sharply, and then increase until the wave functions cease to be reliable.[11] The largest variation of the potential energy occurs for oblate shapes, reflecting the more rapid increase there in $\hbar\omega(\delta)$. We find that, when the kinetic energy predicts a large deformation, the effect of the potential is to decrease δ by about 0.1 and 0.15 for the prolate and oblate minima. Taking account of the similarity of the K-K curves to the deviations away from horizontal of the delta curves indicates that this is an overestimate.

The result (b) in the introduction was determined by trying different sequences of level filling. Note finally that we have not varied $\hbar\omega$ for each nucleus.

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†Present address.

[1]R. Muthukrishnan, Nucl. Phys. A93, 417 (1967).

[2]J. Bar-Touv and I. Kelson, Phys. Rev. 138, B1035 (1965).

[3]A. Bohr and B. Mottelson, unpublished lectures. See also S. A. Moszkowski, in Handbuch der Physik, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39, p. 501, and references cited there for the axially symmetric version.

[4]B. L. Scott and S. A. Moszkowski, Nucl. Phys. 29, 665 (1962). See also T. T. S. Kuo and G. E. Brown, Nucl. Phys. 85, 40 (1965), Sec. 2.1.

[5]S. G. Nilsson, Kgl. Danske Videnskab. Selskab. Mat.-Fys. Medd. 29, No. 16 (1955). To be consistent with T. A. Brody and M. Moshinsky, Tables of Transformation Brackets (Monografías del Institute de Física, Ciudad Mexico, Mexico, 1960), we have used the usual phase convention for the radial wave function (see footnote, p. 35).

[6]Any two-particle matrix element of the delta function is independent of deformation. To verify this, write the delta function in Cartesian coordinates and do each double integral separately, obtaining for each some constant times a scale-factor term. The product of the scale terms is constant by volume conservation.

[7]A. R. Edmonds, Angular Momentum in Quantum Mechanics (Princeton University Press, Princeton, New Jersey, 1957).

[8]Brody and Moshinsky, Ref. 5.

[9]A. Kallio and K. Kolltveit, Nucl. Phys. 53, 87 (1964).

[10]This may result in our discarding possibly important surface terms. This question is now being investigated.

[11]In O^{16} (not plotted) we find the one exception, of interest in its relation to the assumptions made in Ref. 2. Here the potential energy rises almost as fast as the kinetic, and the delta curve is very flat by comparison.

GRAVITATIONAL RADIATION*

J. Weber
Department of Physics and Astronomy, University of Maryland, College Park, Maryland
(Received 8 February 1967)

The results of two years of operation of a 1660-cps gravitational-wave detector are reviewed. The possibility that some gravitational signals may have been observed cannot completely be ruled out. New gravimeter-noise data enable us to place low limits on gravitational radiation in the vicinity of the earth's normal modes near one cycle per hour, implying an energy-density limit over a given detection mode smaller than that needed to provide a closed universe.

Apparatus for measurement of the Riemann tensor and search for gravitational radiation was described some years ago.[1] The space-time derivatives of the gravitational potential induce relative motion between part of an elastic body. If the Riemann tensor has Fourier components in the vicinity of elastic normal modes of quadrupole symmetry, these modes may be observed to have greater than thermal energy. Utilization of resonance improves the signal-to-noise ratio.

High-frequency detector operations. – A large aluminum cylinder has been instrumented so that its compressional mode in the vicinity of 1660 cps can be observed continuously with sensitivity limited by the thermal fluctuations.[2]

Relative displacement of the end faces much smaller than a nuclear radius may be observed, implying observable strains of a few parts in 10^{16}.

Let us imagine the axis of the cylinder to be in the direction x^1 of a normal coordinate system with the pole at the cylinder center of mass. The voltage output of the square-law detector is proportional to the square of the Fourier transform of the Riemann-tensor component R_{1010}. A Riemann-tensor transform over the apparatus bandwidth exceeding 10^{-34} cm^{-2} would give observable excitation over an averaging time roughly equal to the cylinder relaxation time.

