COLLISIONAL EFFECTS IN PLASMAS–DRIFT-WAVE EXPERIMENTS AND INTERPRETATION*

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We report the results of experiments on low-temperature alkali plasmas in strong magnetic fields and their interpretation in terms of collisional drift modes, in which diffusion over the transverse wavelength, resulting from ion-ion collisions,[1] plays an important role.

Collisional drift modes[2] arise in the presence of a density gradient perpendicular to the magnetic field and result from the combined effects of ion inertia,[3] electron-ion collisions, and mean electron kinetic energy along the magnetic-field lines.[4]

Our experiments determine frequencies, amplitudes, and azimuthal mode numbers of steady-state drift waves as functions of magnetic field strength and ion density. Their interpretation is based on a theory which includes the effects of ion-ion collisions on the ion motion. Although the observed relative wave amplitudes are not small, the linearized approximation is expected and found to predict correctly frequencies and the abrupt appearance of certain modes as functions of the various physical parameters (density, magnetic field, temperature, and ion mass).

The principal experimental results are the consistent observations of single-mode steady-state oscillations, which we have identified as drift modes, and, at certain critical values of the magnetic-field strength, sudden changes of both the azimuthal mode number and frequency[5] of the oscillations. We have explained these results by a theory, which predicts abrupt stabilization of a particular mode with decreasing magnetic field as a result of increased diffusion over the transverse wavelength[1] due to ion-ion collisions.[6] This type of diffusion, in fact, suppresses the instability when the ion gyroradius reaches a critical size relative to the transverse wavelength of the mode.

The experimental work has been performed on the Princeton Q-1 device.[7] The plasma consists of ions produced by surface ionization of cesium or potassium atomic beams incident on hot tungsten plates located on both ends of the plasma column and of thermionic electrons emitted from the same plates. The alkali vapor pressure is cryogenically reduced to below 10^{-8} Torr and the residual pressure is maintained at approximately 10^{-7} Torr, so that the plasma is fully ionized. The plasma column is 3 cm in diameter and 128 cm long. Plasma (center) densities ranged from 5×10^{10} to 5×10^{12} cm^{-3}, plasma (ionizer-plate) temperatures from 2100 to 2900°K, and magnetic fields from 2 to 6.5 kG. Electric fields are not applied to the plasma, and the thermionic voltage between end plates is maintained below 5 mV. The waves are detected as either ion-density or plasma-potential fluctuations with Langmuir probes. Special effort was directed towards conclusive identification of the drift wave, since it was recognized that a plasma rotation comparable with the electron diamagnetic velocity is present in Q machines for certain ranges of plasma temperature and density.[8] Our experiments differ from most previous drift-wave work in that the neutral beam was collimated to reduce the ion density at the edge of the ionizer plate where the temperature gradient is large. This procedure spatially separates the effects of temperature and density gradients.[9] The oscillations reported here are confined to the region where only the density gradient is important.

The experiments have shown that the relevant modes are localized in the radial direction with the amplitude maximum at approximately $\frac{1}{3}$ of the plasma radius. Then, for these localized modes we can carry out the theory simulating cylindrical geometry by a one-dimensional "slab" model, with density gradient in the x direction and magnetic field in the z direction. We adopt the electrostatic approximation $\vec{E}=-\nabla\varphi$ valid when $\beta=8\pi p/B^2\ll1$, as is the case here ($\beta\lesssim10^{-6}$). The time-independent electric field (experimentally found to be constant in the region where temperature gradients are negligible) is ignored because it only produces a Doppler shift in the relevant frequencies. We use the following linearized equations adopting a standard notation:

$$nm(\partial\vec{u}_{1i\perp})/\partial t-\mu_\perp\nabla_\perp{}^2\vec{u}_{1i\perp}=-\nabla_\perp p_1+(\vec{J}_1\times\vec{B}/c), \tag{1a}$$

$$-\nabla_\parallel(n_1KT_e-en\varphi_1)-\nu_{ei}nm_eu_{1e\parallel}=0, \tag{1b}$$

$$\partial n_1/\partial t+\vec{u}_{1i\perp}\cdot\nabla n+\vec{u}_i\cdot\nabla n_1=0, \tag{1c}$$

$$\partial n_1/\partial t + \vec{u}_{1e\perp}\cdot\nabla n + \vec{u}_{e\perp}\cdot\nabla n_1 + n\nabla_{\parallel}u_{1e\parallel} = 0, \tag{1d}$$

