While it may be argued that these lower energies are not high enough for the application of Regge theory, even in the energy range of this experiment where an excellent trajectory is obtained, Eq. (1) by itself does not provide a fully satisfactory explanation of the data.

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[1]Preliminary results have been reported by D. O. Caldwell, J. P. Dowd, K. Heinloth, and T. R. Sherwood, in International Symposium on Electron and Photon Interactions at High Energies. Hamburg, 1965 (Springer Verlag, Berlin, Germany, 1965), Vol. 2, p. 178; and D. O. Caldwell, J. P. Dowd, and T. R. Sherwood, Bull. Am. Phys. Soc. 10, 701 (1965).

[2]Details may be found in D. O. Caldwell, J. P. Dowd, K. Heinloth, and M. D. Rousseau, Rev. Sci. Instr. 36, 283 (1965).

[3]The exact laboratory acceptance of the apparatus is given by J. P. Dowd, dissertation, Massachusetts Institute of Technology, 1966 (unpublished).

[4]Experiments fail to show evidence for pion exchange in ρ^0 photoproduction or for the decay $\rho^0 \rightarrow \gamma + \pi^0$; for a discussion of this point see S. D. Drell, in International Symposium on Electron and Photon Interactions at High Energies. Hamburg, 1965 (Springer Verlag, Berlin, Germany, 1965), Vol. 1, p. 71; or A. Krass, Stanford Linear Accelerator Center Technical Note No. TN-65-76, 1965 (unpublished). If A parity holds, this vertex is forbidden and only a π meson can be exchanged in the reaction observed; for a discussion of A parity, see J. B. Bronzan and F. E. Low, Phys. Rev. Letters 12, 522 (1964).

[5]K. Schilling, Deutsches Elecktronen-Synchrotron Report No. DESY 66/9, 1966 (unpublished).

[6]G. Zweig, Nuovo Cimento 32, 659 (1964), and private communication; the factor of t^2 was inadvertently omitted in the paper, and a factor $1/\Gamma^2(\alpha+1)$ has been included here in accordance with more recent work.

[7]G. Kramer and P. Stichel, Z. Physik 178, 519 (1964), and private communication.

[8]V. B. Elings, K. J. Cohen, D. A. Garelick, S. Homma, R. A. Lewis, P. D. Luckey, and L. S. Osborne, Phys. Rev. Letters 16, 474 (1966).

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SCATTERING LENGTHS IN THE QUARK MODEL

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A set of predictions concerning scattering lengths is obtained by means of the quark model. The results, however, hold also for some form of U(6)⊗U(6) invariance at rest.

We wish to present a remarkably well-satisfied set of relations between a large number of scattering lengths of nonresonant hadronic amplitudes. These were first obtained from a quark model with single rearrangement, but other derivations also lead to the same results.

Consider s-wave scattering between strongly interacting particles. The basic formula is simply derived from the well-known equation for low-energy scattering: $-k\cot\delta = a$. Since we are considering scattering from bound quarks, we compare different quark amplitudes taken at the same velocity. Such an approach was first pointed out by Fermi[1] and leads to the relation

$$a = -(1/k)\tan\delta = -m_\gamma \beta, \qquad (1)$$

where m_γ is the reduced mass of the system undergoing scattering and β contains the dynamical information due to the forces. In the rearrangement model[2] we consider a permutation of two quarks, one in each hadron, without changing any of the internal quantum numbers, including spin. The transition amplitude is assumed to be proportional to the number of such single permutations which transform the initial state into the final state. If no such permutation is possible, the amplitude is 0. The rearrangement hypothesis assumes that all possible configurations carry equal weight; hence we cannot apply this method to channels dominated by low-lying s-wave resonances. We therefore exclude the nucleon-nucleon system in both isospin channels, the $(\bar{K}N)_{I=1}$ because of the Dalitz-Tuan resonance, and the $(\Xi N)_{I=0}{}^{S=0}$,[3] and we keep the $(\pi\pi)_{I=0}$ since it is not yet established whether or not a resonance exists in this state.

