since values exceeding these would give more gravimeter noise than that currently observed.

The energy density is not a well-defined quantity in general relativity. If we choose the rest frame of an assumed radiator, a gravitational-radiation mass density is implied by Eq. (9) with a limit

$$\rho < 8\times 10^{-31}\ \mathrm{g\ cm^{-3}} \qquad (10)$$

over a given earth mode near the lowest ones. A power spectrum of the gravitational-radiation mass density $\rho_p(\omega)$ is implied with limit

$$\rho_p(\omega) < 2\times 10^{-25}\ \mathrm{g\ cm^{-3}\ rad^{-1}\ sec.} \qquad (11)$$

The values given by Eqs. (9)-(11) are believed to have significance because detection sensitivity better than the energy density required to give a closed universe has been achieved over single detection modes. Stated in other terms, if a given mechanical degree of freedom has a threshold such that its excitation implies energy density much greater than can be ruled out on cosmological grounds, then it is not a satisfactory detector of background radiation; the earth's modes and the high-frequency detector do have sufficiently low thresholds for this purpose.

The gravitational-wave detectors record isolated events which are not detected by seismometers, gravimeters, tilt meters, or devices responsive to only electromagnetic fields, of types currently in use. The new limits on gravitational radiation are sufficiently low to be of interest for cosmology.

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[1]For bibliography and detailed analyses, see J. Weber, General Relativity and Gravitational Waves (Interscience Publishers, Inc., New York, 1961), Chaps. 7 and 8; in Gravitation and Relativity, edited by Hong-Yee Chiu and W. F. Hoffmann (W. A. Benjamin, Inc., New York, 1964), Chap. 5; in Proceedings of the International Enrico Fermi School of Physics, Course XX (Academic Press, Inc., New York, 1962), p. 116; and Relativity Groups and Topology (Gordon and Breach Publishers, Inc., New York, 1964), p. 865.

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[3]J. Weber, to be published.

[4]J. Weber and J. V. Larson, J. Geophys. Res. 71, 6005 (1966).

K_1^0-K_2^0 MASS DIFFERENCE AND LOW-ENERGY π-π DYNAMICS

Ronald Rockmore* and Tsu Yao
Department of Physics, Rutgers, The State University, New Brunswick, New Jersey
(Received 20 January 1967)

Recently, experimental results[1] indicating that $\Delta m = m(K_1^0) - m(K_2^0) = -0.5/\tau_1$, where τ_1 is the lifetime of the K_1^0, have spurred a recalculation of this quantity by Truong[2] in terms of the weak self-energy model introduced by Barger and Kazes[3] and later developed by Nishijima.[4] One of the purposes of this note is to show that if one accepts the premises on which Truong's calculation is based,[5] his conclusion that[2] "the possibility of a di-pion resonance ($J^{PG} = 0^{++}$) is suggested" is quite vitiated; our conclusion follows from a careful examination of the so-called "dynamical correction term" in the self-energy model, which, according to Ref. 2, results only from the usual left-hand cut of the numerator function, $N(\nu)$, where $\nu = \frac{1}{4}s - 1$[6]; not only is it argued there that the contribution from that cut may be neglected, but the possibility of numerator zeros, which will give rise to pole-like contributions in this model, is entirely overlooked. (The reliance in Ref. 2 on unrealistic S-wave π-π scattering models with constant numerator functions is, of course, to blame.) In a more physically based model of S-wave π-π scattering considered in detail below we find such a zero occurring quite naturally (and for small λ, rather near the physical threshold, $\nu = 0$); the resulting contribution from it makes for a significant correction to the main term of Ref. 2. Thus we find the "approximate formula" of Ref. 2,[7]

$$2\tau_1 \Delta m \approx -\cot\delta_0^{\,0}(\nu_K), \qquad (1)$$

to be dubious.