$$\vec{u}_{1e\perp} = -\nabla\varphi_1\times\vec{B}cB^{-2} + KTc(eB^2n)^{-1}\nabla n_1\times\vec{B}, \tag{1e}$$

$$J_{1\parallel} = -enu_{1e\parallel},\quad \nabla_{\perp}\cdot\vec{u}_{1i\perp} = 0,\quad \nabla\cdot\vec{J}_1 = 0,$$

$$\vec{u}_{i,e} = KTc(\mp eB^2n)^{-1}\nabla n\times\vec{B}. \tag{1f}$$

Perturbed quantities are indicated by the subscript 1. Ion motion and electron inertia along the lines of force have been ignored.[10] The effects of ion-ion diffusion across the field enter through the coefficient $\mu_{\perp}$ which is given by $\mu_{\perp} = \frac{1}{4}(nKT\nu_i/\Omega_i^2h_{\pi}^2)$ for our experimental conditions,[11] with Ω_i the ion gyrofrequency and h_{π} a dimensionless coefficient tabulated in Ref. 11. For simplicity we assume a WKB-type solution $\varphi = \varphi_1 \exp(i\int k_x dx + ik_y y + ik_{\parallel}z + i\omega t)$ and consider modes such that $k_x \gg n^{-1}(dn/dx)$. This implies mode localization in the x direction and will restrict us to modes with azimuthal mode number $m > 1$, since those with $m = 1$, equivalent to an off-axis shift of the whole plasma column, are less localized and cannot be simulated in a plane geometry. Moreover, in the experiments, $k_{\parallel} \approx \pi/L$, where L is the length of the plasma column. By expressing $\vec{J}_{1\perp}$ and $\vec{J}_{1\parallel}$ in terms of φ, one derives for $T_e = T_i$ the dispersion relation in its simplest form,

$$i(\omega + k_y v_d)\left(\frac{k_{\parallel}^2KT}{m_e\nu_{ei}} - \frac{\nu_{ii}b^2}{4}\right) = \left(\omega - k_y v_d - \frac{i\nu_{ii}b}{2}\right)b\omega, \tag{2}$$

where $v_d = -(1/n)(dn/dx)(cKT/eB)$ is the electron diamagnetic velocity, $b = \frac{1}{2}(k_x^2 + k_y^2)a_L^2$ is assumed to be smaller than unity, $a_L = (2KT/M)^{1/2}/\Omega_i$ is the ion Larmor radius, and $\nu_{ei} = \frac{1}{2}(M/m_e)^{1/2}\nu_{ii}$, taking for ν_{ii} the definition of Ref. 11. Here we have neglected terms of order $k_{\parallel}^2KT(m_e\nu_{ei}k_yv_d)^{-1}$ in comparison with 1. This dispersion relation is a quadratic equation whose roots are easily evaluated numerically, and reveals two important points. First, there exists a critical value of b given by $b_c^2 = 4k_{\parallel}^2KT(m_e\nu_{ei}\nu_{ii})^{-1}$ such that the linearized growth rate is positive only for $b < b_c$. In connection with this, we display in Fig. 1 some experimental results for fixed neutral-beam flux and plasma temperature, i.e., for approximately constant ion density. Note that for most values of magnetic field only one single mode is detected, but in the mode transition regions two separate modes are observed. The rapid rise of $\gamma = -\mathrm{Im}(\omega)$ with increasing magnetic field, once the condition $b < b_c$ is satisfied, is evident. The second point is that the maximum growth rate is found to correspond to frequencies $\mathrm{Re}(\omega) \approx -0.5k_yv_d$. In particular, we find that for $b < b_c$, the growth rate γ is maximized for the dimensionless parameter $\Sigma_0 = k_{\parallel}^2KT \times (m_e\nu_{ei}k_yv_db)^{-1}$ slightly above unity, m_e being the electron mass. At this point, the magnitude of γ is $\sim 0.2k_yv_d$ and is comparable with the instability frequency. Figure 1(b) shows that the observed frequencies are proportion-

