VARIATIONAL CALCULATIONS IN LIGHT NUCLEI WITH REALISTIC NUCLEON-NUCLEON INTERACTIONS: GROUND-STATE DEFORMATIONS*

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In this paper we present the results of variational calculations for the ground-state deformation of even-even nuclei, $8 \leq A \leq 36$. We find that (a) the potential energy depends weakly on deformation when compared with the corresponding variations in the kinetic energy, tending, especially for oblate shapes, to reduce the deformation a little; and (b) the magnitude of the potential energy is greatest when precisely those levels are filled for which the kinetic energy is minimized. This suggests a picture in which it is primarily the behavior of the kinetic energy that determines the ground-state deformations in this region.

Kinetic-energy model.—Further evidence for this conjecture is presented in Fig. 1. The intrinsic quadrupole moments Q_0 and the asymmetry parameter Q_2 that were obtained in the Hartree-Fock (H-F) calculations[1,2] are compared with the kinetic-energy model, Eqs. (1)-(4) below.

In the kinetic-energy model[3] the neutrons and protons fill the lowest single-particle states in an anisotropic oscillator potential subject to the volume conservation constraint, $\omega_x\omega_y\omega_z = \omega^3$, at all deformations ($\hbar\omega = 41/A^{1/3}$ MeV). We may write the kinetic energy as

$$\langle K\rangle = \sigma_x\hbar\omega_x + \sigma_y\hbar\omega_y + \sigma_z\hbar\omega_z, \qquad (1)$$

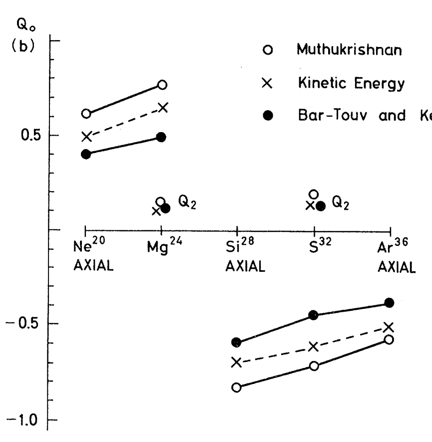

FIG. 1. The ground-state qudrupole moments Q_0 and the asymmetry parameters Q_2 obtained in H-F calculations are compared with the values predicted from kinetic energy alone.

where, e.g., σ_x is the sum $\frac{1}{2}\sum(n_x+\frac{1}{2})$ over the x-oscillator quanta in the occupied states. When (1) is minimized with respect to the deformation parameters, we obtain

$$\sigma_i\hbar\omega_i = \tfrac{1}{3}K_{\min} = [\sigma_x\sigma_y\sigma_z]^{1/3}\hbar\omega \quad (i=x,y,z), \qquad (2)$$

$$Q_0 = [2\sigma_z^2 - \sigma_x^2 - \sigma_y^2]q_0, \qquad (3)$$

and

$$Q_2 = 2[\sigma_x^2 - \sigma_y^2]q_0, \qquad (4)$$

where $q_0 = 3\hbar^2/MK_{\min}$. In order to obtain identical signs for Q_2 in Mg^{24} and S^{32} we have made use of the freedom to order the σ's arbitrarily.

In Ref. 1 the H-F equations were solved for all nucleons using the anistropic oscillator basis. In view of the relatively flat behavior of the potential energy, the agreement between the kinetic-energy model and Ref. 1 is not surprising. The similar results of Ref. 2, in which the H-F equations are restricted to the nucleons in the $2s$-$1d$ shell, the O^{16} core being regarded as inert, constitutes a fascinating puzzle in the s-d shell.

It is worth noting that for $A \leq 40$, wherever the asymmetric deformations lie lowest (as is usual for the non-$4n$ nuclei) there is ordinarily a symmetric minimum within 1 MeV. Such small differences may easily be compensated by the potential energy lost in breaking time-reversed pair states. The exceptions are $Mg^{24,28}$ where the difference is 1.8 MeV.

