SINGULAR STRING OF MAGNETIC MONOPOLES

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The significance of the "string" attached to the Dirac magnetic monopole is reviewed. A mechanism proposed by Schiff, to explain the absence of free quarks in nature, is shown to be invalid.

Recently a very ingenious theory was proposed by Schiff[1] to account for the fact that quarks, carrying one-third integral charges, appear to move freely within hadrons, but cannot escape as individuals. Schiff's argument is based on an old idea of Dirac,[2] who showed that electric and magnetic charges can coexist in quantum theory only if the product of any electric charge e and any magnetic charge g satisfies $eg/\hbar c=\frac{1}{2}n$, where n is an integer. More recently, Schwinger[3] sharpened Dirac's result and found $eg/\hbar c=n$ (or possibly $2n$). It follows that if our universe contains even a single magnetic pole of strength g, then all electric charges must be integral multiples of $\hbar c/g$. Now, Schiff[1] argued that if the Dirac magnetic monopole has finite extension R, the consequent quantization of charge applies only to clusters of particles of size R, but not to the individual particles (quarks) within such clusters. The purpose of the present note is to show that Schiff's argument is erroneous.

First, let us recall how quantum theory restricts $eg/\hbar c$ to integral values, while there is no such restriction in classical theory. The reason is that in quantum theory it is necessary, in general, to introduce the electromagnetic potentials $\vec{A}$ (the fields $\vec{E}$ and $\vec{B}$ are not sufficient).[4] If, for simplicity, we consider the magnetic monopole as a classical object (as Schiff does), we must first solve the equation curl $\vec{A}=\vec{B}$, which is always possible for a simply connected domain in which div $\vec{B}=0$, i.e., in which there is no magnetic charge. It is however impossible to solve the above equation in the multiply connected domain surrounding a magnetic pole. If a formal solution is sought, it turns out that $\vec{A}$ is singular along a line passing through the pole (the Dirac "string"). Along that line, $\vec{B}$ behaves as a delta function,[3] in such a way that the total magnetic flux across any closed surface is zero (see Fig. 1).[5]

Obviously the Dirac string (the orientation of which is arbitrary) is a mathematical object, not a physical one. It is the function of charge quantization to remove this anomaly and restore the physical equivalence of all space-time points, even though manifest rotational invariance has been destroyed.[3] Actually, what is done by charge quantization is to ascertain that the phase shift around the singular string is $2\pi n$ and therefore is unobservable. Different orientations of the string can then be related by innocuous gauge transformations.

Schiff's theory[1] is based on the following argument: If the magnetic pole has a finite size R, the singular "string" becomes a "bundle," and then the Dirac-Schwinger theory applies only to clusters of particles of size R (or larger). On the other hand, there is no charge quantization for particles separated by less than R, because the "bundle" cannot be squeezed between them so as to scramble their phases.

The fallacy of this argument is the following: Even an extended magnetic monopole leads to an $\vec{A}$ with a singular "string" (not with a "bundle"). This is a straightforward consequence of solving curl $\vec{A}=\vec{B}$ for a spherically symmetric $\vec{B}$.[6,7] As stated above, this string is a purely mathemetical object (thanks to charge quantization). It arises solely because of the multiple connectedness of space around the magnetic pole, and it is completely meaningless to smear it into a bundle.

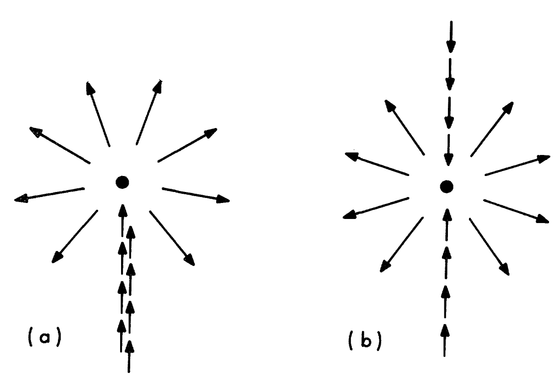

FIG. 1. Schematic description of (a) Dirac's string and (b) Schwinger's string, showing why the Dirac monopole may be half as big as that of Schwinger.

