(0\pm6.5)\times10^6\ \mathrm{sec}^{-1},$$

$$y_{3\pi}=(0\pm3.5)\times10^6\ \mathrm{sec}^{-1}. \quad (11)$$

It is now a well-defined problem to compute the probability distribution of the values of

Table I. Parameters determined from existing experiments. The average values of these parameters are in general agreement with those listed in Ref. 2. In version *A*, we assume $y_{\text{lep}}=0$ and $y_{3\pi}=0$. In version *B*, we assume $y_{\text{lep}}=(0\pm 6.5)\times 10^6\ \text{sec}^{-1}$, and $y_{3\pi}=(0\pm 3.5)\times 10^6\ \text{sec}^{-1}$. Corrections for the contribution from $2i\,\text{Re}A_2\,\text{Im}A_2/[A_0^2+i(m_S-m_L)]$ to ϵ have been made only in the entries $\hat{\epsilon}$, Reϵ, Imϵ for solution II assuming $|\text{Re}A_2/A_0|\lesssim 0.05$.[a] Simultaneously multiplying $\text{Im}A_2/A_0$ by -1 and adding 180° to $\delta_2-\delta_0$ will give us another set of solutions.

Version	Solution	$\hat{\eta}_{00}$ (deg)	$10^3\,\text{Im}A_2/A_0$	$\delta_2-\delta_0$ (deg)	$10^3\|\epsilon\|$	$10^3\,\text{Re}\epsilon$	$10^3\,\text{Im}\epsilon$	M_i ($10^7\ \text{sec}^{-1}$)	$\hat{\eta}_{00}-\hat{\eta}_{+-}$ (deg)	$\hat{\epsilon}$ (deg)
A	I	50^{+11}_{-10}	$1.45^{+0.26}_{-0.25}$	-30^{+21}_{-25}	$5.80^{+0.40}_{-0.29}$	$4.24^{+0.29}_{-0.21}$	$3.95^{+0.28}_{-0.19}$	$-4.60^{+0.21}_{-0.31}$	16^{+24}_{-23}	43 ± 1
	II	216^{+10}_{-11}	$3.22^{+0.22}_{-0.28}$	-55^{+12}_{-12}	$0.69^{+0.48}_{-0.40}$	<0	<0	$0.54^{+0.22}_{-0.32}$	182^{+3}_{-5}	Third quadrant
B	I	50^{+20}_{-19}	$1.45^{+0.40}_{-0.33}$	-30^{+32}_{-39}	$5.80^{+0.51}_{-0.47}$	$4.24^{+0.70}_{-0.74}$	$3.95^{+0.73}_{-0.76}$	$-4.60^{+0.40}_{-0.31}$	16^{+33}_{-32}	43^{+11}_{-10}
	II	216^{+19}_{-20}	$3.22^{+0.28}_{-0.29}$	-55^{+18}_{-18}	$0.69^{+0.62}_{-0.47}$	<0.75	<0.75	$0.54^{+0.31}_{-0.40}$	182^{+9}_{-10}	

[a]Note added in proof.—In the equation for ϵ, we have neglected a term $2i\,\text{Re}A_2\,\text{Im}A_2/[A_0^2+i(m_S-m_L)]$ [L. Wolfenstein, Nuovo Cimento 42, 17 (1966)]. There is no good experimental data on either the magnitude or the sign of $\text{Re}A_2$. The difficulty in measuring it to a high degree of accuracy is pointed out by B. W. Lee and C. N. Yang, to be published. Besides, it is likely that $\text{Re}A_2/A_0\lesssim 0.05$. Inclusion of this term (i) will be important only in the determination of $\hat{\epsilon}$, Reϵ, Imϵ for solution II, (ii) will change the average values of the other parameters by at most a few percent, and (iii) will have essentially no effects on the error estimate. Therefore, in this table only entries $\hat{\epsilon}$, Reϵ, Imϵ for solution II include this correction. We would like to thank Professor L. Wolfenstein for calling this point to our attention.

$|\eta_{00}|$ and $\hat{\eta}_{00}$. To facilitate the computation, however, we observe that the complex vectors η_{+-}, η_{00}, ϵ are all more or less parallel to $\exp(i\alpha)$. The quantity α is defined by

$$-\alpha=\tan^{-1}[(m_S-m_L)/A_0^2]=-43°. \tag{12}$$

The large errors, i.e., those involved in the knowledge of $\hat{\eta}_{+-}$ and y, are perpendicular to this direction. Therefore it is easier to compute the real and imaginary parts of $\eta_{00}\exp(-i\alpha)$. It follows from Eq. (4) that

$$\text{Re}[\eta_{00}\exp(-i\alpha)]=\frac{-3M_i}{2[A_0^4+(m_S-m_L)^2]^{1/2}}-2\,\text{Re}\eta_{+-}\exp(-i\alpha)$$

and

$$\text{Im}[\eta_{00}\exp(-i\alpha)]=\frac{3(y_{\text{lep}}+y_{3\pi})}{2[A_0^4+(m_S-m_L)^2]^{1/2}}-2\,\text{Im}\eta_{+-}\exp(-i\alpha). \tag{13}$$