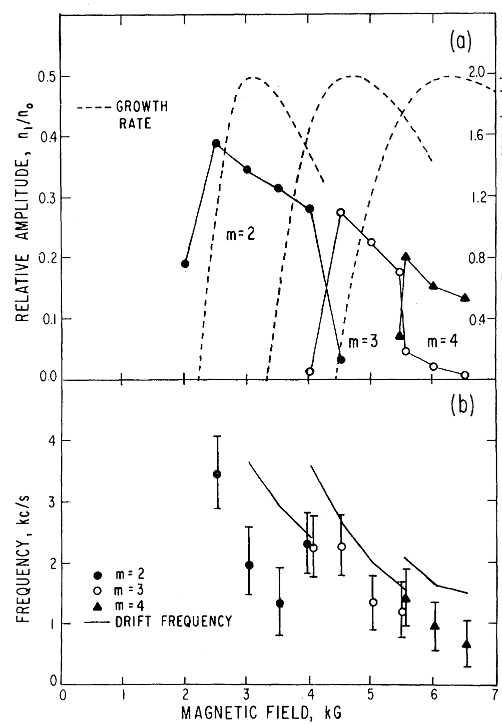

FIG. 1. (a) Observed oscillation amplitudes are compared with theoretical growth rates as a function of magnetic field strength for various azimuthal mode numbers. The absolute value of the magnetic field strength for the theoretical (slab model) curves has been scaled by a factor of ~1.5 to give a good fit to the data. The relative amplitude is defined as the ratio of the maximum density fluctuation to the central density. (b) The oscillation frequency (after subtraction of the rotational Doppler shift) is compared with the drift frequency $\nu_d = k_yv_d/2\pi$ as a function of the magnetic field strength. The drift frequency, which has an uncertainty of ± 0.5 kc/sec, is computed from the experimental values of k_y, T, and $n^{-1}(dn/dx)$. The data are for a potassium plasma, $n_0 = 3.5\times10^{11}$ cm^{-3}, $T = 2800$°K.

al to, but less than, $k_y v_d$. These considerations indicate that the criterion[12] $b = b_c$, which can be written as

$$\frac{(k_x^{\ 2}+k_y^{\ 2})^{1/2}}{B} = \frac{k_\perp}{B} = \left(\frac{4e^4 k_\parallel^{\ 2}}{M^2c^4KTm_e\nu_{ei}\nu_{ii}}\right)^{1/4} \propto \left(\frac{T}{n}\right)^{1/2}\frac{1}{M^{3/8}}, \qquad (3)$$

describes the onset of the single modes. In Fig. 2(a) we plot the experimental values at instability onset of $B/k_\perp$ vs $n^{1/2}$ and find agreement between the experiments and this theoretical prediction, including the numerical coefficient. In addition, other measurements have shown dependence on plasma temperature and ion mass as predicted by the transition criterion, Eq. (3). In Fig. 2(b), the radial extent of the oscillation is shown as a function of magnetic field. The extent of the oscillation in the radial direction was found to increase for decreasing values of m, as expected from the full analysis of the normal-mode equation and corresponding to $(\partial/\partial x) \sim k_y$. The position of the amplitude maximum does not coincide with the position of maximum density gradient and may be determined by the radial dependence of the growth rate, taking into account both the variations of n and dn/dx.

It is evident from our treatment that the linearized approximation is inadequate to explain the large experimental amplitudes (typically $n_1/n_0 \sim e\varphi/KT \sim 20\,\%$) at which the growth of the waves ceases. However, there are good arguments[13] for predicting that the saturation stage is reached when the perturbed $(\vec{E}\times\vec{B})c/B^2$ drift velocity becomes of the order of the diamagnetic velocity (in our case $\sim 2\times10^3$ cm/sec). This point is confirmed by the experiment and, in addition, it is shown [see Fig. 1(a)] that the pattern of the measured amplitudes follows closely that of the calculated growth rates as a function of the magnetic field. If we consider this as an indication that at the saturation stage the amplitude is proportional to a power of the growth rate,[14] we can associate the observed frequency with that of the maximum growth rate.[15]

On the other hand, if we make use of a quasilinear approximation[16] in treating the problem and include higher order terms in the dispersion relation (see Ref. 12), we observe that at the saturation level the frequency of oscil-