I am indebted to Mr. A. Casher for a discussion of this problem.

[1]L. I. Schiff, Phys. Rev. Letters 17, 714 (1966).
[2]P. A. M. Dirac, Proc. Roy. Soc. (London) A133, 60 (1931); Phys. Rev. 74, 817 (1948).
[3]J. Schwinger, Phys. Rev. 144, 1087 (1966).
[4]It is possible to avoid the use of potentials only at the expense of introducing nonlocal variables. See, e.g., S. Mandelstam, Ann. Phys. (N.Y.) 19, 1 (1962). This nonlocal formalism also leads to charge quantization, as shown by N. Cabibbo and E. Ferrari, Nuovo Cimento 23, 1147 (1962).
[5]A "realistic" model of the Dirac string has been constructed by R. A. Ferrell and J. J. Hopfield, Physics 1, 1 (1964).
[6]Note that $\vec{A}$ can be defined only at places where div $\vec{B}=0$. It is therefore meaningless to ask for the value of $\vec{A}$ inside the extended magnetic pole.
[7]Any $\vec{B}$ which is not spherically symmetric can be analyzed into a pole, a dipole, a quadrupole, etc. Only the pole term gives rise to the singular line.

RADIO PULSES FROM COSMIC-RAY AIR SHOWERS*

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Radio pulses from air showers were detected in conjunction with the Bolivian Air-Shower Joint Experiment air-shower array, which permitted measurement of shower size, arrival direction, and core location. For the first time, good correlation has been found between the shower arrival direction and the antenna pattern and between the shower size and the frequency of radio pulses; some correlation has been found between the radio-pulse height and the shower size and between the shower core location and the antenna position.

Radio pulses associated with extensive air showers have been detected by several groups in England and Ireland.[1-5] Of these, only Allan and Jones worked in conjunction with a shower array capable of determining arrival directions and other shower parameters. They found some correlations between shower parameters and radio pulses, but they state that their radio showers were not especially distinguished from the others by size, location, or direction of the shower axis. In this paper we report preliminary results that do show some such distinction.

Our experiment was carried out at an altitude of 5200 m and a latitude of 16°S in conjunction with the Bolivian Air-Shower Joint Experiment (BASJE) air-shower array,[6] which now has a diameter of 300 m (Fig. 1) and which measures many shower parameters. Among these is the arrival direction, which is determined with an accuracy of 2.5°.[7]

The BASJE array was triggered on two types of events:

(a) On "large showers," namely, whenever there were more than 15 particles in any two of the five scintillators in the outermost ring (*D* ring) and more than 30 particles in the scintillator at the center of the array. This trig-

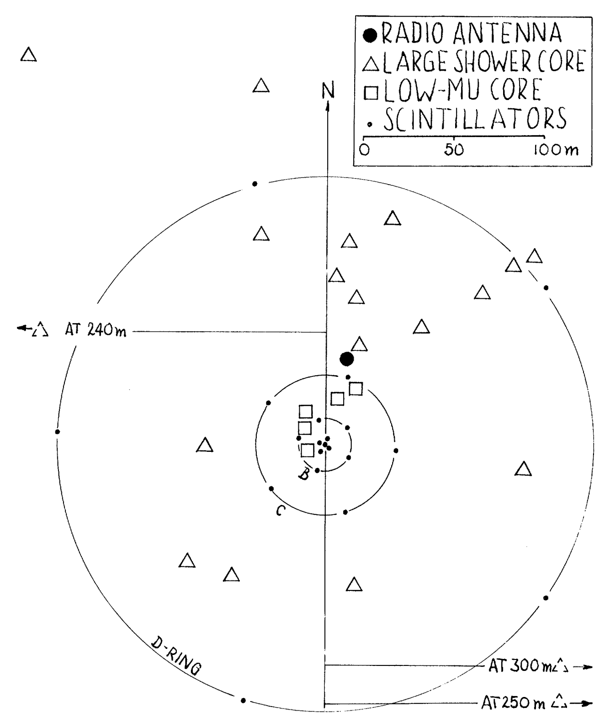

FIG. 1. Impact points of the radio-shower cores relative to the radio antenna and the BASJE scintillators. The radio antenna was 50 m N of the center of the array.