The antenna has the directivity pattern of an axial quadrupole, the orientation of which rotates with the earth. If radiation is incident from some given direction, maxima would be seen twice each sidereal day. No such effects have been observed.

An absolute calibration of the detector using a noise source and a locally generated gravitational field was carried out in collaboration with Sinsky and established the fact that the noise temperature of the cylinder is close to room temperature. This therefore implies that no large isotropic gravitational radiation is contributing to the noise output. These data imply the limit

$$\rho < 10^{-28} \text{ g cm}^{-3}$$

for the mass density ρ of gravitational radiation over the integrated bandwidth of the detector. The bandwidth $\Delta\omega \approx 10^{-1}$ rad sec^{-1} with $\omega \approx 10^4$ rad sec^{-1}.

Observation of events. – The high-frequency detector has been operating with good sensitivity and isolation since about January 1965. During the course of the dynamical gravitational-induction-field experiment, the acoustic and electromagnetic isolation of the detector were improved sufficiently so that 100 W of electromagnetic-acoustic power in the same room as the detector results in less than 10^{-23} W at the detector output. Extreme precautions have been taken to isolate the detector from line-voltage fluctuations, with storage-battery power employed for all critical potentials.

Nonetheless, the gravitational-wave detector responds to some large forces not having their origin in gravitational radiation. Certain kinds of earthquakes, violent local earth motion, large magnetic field fluctuations, unusually intense sound, and tilting of the apparatus platform all will excite the detector. In addition, we find that the relaxation processes associated with temperature changes also occasionally excite the detector. To assist in identification of events not having their origin in gravitational radiation, we have at the detector location a low-frequency seismometer (one cycle in 30 sec to one cycle per second), a high-frequency seismometer at the detector frequency, east-west and north-south tilt meters, a gravimeter which responds to changes in vertical acceleration g exceeding a part in 10^{10}, and a continuous recorder of line-voltage fluctuations. The room temperature is controlled to a fraction of a degree.

Noise power is recorded continuously. By an event we mean a sudden increase in output exceeding 5 times the mean noise deviation power. The events that have been observed without simultaneous occurrences on the seismometer, the gravimeter, and the tilt meters are shown in Table I.

Recently two additional detectors became operational. One is a 1660-cps instrument located on a concrete pier 3 km from the large detector. The other one is a 1120-cps instrument at the main detector site.

The second 1660-cps instrument has a cross section an order smaller, a larger bandwidth, a higher noise temperature, and completely different electronics from the large cylinder. The last three events listed in the table were coincident occurrences on both 1660-cps instruments. The lack of a 1120-cps coincidence rules out wide-band electromagnetic disturbances as the origin.

Table I. Events observed without simultaneous occurrences on the seismometer, the gravimeter, and the tilt meter.

Time[a]	Date
0924	21 September 1965
2342	5 August 1966
1015	7 August 1966
1645	22 November 1966
0130	2 December 1966
0720	17 December 1966
0140	20 December 1966
1730	20 January 1967
2309	22 January 1967
1320	17 February 1967

[a]All times are Greenwich Time.

As we remarked earlier, the large detector is unusually well isolated. The peaks discussed here occur about one minute out of every month for the large detector and about one percent of the time for the small detector. Thus the probability of a coincident occurrence is less than 10^{-6} if these events are of random character. Absorption of energy exceeding several kT over the large-detector relaxation time of 30 sec will result in a peak. If we calculate energy flux in the rest frame of an assumed radiator, the cross section of the large detector is about 3×10^{-19} cm^2. The indicated large-detector peaks would suggest a flux of about 2×10^4 erg cm^{-2} sec^{-1}. This is so large that observable astrophysical effects would be expected. Since none have been reported, an origin in gravitational radiation appears very unlikely. Perhaps some seismic events are not being observed by the gravimeter-seismometer-tilt installation. This possibility is being explored further.

No peaks unaccompanied by seismic effects and exceeding 20 mean power deviations have been seen on the large detector.