After first making the straightforward extension of the derivation of the complete expression for $2\tau_1\Delta m$ given in Ref. 2 to the case where there is a numerator zero at $\nu=\bar{\nu}<0$,

$$2\tau_1\Delta m+\cot\delta_0{}^0(\nu_K) = \left(\frac{\nu_K+1}{\nu_K}\right)^{1/2}\frac{|D_0{}^0(\nu_K)|^2}{(\nu_K-\bar{\nu})N_0{}^{0\prime}(\bar{\nu})D_0{}^0(\bar{\nu})}-\left(\frac{\nu_K+1}{\nu_K}\right)^{1/2}\frac{1}{2\pi i}\oint_{\text{left-hand cut}}\frac{d\nu'}{\nu'-\nu_K}\frac{|D_0{}^0(\nu_K)|^2}{N_0{}^0(\nu')D_0{}^0(\nu')}, \quad (2)$$

we are ready to consider a model of low-energy π-π dynamics. For $\lambda=0$, our model is that of single ρ exchange in the approximation of zero width; in this approximation the S-wave π-π interaction is a "derived" interaction, it being the s-channel remnant of t- and u-channel vector-meson exchange. One has

$$-32\pi M_{\text{Born}}(s,t,u) = \delta_{ab}\delta_{cd}f_\rho{}^2\left[\frac{t-s}{u-m_\rho{}^2}+\frac{t-u}{s-m_\rho{}^2}\right]+\delta_{ad}\delta_{cb}f_\rho{}^2\left[\frac{u-s}{t-m_\rho{}^2}+\frac{u-t}{s-m_\rho{}^2}\right]+\delta_{ac}\delta_{bd}f_\rho{}^2\left[\frac{s-u}{t-m_\rho{}^2}+\frac{s-t}{u-m_\rho{}^2}\right]. \quad (3)$$

The amplitude $M(s,t,u)$ clearly satisfies Adler's consistency condition,[8]

$$M_{\text{Born}}(1,1,1)=0; \quad (4)$$

it is just the crossing-symmetric generalization of Sakurai's[9] connection of vector-meson dominance with current algebra.[10] [As we have remarked above, we have[11]

$$M_{\text{Born}}(\tfrac{4}{3},\tfrac{4}{3},\tfrac{4}{3})=0=-\lambda.] \quad (5)$$

The S-wave projection of this "Born" amplitude,[12]

$$[N_0{}^I(\nu)]_{\text{Born}}=-\beta_{I1}\frac{f_\rho{}^2}{4\pi}\left\{\frac{(2\nu+1+\frac{1}{4}m_\rho{}^2)}{\nu}Q_0\left(1+\frac{m_\rho{}^2}{2\nu}\right)-\frac{1}{2}\right\}, \quad (6)$$

has a zero surprisingly near the symmetry point, ν_0, at $\nu=\nu_0+0.014$, and yields zero-energy scattering lengths,

$$a_{S0}=\frac{2}{m_\rho{}^2}\left(\frac{f_\rho{}^2}{4\pi}\right)=-2a_{S2}, \quad (7)$$

which are rather close to those predicted by the current algebra.[10]

For a quantitative examination of the "correction term" [the right-hand side of Eq. (2)], we have utilized Desai's[13] one-pole N/D calculation of S-wave π-π scattering with current input ρ parameters.[14] (For $\lambda=0$, this tractable model is the one-pole approximation to the appropriate partial-wave projection of the simple ρ-exchange model discussed above.) Moreover, it should be noted that, in this model, where the usual left-hand cut is replaced by a pole, the numerator function,

$$N_0{}^0(\nu)=-5\lambda+(\nu-\nu_0)\frac{\omega_{S0}+\nu_0}{\omega_{S0}+\nu}B_0, \quad (8)$$

has a single zero, which, in the range of λ we have considered, always lies to the left of the physical threshold. Because of the small positive slope of the numerator function at this point, we find that even a distant zero produc-

Table I. Calculations relating to Eq. (2) based on Desai's model of S-wave, $I=0$, π-π scattering.

λ	$a_{S0}\times m_\pi$	Main term	Correction term	$2\tau_1\Delta m$
	$f_\rho{}^2/4\pi=2.16$,	$\omega_{S0}=14.5$		
0.01	0.08	−2.16	1.59	−0.57
0	0.14	−1.90	1.39	−0.51
−0.10	0.84	−1.61	0.82	−0.79
−0.15	1.31	−1.63	0.13	−1.50
	$f_\rho{}^2/4\pi=2.4$,	$\omega_{S0}=13.3$		
0.01	0.10	−1.84	1.39	−0.45
0	0.16	−1.72	1.24	−0.48
−0.10	0.87	−1.49	0.82	−0.67
−0.15	1.36	−1.53	0.35	−1.18

es a significant correction (of opposite sign). The relevant numerical results have been displayed in Table I.