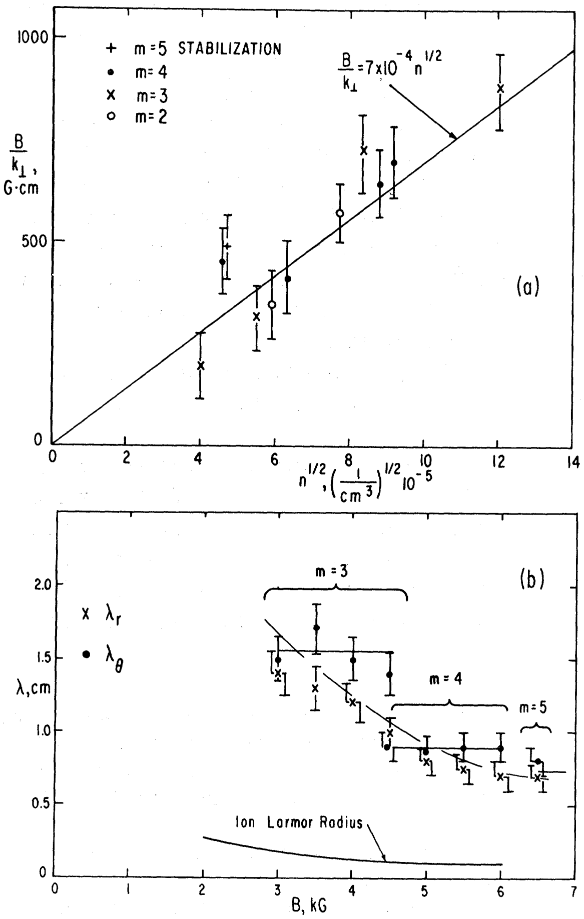

FIG. 2. (a) The ratio of magnetic field strength to perpendicular wave number is plotted versus the square root of the density for the stabilization points of several modes. Theory [Eq. (3)] gives a proportionality factor of 9.7×10^{-4}. (b) The measured radial (λ_r) and azimuthal (λ_θ) wavelengths of the perturbation are displayed as a function of the magnetic field.

lation is $|\omega| \lesssim k_y v_d$. In this connection we notice that more detailed measurements than those given in Fig. 1(b) have shown $\omega/k_y v_d \approx -\frac{1}{2}$.

The observation of one single mode at a given magnetic field is explained, within the limits of the linearized theory, by the fact that only one mode has appreciable growth rate outside the mode-transition regions.

Several important considerations of general nature arise from this work. First, the agreement of the theoretically predicted frequencies with observations, coupled with the fact that the mode amplitudes maximize when $\Sigma_0 \approx 1$, implies that the large growth rates given by the linearized approximation $\gamma \approx 0.2k_y v_d \sim \mathrm{Re}(\omega)$

can occur at proper values of density and magnetic field even though the transverse wavelength is larger than the ion gyroradius. This feature is not shown by the collisionless drift waves and can make the collisional ones suitable to explain anomalous Bohm-type diffusion.[13] Secondly, the role of ion-ion collisions, which can stabilize collisional interchange modes in plasma confinement configurations having magnetic shear,[1,17] has been demonstrated experimentally in this simple geometry in agreement with the theoretical prediction.

Additional experimental and theoretical data concerning this interpretation will be reported elsewhere.

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[9]In the region of large temperature gradient, at the edge of the ionizer plate, at least two effects complicate drift-wave measurements: the possibility of excitation of temperature-gradient drift waves propagating with the ion diamagnetic velocity and high, nonuniform, plasma rotation caused by a varying radial electric field. Because of this latter difficulty, identification of drift waves as either density or temperature gradient waves appears to be greatly hindered. In our experiments, the direction of the steady-state $\vec{E}\times\vec{B}$ rotation is the same as that of the electron diamagnetic velocity.

[10]We have derived the dispersion relation by different methods such as that given in Ref. 1, or using a guiding center description. The one adopted here has the advantage of being easily modified to include corrections associated with the ratio of ν_{ii}/Ω_i by using the pressure tensor given in this reference. In Eq. (1), we have omitted the terms $(\vec{u}\cdot\nabla\vec{u})_1+\nabla\cdot\pi_1$ due to the ion diamagnetic velocity in equilibrium and to the pressure tensor term π_1 associated with finite Larmor-radius effects, because their contribution to $\nabla\cdot\vec{J}_1$ cancels exactly [see B. Coppi, Phys. Rev. Letters 12, 417 (1964)].

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[12]We have investigated analytically and numerically corrections to the dispersion relation Eq. (2) resulting from higher order terms in b. These corrections do not qualitatively affect Eq. (3) provided the parameter $\alpha = k_\parallel{}^2(d\ln n/dx)^{-2}(M/m_e)^{1/2}$ is less than unity. However, for values of α near unity, the maximum growth rate is substantially reduced and the frequency at maximum growth rate is closer to $-k_y v_d$. For our experiment, $\alpha\approx\frac{1}{2}$.

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[15]We note that the experimental mode-amplitude behavior is in agreement with the theoretical growth rates given here, and is not compatible with expressions showing a monotonic increase with $k_\perp/B$. [F. F. Chen, Phys. Fluids 8, 1323 (1965).]

[16]This is a result of work by C. R. Oberman and one of us (B.C.).

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