Potential energy calculation.—We start from a Slater determinant built up of neutron and proton single-particle states filling the axially symmetric oscillator potential. The antisymmetrized two-particle matrix elements $\langle\alpha\beta| \times V|\alpha\beta\rangle$ are evaluated by transforming into the relative coordinates, where the difficulty of the hard core of the potential can be overcome by the separation method.[4] It is useful to approximate the single-particle states with an extended version of the Nilsson[5] expansion in spherical oscillator states,

$$|\alpha\rangle \equiv |[NN_Z\Lambda]\Omega\rangle = \sum_{nlj} A_{nlj}^{[\alpha]}[\varphi_{nl}(\vec{r})\chi(\vec{\sigma})]_\Omega^j, \qquad (5)$$

in which the $A = A(\delta)$ are determined with the $\Delta N = \pm 2$ off-diagonal matrix elements of $r^2 Y_{20}$ included. If the wave functions were exact, the delta-function matrix elements would be independent of deformation.[6] We have used this fact to determine that in the expansion (5), states up to $N = 2n + l = 6$ should be included for the approximation to be reliable over the deformations of interest. We may express the two-particle states $|\alpha\beta\rangle$ in terms of the wave functions in the relative and c.m. coordinates, with the transformation coefficients

$$B_{nlS\mathcal{J}NLJ}^{[\alpha\beta]}(\delta) = \sum_{q_a q_b} (j_a \Omega_a j_b \Omega_b | JM) A_{q_a}^{[\alpha]} A_{q_b}^{[\beta]} \sum_{\mathcal{L}} (-1)^{l+L-\mathcal{L}} (n_a l_a n_b l_b | nlNL)_{\mathcal{L}}$$
$$\times ([Ll]^{\mathcal{L}} S | L[lS]^{\mathcal{J}})_J ([l_a \tfrac{1}{2}]^{j_a} [l_b \tfrac{1}{2}]^{j_b} | [l_a l_b]^{\mathcal{L}} [\tfrac{1}{2}\tfrac{1}{2}]^{S})_J, \quad (6)$$

where q_a represents the $(n_a l_a j_a)$, and the angular-momentum and Moshinsky transformations are defined as by Edmonds[7] and Brody and Moshinsky.[8] We shall consider the Kallio-Kolltveit (K-K) potential[9] which acts only in relative s states. Furthermore, we assume that the interaction does not depend upon the local density,[10] perform the sum over the c.m. quantum numbers, and express the matrix element as the sum over products,

$$\langle \alpha\beta | V_{K\text{-}K} | \alpha\beta \rangle = \sum_{T \geq |M_T|} (\tfrac{1}{2} m_{\tau_a} \tfrac{1}{2} m_{\tau_b} | T M_T)^2 \sum_{nn'S} [1 - (-1)^{S+T}] C_{nn'S}^{[\alpha\beta]} R_{nn'S}, \quad (7)$$

of the radial integrals $R_{nn'S}$ and the geometric factor

$$C_{nn'S}^{[\alpha\beta]}(\delta) = \sum_{NLJ} B_{nlS\mathcal{J}NLJ}^{[\alpha\beta]} B_{n'l'S\mathcal{J}NLJ}^{[\alpha\beta]}, \quad (8)$$

evaluated with $l = l' = 0$.

The deformation manifests itself directly in the coefficients (8), but also in the radial integrals which must be evaluated for each nucleus and deformation at the value of $\hbar\omega(\delta)$ determined by volume conservation [Eq. (4), Ref. 5]. The separation distances were determined in each case for all $n \leq 6$, and their average values used in evaluating the integrals. This refinement proved unnecessary owing to the fact that for $n \geq 2$, where the separation distances can increase rapidly with $\hbar\omega$, the factors (8) are already very small.