Normal modes of the earth.—The use of the earth's normal modes[1] and those of the moon[3] offers the possibility of observations in the range from about one cycle per hour upwards in frequency with cross sections up to 10^4 m^2 for the earth and roughly 50 m^2 for the moon.

At resonance a Riemann tensor component $R_{0i0}{}^{j}=B\sin(\omega t-kx)$ may induce a surface displacement ξ^j of the order[1] of

$$\xi^j\approx\frac{Brc^2}{\omega^2}\left[\frac{8Q}{\pi^2}\cos(\omega t-kx_c)+\sin(\omega t-kx_c)\right]. \qquad (1)$$

In Eq. (1), Q is the quality factor, r is the radius, and ω is the angular frequency. Our elastic body has dimensions roughly a wavelength of sound, small compared with a gravitational wavelength. Its center of mass is assumed to be located at x_c.

Let us assume that observations are being made with a gravimeter on the surface of the earth or moon. This instrument measures acceleration, g. Motion of the surface results in a change Δg given by

$$\Delta g=-\frac{2GM}{r^3}\xi+\frac{d^2\xi}{dt^2}. \qquad (2)$$

In Eq. (2), the first term on the right is the change in g associated with change in distance from the center; the second term is the acceleration resulting from motion relative to the center of mass. We are using normal coordinates, with the pole at the center of mass.

For harmonic motion at resonance, Eqs. (1) and (2) give

$$\langle(\Delta g)^2\rangle\approx\frac{B^2r^2c^4}{2}\left[\left(1+\frac{8Q}{\pi^2}\right)\left(1+\frac{2g}{\omega^2r}\right)\right]^2. \qquad (3)$$

For a continuous spectrum of radiation, Eq. (3) would be replaced by an integral over the power spectrum of the Riemann tensor. For a mode with a reasonably large Q (exceeding 10), Eq. (3) becomes

$$\langle(\Delta g)^2\rangle\approx\langle B^2\rangle r^2c^4\left[\left(1+\frac{8Q}{\pi^2}\right)\left(1+\frac{2g}{\omega_0{}^2r}\right)\right]^2. \qquad (4)$$

In Eq. (4), ω_0 is the mode angular frequency and $\langle B^2\rangle$ is the integral of the power spectrum of B over the mode. Equation (4) is also valid if $\langle(\Delta g)^2\rangle$ and $\langle B^2\rangle$ are replaced by their power spectra.

The noise output of a gravimeter was recently studied by Weber and Larson.[4] They found no evidence of earth normal-mode excitation during quiet periods. The noise-power spectrum in the vicinity of one cycle per hour (angular frequency units) was found to be

$$g_p(\omega)\approx6.9\times10^{-14}\ \mathrm{gal}^2\ \mathrm{rad}^{-1}\ \mathrm{sec}. \qquad (5)$$

The earth modes in the vicinity of one cycle every 54 min have $Q\approx400$. Therefore, the bandwidth $\Delta\omega$ is given by

$$\Delta\omega=\omega/Q=4.86\times10^{-6}\ \mathrm{rad\ sec}^{-1}. \qquad (6)$$

Equations (4) and (5) imply that a mean-squared Riemann tensor over a 54-min mode would excite it above the noise level if a Riemann-tensor power spectrum $R_p(\omega)$ were present with

$$R_p(\omega)>6.0\times10^{-79}\ \mathrm{cm}^{-4}\ \mathrm{rad}^{-1}\ \mathrm{sec}. \qquad (7)$$

This implies that the following approximate limits may be placed on continuous background gravitational radiation in the vicinity of one cycle per hour:

$$R_p(\omega)<6\times10^{-79}\ \mathrm{cm}^{-4}\ \mathrm{rad}^{-1}\ \mathrm{sec}, \qquad (8)$$

and Eq. (8) suggests that a mean-squared Riemann tensor over one of the earth's low frequency modes has the approximate limit

$$\langle R^2\rangle<3\times10^{-84}\ \mathrm{cm}^{-4}, \qquad (9)$$

since values exceeding these would give more gravimeter noise than that currently observed.