Referring to Fig. 1, we approximate the experimental knowledge of η_{+-} as

$$\text{Re}[\eta_{+-}\exp(-i\alpha)]=(1.92\pm 0.11)\times 10^{-3}$$

$$\text{Im}[\eta_{+-}\exp(-i\alpha)]=-(0.30\pm 0.44)\times 10^{-3}. \tag{14}$$

Excluding the new measurements of $|\eta_{00}|$ it is obvious from the geometrical construction of Fig. 1 that known experiments do not yield any information on the real part of $\eta_{00}\exp(-i\alpha)$ but give a Gaussian distribution on its imaginary part, namely,

$$\text{Im}[\eta_{00}e^{-i\alpha}]=(0.61\pm 1.65)\times 10^{-3}. \tag{15}$$

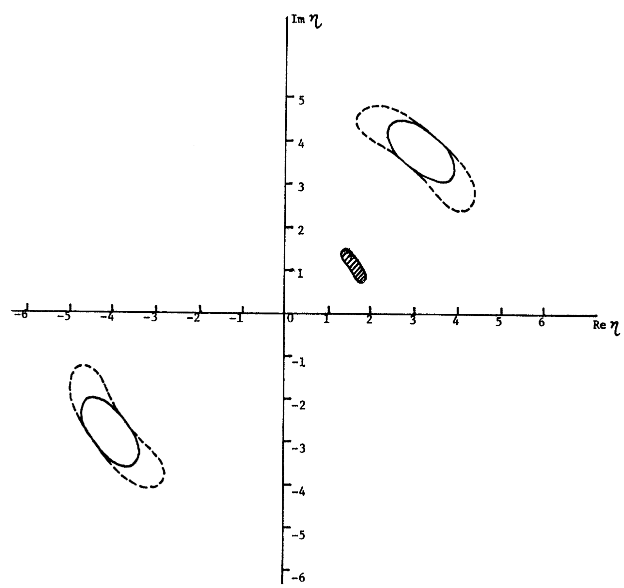

FIG. 1. η_{00}, η_{+-}, and their errors determined from existing experiments. These are expressed in units of 10^{-3}. The shaded area contains the allowed values of η_{+-} within 0.6 standard deviation. There are two solutions for η_{00}. Assuming $y_{\text{lep}}=0$, $y_{3\pi}=0$, the region enclosed by solid curve contains the allowed values of η_{00} within 0.6 standard deviation. If we assume $y_{\text{lep}}=(0\pm 6.5)\times 10^6\ \text{sec}^{-1}$, $y_{3\pi}=(0\pm 3.5)\times 10^6\ \text{sec}^{-1}$, then the region enclosed by dotted curve contains the allowed values of η_{00} within 0.6 standard deviation.

Combining these with the new measurements of $|\eta_{00}|$, we obtain the further entries in Table I.

It is clear from these discussions that the largest errors reside in those of the angles of η_{+-} and η_{00}. More accurate measurements of $\hat{\eta}_{+-}$, and measurements of $\hat{\eta}_{00}$ or $\hat{\eta}_{00}-\hat{\eta}_{+-}$ or $\mathrm{Re}\epsilon$, would serve to narrow the degree of uncertainty.

It is interesting to notice that

$$\left|\mathrm{Im}\frac{A_2}{A_0}\right| \leqslant 3.2\times10^{-3}. \quad (16)$$

Comparing this with the corresponding K^+ decay rate

$$\left|\frac{A_2^+}{A_0}\right| = 0.055, \quad (17)$$

one sees that it is likely that $\mathrm{Im}A_2/\mathrm{Re}A_2 \sim 10^{-1}$. In other words[11] the $|\Delta I| > \frac{1}{2}$ amplitude seems still largely *CP* conserving.

We would like to thank Professor C. N. Yang for suggesting this investigation and for his continuous guidance.

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TESTS FOR *CP* NONINVARIANCE WITH $|\Delta T| > \frac{1}{2}$ IN THE PARTIAL RATES FOR $K^\pm \to \pi^0 + \pi^\pm$ AND $K^\pm \to \gamma + \pi^0 + \pi^\pm$

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(Received 16 January 1967)

CP noninvariance with $|\Delta T| > \frac{1}{2}$ and *CPT* invariance implies that the partial rates, $r_\pm$, for $K^\pm \to \pi^0 + \pi^\pm$, and also the partial rates, $r_\pm{}^1$ for $K^\pm \to \gamma + \pi^0 + \pi^\pm$, will be unequal. An approximate phenomenological analysis is formulated, and suggests the possibility of $|1-(r_-/r_+)| \cong 10^{-3}$ and $(r_-{}^1/r_+{}^1) \cong 3$.

The recent discovery[1,2] that the rate for $K_L{}^0 \to 2\pi^0$ is significantly larger than one-half of the rate for $K_L{}^0 \to \pi^+ + \pi^-$ implies that *CP*-nonconserving nonleptonic decay amplitudes occur which violate the nonleptonic $|\Delta T| = \frac{1}{2}$ rule.[3,4] The origin of such relatively small amplitudes may be instrinsic to the weak interaction,[3,4] or they may arise from an electromagnetic correction to the weak interaction,[5] if the electromagnetic interaction is not *CP*-invariant.[6,7] Of course, ordinary electromagnetic interactions must give rise to small corrections to weak interactions which violate the nonleptonic $|\Delta T| = \frac{1}{2}$ rule.[8]

The decays $K^\pm \to \pi^0 + \pi^\pm$ must, of course, proceed into a pure $T = 2$ state with $|\Delta T| = \frac{3}{2}, \frac{5}{2}$.