Consider first meson-baryon scattering. Because the antiquark must remain in the meson, the only possible rearrangement is an interchange of the quark of the meson with one of the three quarks of the baryon. Since the $\bar{K}$ contains only a λ quark which cannot rearrange, we obtain $\beta_I{}^{\bar{K}N}=0$ (the upper indices describe the particle channel and the lower, the isospin).[4] Also $\beta_{3/2}{}^{\pi N}=\beta_1{}^{KN}$, since both cases involve the same rearrangement of a p quark with the quarks in

Table I. Comparison of theory and experiment.

Process	β	m_γ/m_π	a	Experiment	References
$(\pi N)_{I=3/2}$	1	1.21	-0.088[a]	-0.088 ± 0.004	b
$(\pi N)_{I=1/2}$	1/4	1.21	-0.022	0.171 ± 0.008	b
$(KN)_{I=0}$	0	3.24	0	-0.078 ± 0.04 0.03 ± 0.03	c d
$\mathrm{Re}(\bar{K}N)_{I=1}$	0	3.24	0	0.004 ± 0.002	e
$(KN)_{I=1}$	1	3.24	-0.23	-0.22 ± 0.01	f
$(\pi\Sigma)_{I=2}$	1	1.31	-0.096		
$(\Lambda N)^{S=1}$	3	5.13	-1.12	-1.26	g
$(\Lambda N)^{S=0}$	3	5.13	-1.12	-1.41	g
$(\Sigma N)_{I=3/2}{}^{S=1}$	31/9	5.24	-1.28		h
$(\Sigma N)_{I=3/2}{}^{S=0}$	-1	5.24	$+0.38$		
$(\Sigma N)_{I=1/2}{}^{S=1}$	$-5/9$	5.24	$+0.21$		
$(\Sigma N)_{I=1/2}{}^{S=0}$	3	5.24	-1.13		
$(\Xi N)_{I=1}{}^{S=1}$	7/9	5.48	-0.30		
$(\Xi N)_{I=1}{}^{S=0}$	5/3	5.48	-0.65		
$(\Xi N)_{I=0}{}^{S=1}$	1/3	5.48	-0.13		
$(\pi\pi)_{I=2}$	1	0.70	-0.06	-0.06	i
$(\pi\pi)_{I=0}$	$-5/3$	0.70	$+0.11$	0.20	i
$(\pi K)_{I=3/2}$	1	1.09	-0.08		

[a]This is our fitted value.
[b]J. Hamilton and W. S. Woolcock, Rev. Mod. Phys. 35, 737 (1963).
[c]R. Chand, to be published.
[d]V. J. Stengler et al., Phys. Rev. 134, B1111 (1964).
[e]J. A. Kadyk, Yona Oren, Gerson Goldhaber, Sulamith Goldhaber, and George H. Trilling, Phys. Rev. Letters 17, 599 (1966); J. K. Kim, Phys. Rev. Letters 14, 29 (1965).
[f]S. Goldhaber et al., Phys. Rev. Letters 9, 135 (1962).
[g]G. Alexander et al., Phys. Letters 19, 715 (1966). These numbers are now probably smaller. G. Alexander et al., to be published. We thank G. Alexander for discussions on the experimental situation. No errors are quoted in the papers because they are strongly correlated.
[h]There are some results on ΣN but they are still preliminary. See V. Hepp, thesis, University of Heidelberg, 1966 (unpublished).
[i]S. Weinberg, Phys. Rev. Letters 17, 616 (1966). Only the small $I=2$ scattering length seems to have experimental backing. The number quoted for $I=0$ is the current algebra prediction.

a physical proton. The other cases require some calculation, but the principle is the same. The baryon-baryon case is somewhat more complicated because of the spin couplings and because all the quarks in each baryon can participate in the rearrangement. We get $\beta^{\Lambda N} = 3\beta_{3/2}{}^{\pi N}$ in contrast to the ratio $\frac{3}{2}$ obtained for asymptotic high-energy scattering by "quark counting," because here only quarks participate and the antiquark is not counted. For meson-meson scattering the calculation is very simple and antiquark rearrangements must be counted, too.