The energy density is not a well-defined quantity in general relativity. If we choose the rest frame of an assumed radiator, a gravitational-radiation mass density is implied by Eq. (9) with a limit

$$\rho < 8\times 10^{-31}\ \text{g cm}^{-3} \qquad (10)$$

over a given earth mode near the lowest ones. A power spectrum of the gravitational-radiation mass density $\rho_p(\omega)$ is implied with limit

$$\rho_p(\omega) < 2\times 10^{-25}\ \text{g cm}^{-3}\ \text{rad}^{-1}\ \text{sec}. \qquad (11)$$

The values given by Eqs. (9)-(11) are believed to have significance because detection sensitivity better than the energy density required to give a closed universe has been achieved over single detection modes. Stated in other terms, if a given mechanical degree of freedom has a threshold such that its excitation implies energy density much greater than can be ruled out on cosmological grounds, then it is not a satisfactory detector of background radiation; the earth's modes and the high-frequency detector do have sufficiently low thresholds for this purpose.

The gravitational-wave detectors record isolated events which are not detected by seismometers, gravimeters, tilt meters, or devices responsive to only electromagnetic fields, of types currently in use. The new limits on gravitational radiation are sufficiently low to be of interest for cosmology.

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[1]For bibliography and detailed analyses, see J. Weber, General Relativity and Gravitational Waves (Interscience Publishers, Inc., New York, 1961), Chaps. 7 and 8; in Gravitation and Relativity, edited by Hong-Yee Chiu and W. F. Hoffmann (W. A. Benjamin, Inc., New York, 1964), Chap. 5; in Proceedings of the International Enrico Fermi School of Physics, Course XX (Academic Press, Inc., New York, 1962), p. 116; and Relativity Groups and Topology (Gordon and Breach Publishers, Inc., New York, 1964), p. 865.

[2]J. Weber, Phys. Rev. Letters 17, 1228 (1966).

[3]J. Weber, to be published.

[4]J. Weber and J. V. Larson, J. Geophys. Res. 71, 6005 (1966).

K_1^0-K_2^0 MASS DIFFERENCE AND LOW-ENERGY π-π DYNAMICS

Ronald Rockmore* and Tsu Yao
Department of Physics, Rutgers, The State University, New Brunswick, New Jersey
(Received 20 January 1967)

Recently, experimental results[1] indicating that $\Delta m = m(K_1^0)-m(K_2^0) = -0.5/\tau_1$, where τ_1 is the lifetime of the K_1^0, have spurred a recalculation of this quantity by Truong[2] in terms of the weak self-energy model introduced by Barger and Kazes[3] and later developed by Nishijima.[4] One of the purposes of this note is to show that if one accepts the premises on which Truong's calculation is based,[5] his conclusion that[2] "the possibility of a di-pion resonance ($J^{PG} = 0^{++}$) is suggested" is quite vitiated; our conclusion follows from a careful examination of the so-called "dynamical correction term" in the self-energy model, which, according to Ref. 2, results only from the usual left-hand cut of the numerator function, $N(\nu)$, where $\nu = \frac{1}{4}s-1$[6]; not only is it argued there that the contribution from that cut may be neglected, but the possibility of numerator zeros, which will give rise to pole-like contributions in this model, is entirely overlooked. (The reliance in Ref. 2 on unrealistic S-wave π-π scattering models with constant numerator functions is, of course, to blame.) In a more physically based model of S-wave π-π scattering considered in detail below we find such a zero occurring quite naturally (and for small λ, rather near the physical threshold, $\nu = 0$); the resulting contribution from it makes for a significant correction to the main term of Ref. 2. Thus we find the "approximate formula" of Ref. 2,[7]

$$2\tau_1 \Delta m \approx -\cot\delta_0^{\,0}(\nu_K), \qquad (1)$$

to be dubious.

After first making the straightforward extension of the derivation of the complete expression for $2\tau_1\Delta m$ given in Ref. 2 to the case where