Representative results are illustrated in Fig. 2. The dashed curves are for the interaction $[0.865 + 0.135\sigma_1 \cdot \sigma_2]\delta(\vec{r}_1 - \vec{r}_2)$ rescaled to give the same value as the K-K force at zero deformation. The potential energies are relatively flat over the region in which the kinetic energies decrease

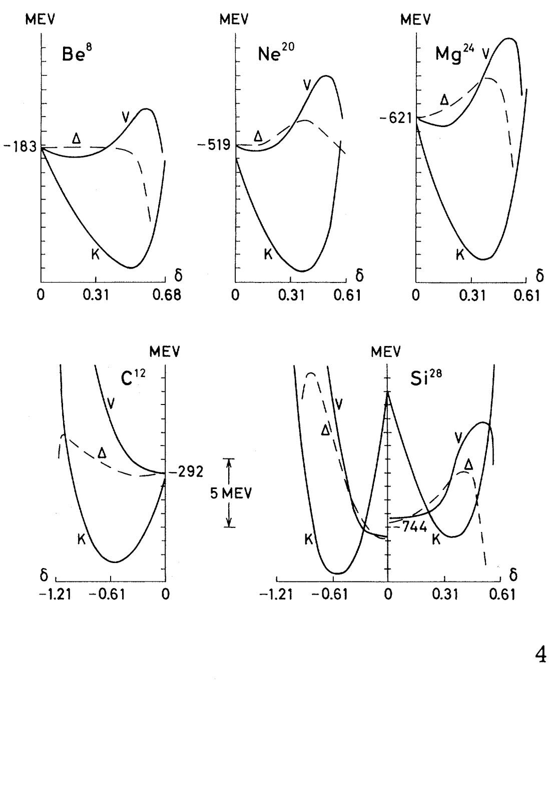

FIG. 2. Kinetic and potential energies for representative cases drawn from the $4n$ nuclei. Total-energy curves are left out to avoid cluttering, and only the magnitude of the potential energy at zero deformation is indicated. The scale is uniform with 1 MeV per division. The dashed lines represent the delta-function interaction. In Si^{28} the ground state is oblate but the total energy lies only 2 MeV lower than the prolate minimum; for the other nuclei shown here the difference exceeds 5 MeV. The sequence of occupied single-particle states differs for the oblate and prolate shapes. This leads to the difference in the potential energies at zero deformation in Si^{28}.

sharply, and then increase until the wave functions cease to be reliable.[11] The largest variation of the potential energy occurs for oblate shapes, reflecting the more rapid increase there in $\hbar\omega(\delta)$. We find that, when the kinetic energy predicts a large deformation, the effect of the potential is to decrease δ by about 0.1 and 0.15 for the prolate and oblate minima. Taking account of the similarity of the K-K curves to the deviations away from horizontal of the delta curves indicates that this is an overestimate.

The result (b) in the introduction was determined by trying different sequences of level filling. Note finally that we have not varied $\hbar\omega$ for each nucleus.

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[6]Any two-particle matrix element of the delta function is independent of deformation. To verify this, write the delta function in Cartesian coordinates and do each double integral separately, obtaining for each some constant times a scale-factor term. The product of the scale terms is constant by volume conservation.

[7]A. R. Edmonds, Angular Momentum in Quantum Mechanics (Princeton University Press, Princeton, New Jersey, 1957).

[8]Brody and Moshinsky, Ref. 5.

[9]A. Kallio and K. Kolltveit, Nucl. Phys. 53, 87 (1964).

[10]This may result in our discarding possibly important surface terms. This question is now being investigated.

[11]In O^{16} (not plotted) we find the one exception, of interest in its relation to the assumptions made in Ref. 2. Here the potential energy rises almost as fast as the kinetic, and the delta curve is very flat by comparison.

GRAVITATIONAL RADIATION*

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The results of two years of operation of a 1660-cps gravitational-wave detector are reviewed. The possibility that some gravitational signals may have been observed cannot completely be ruled out. New gravimeter-noise data enable us to place low limits on gravitational radiation in the vicinity of the earth's normal modes near one cycle per hour, implying an energy-density limit over a given detection mode smaller than that needed to provide a closed universe.

Apparatus for measurement of the Riemann tensor and search for gravitational radiation was described some years ago.[1] The space-time derivatives of the gravitational potential induce relative motion between part of an elastic body. If the Riemann tensor has Fourier components in the vicinity of elastic normal modes of quadrupole symmetry, these modes may be observed to have greater than thermal energy. Utilization of resonance improves the signal-to-noise ratio.

High-frequency detector operations. —A large aluminum cylinder has been instrumented so that its compressional mode in the vicinity of 1660 cps can be observed continuously with sensitivity limited by the thermal fluctuations.[2]