The best reconciliation of the Desai model[13] with current data[1] on the K_1^0-K_2^0 mass difference would appear to indicate values of λ between -0.1 and -0.15 in the range of acceptable values[14] of $f_\rho^2/4\pi$, and a large S-wave, $I=0$, scattering length as well. However, one is likely straining the reliability of the model to draw so quantitative an inference. At the same time, one sees that this more realistic model does not require a large π-π phase shift[15] at energies in the neighborhood of the K-meson mass as has been suggested.[2]

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[1]Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, California, 1966 (to be published).

[2]T. N. Truong, Phys. Rev. Letters 17, 1102 (1966).

[3]V. Barger and E. Kazes, Phys. Rev. 124, 279 (1961).

[4]K. Nishijima, Phys. Rev. Letters 12, 39 (1964).

[5]Briefly, these are that (a) the effect of CP noninvariance, (b) the contributions of the leptonic and three-pion ($K_2^0 \to 3\pi$) modes, and (c) the pion and η^0 pole contributions to the self-energy of K_2^0 can be neglected.

[6]We work in units $m_\pi = 1$.

[7]We set $\nu_K = \frac{1}{4}(m_K^2/m_\pi^2) - 1$; as usual, we take

$$\left(\frac{\nu}{\nu+1}\right)^{1/2} \cot\delta_0^{\,I}(\nu) = \mathrm{Re}[A_0^{\,I}(\nu)]^{-1} = \frac{\mathrm{Re}D_0^{\,I}(\nu)}{N_0^{\,I}(\nu)},$$

where

$$D_0^{\,I}(\nu) = 1 - \frac{\nu-\nu_0}{\pi}\int_0^\infty d\nu' \left(\frac{\nu}{\nu'+1}\right)^{1/2} \frac{N_0^{\,I}(\nu')}{(\nu'-\nu_0)(\nu'-\nu)},$$

with $\nu_0 = -\frac{2}{3}$.

[8]S. L. Adler, Phys. Rev. 137, B1022 (1965); 139, B1638 (1965).

[9]J. J. Sakurai, Phys. Rev. Letters 17, 552 (1966).

[10]However, in contrast to Weinberg's results [S. Weinberg, Phys. Rev. Letters 17, 616 (1966)], in this model we find $A=0$, $B=-\frac{1}{2}C$.

[11]It is interesting that in Weinberg's model one finds for λ the minuscule value $\lambda = -0.0028(f_\rho^2/4\pi)$.

[12]$\beta_{01}=1$, $\beta_{21}=-\frac{1}{2}$. See, for example, L. A. P. Balasz, Phys. Rev. 128, 1939 (1962).

[13]B. R. Desai, Phys. Rev. Letters 6, 497 (1961).

[14]J. J. Sakurai, Phys. Rev. Letters 17, 1021 (1966).

[15]Indeed for $f_\rho^2/4\pi = 2.16$, and $\lambda = 0$, the unitarization provided by Desai's model reduces the Born phase shift, $\delta_0^{\,0}(\nu_K)$, very slightly.

S-WAVE $\pi\pi$ SCATTERING AND THE K_1^0-K_2^0 MASS DIFFERENCE*

Kyungsik Kang
Department of Physics, Brown University, Providence, Rhode Island

and

David J. Land†
Radiation Physics Division, U. S. Naval Ordnance Laboratory, Silver Spring, Maryland

(Received 15 February 1967)

The result of recent measurements of the K_1^0-K_2^0 mass difference has been used as a test of various solutions to s-wave $\pi\pi$ scattering which are obtained by solving the full N/D equations with various given forms of the driving force. Certain features of these solutions with regard to the evaluation of the mass difference are also pointed out.

In view of the fact that several recent measurements of the K_1^0-K_2^0 mass difference have resolved some of the previous experimental uncertainties,[1] it is of some interest to consider this problem again theoretically.[2-5] These measurements indicate a value of $\Delta M \equiv M(K_1^0) - M(K_2^0) \simeq -0.5\tau_1^{-1}$, where τ_1 is the lifetime of the K_1^0 meson. In the present work, we use the K_1^0-K_2^0 mass difference as a test of various solutions to the problem of $I=0$, s-wave $\pi\pi$ scattering. We obtain some exact solutions for the s-wave amplitude by solving the full N/D equations with various given forms of the driving force. We then discuss certain features of these solutions with regard to the evaluation ΔM.

Since the K_1^0-K_2^0 mass difference is produced by the weak interactions, the problem reduces to a calculation of the self-energies of the K_1^0 and K_2^0 mesons due to the weak interactions. From experiments, the lifetime of K_1^0 is of the order of 10^{-10} sec while that of K_2^0 is of