An equivalent formulation that leads to the same results is obtained from the same additivity assumption that leads to the high-energy scattering results. The use of SU(3) for the two-body quark amplitudes leads to relations analogous to those in the high-energy case.[5] The rearrangement predictions satisfy these relations, but are stronger. They are obtained in the additivity model by making the additional assumption of universal $U(6)\otimes U(6)$ exchange which expresses all two-body quark amplitudes in terms of a single parameter. The amplitudes are obtained by exchange of 72 boson states classified in the adjoint representation of $U(6) \otimes U(6)$ and universally coupled to the quarks; i.e., the couplings are proportional to the matrix elements of the corresponding $U(6)\otimes U(6)$ generator.[6] It can easily be seen that in this scheme there is no quark-antiquark scattering and that the results coincide with the ones obtained by one-body rearrangement. The results are displayed in Table I and the agreement is excellent with the only exception of $(\pi N)_{1/2}$. It is perhaps worth emphasizing the strong dependence on kinematics in the low-energy region as compared with the corresponding high-energy case.[7] Besides the agreement with the measured quantities there is a large body of predictions that can be tested soon. For example, though many baryon-baryon scattering lengths are one order of magnitude larger than in the meson-baryon case, the $(\Xi N)_{I=0}{}^{S=1}$ is of the same order of magnitude as $(\pi N)_{I=\frac{3}{2}}$. This comes about because of the inability of the λ quark to rearrange and because of particular combination of spin couplings. Notice also that the $\frac{3}{2}$ ratio of high-energy cross sections goes down to about 1/200, in agreement with experiment.[8] However, all these cross sections are well below their unitarity limits giving us confidence that we are far from resonances.

Our results are compatible with those calculated from current algebra.[9] However, current algebra has not been able to calculate low-energy baryon-baryon processes. It is important to remember that most high-energy scattering results have been also obtained from current algebra as well.[10] Moreover, our second approach described in this note requires only $U(6)\otimes U(6)$ symmetry[11] for classification of states and couplings at rest and it might be considered a hint that these results can be obtained from current algebra without any need for quarks.

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[1]See, for example, any book dealing with scattering of bound neutrons at low energy.

[2]Rearrangement was introduced by H. R. Rubinstein and H. Stern, Phys. Letters 21, 447 (1966); notice however, that in that problem, nucleon-antinucleon annihilation, there is total and not one-body rearrangement.

[3]B. Diu and H. R. Rubinstein, Nuovo Cimento 38, 831 (1965).

[4]We refer to the real part of the scattering length when dealing with exotermic channels. Also, we use the $\mathcal{P}$, $\mathfrak{N}$, and λ quark notation for the fundamental triplet.

[5]H. J. Lipkin and F. Scheck, Phys. Rev. Letters 16, 71 (1966). We obtain Eqs. (3a)-(3c).

[6]A general description of this elegant technique has been obtained by H. J. Lipkin, to be published.

[7]This is due to the strong energy dependence of the low-energy scattering as compared with the flat dependence of the high-energy cross sections. See P. B. James and H. D. D. Watson, Phys. Rev. Letters 18, 179 (1967).

[8]Independent of this model, it is a striking physical fact that there is such an enormous range of values for cross sections in nonresonant cases.

[9]See, for example, Y. Tomozawa, Nuovo Cimento 46A, 707 (1966). The K-meson current-algebra results are not very good, but this is probably due to the use of the hypothesis of partially conserved axial-vector currents for K mesons.

[10]These results were derived by N. Cabibbo, L. Horwitz, and Y. Ne'eman, Phys. Letters 22, 336 (1966), assuming an algebra for Regge residues. They have been recently derived under much weaker assumptions by H. R. Rubinstein and G. Veneziano, to be published.

[11]R. Dashen and M. Gell-Mann, Phys. Letters 17, 142 (1965).