of oxygen is burnt. This amount of energy is just enough to expand the star and produce oscillations around a dynamically stable configuration. The oscillations decay and the star evolves slowly once again towards a dynamically unstable state. The process is repeated again and again until oxygen is almost completely consumed from the central regions of the star. The investigation of these relaxation oscillations and the further evolution of $30M_\odot$ stars are in progress.

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SUPERCONVERGENCE RELATIONS IN THE PRESENCE OF REGGE CUTS*

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(Received 17 February 1967)

Recently several authors[1] have exploited superconvergence relations (SCR) to obtain relations between the masses and partial widths of the hadrons. The concept of superconvergence, which was first pointed out by de Alfaro, Fubini, Rossetti, and Furlan,[2] has as its basis the fact that an amplitude A which is odd under crossing symmetry and which satisfies

$$A(s,t) \lesssim s^{-1-\epsilon}, \quad \epsilon > 0, \tag{1}$$

and

$$A(s,t) = \frac{1}{\pi}\int \frac{A_s(s',t)ds'}{s'-s} + \frac{1}{\pi}\int \frac{A_u(s',t)ds'}{s'-u} \tag{2}$$

will then satisfy

$$\int A_s(s',t)ds' = 0. \tag{3}$$

In Eqs. (2) and (3), s, u, and t are the usual Mandelstam variables and $A_{s,u}(s,t)$ are the absorptive parts with respect to the s and u reactions. We have used the $s \leftrightarrow u$ crossing relation $A_s = -A_u$ in obtaining Eq. (3).

The asymptotic bound in Eq. (1) is obtained with the assumption of the Regge asymptotic behavior; the power of s, in Eq. (1), would be given by $\alpha - n$ (α is the Regge trajectory in question and n is determined by the helicities in the t reaction). In channels where the helicity is not sufficiently large, high isospin will be helpful in obtaining the asymptotic behavior in Eq. (1). Since there are no $I=2$ Regge trajectories with $\alpha_{I=2} > 0$, Eq. (1) is valid for a process where there is at least 1 unit of helicity flip in the t channel ($I=2$).

In this note we would like to point out that even though there is no evidence for $I=2$ Regge trajectories with $\alpha_{I=2} > 0$, there are Regge cuts with $I=2$ that will generally invalidate the $I=2$ superconvergence relations. It will be shown that there exists cuts with the property that $\alpha_{\rm cut} > 0$.

Convincing arguments for the existence of cuts in the angular momentum plane have been advanced by Mandelstam.[3] The leading cut for an $I=2$ amplitude would arise from the exchange in the t channel of two $I=1$ Regge trajectories, Fig. 1. Of the known hadron trajectories the $I=1$ trajectory α_ρ with the quantum numbers of the ρ meson would be the highest. The Regge cut coming from the two-ρ intermediate state is necessary to mask an essential

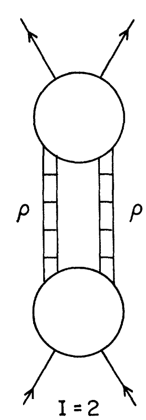

FIG. 1. Feynman diagram with the $I=2$ Regge cut.

singularity at the wrong signature point $J=1$ in the reaction $\pi+\pi\to\rho+\rho$. This cut would affect any process that is coupled by unitarity to the two-ρ system. Particular processes where this $I=2$ cut should be present are $\pi^-+\Sigma^+\to\pi^++\Sigma^-$, $\pi^-+p\to\pi^++N^{*-}$, and $\pi^-+\rho^+\to\pi^++\rho^-$. As an example we will discuss pseudoscalar-baryon scattering where the amplitude $h_{+-}(s,t)/\sin\psi$ (ψ is the c.m. scattering angle and h_{+-} is the helicity-flip amplitude of the t reaction) is a favorable candidate for an $I=2$ superconvergence relation.[4]

The asymptotic behavior of $h_{+-}(s,t)/\sin\psi$ in the presence of the $I=2$ Regge cut, Fig. 1, would be given by[3,5]

$$h_{+-}(s,t)/\sin\psi \xrightarrow[s\to\infty]{} b\exp\{[2\alpha_\rho(\tfrac{1}{4}t)-2]\ln s\}/\ln s, \quad (4)$$

where b is related to the discontinuity of the cut. At $t=0$ we have the following three possibilities:

$$\text{for } \alpha_\rho(0)<\tfrac{1}{2},\quad h_{+-}/\sin\psi\to bs^{-1-\epsilon}/\ln s;$$

$$\text{for } \alpha_\rho(0)=\tfrac{1}{2},\quad h_{+-}/\sin\psi\to b/s\ln s;$$

$$\text{for } \alpha_\rho(0)>\tfrac{1}{2},\quad h_{+-}/\sin\psi\to bs^{-1+\epsilon}/\ln s. \quad (5)$$

Therefore the existence of the $I=2$ SCR will depend crucially upon the value of α_ρ. Present analysis of high-energy π^-p charge-exchange scattering[6] indicates a value of $\alpha_\rho(0)\approx\frac{1}{2}$ with the limits $0.45\lesssim\alpha_\rho(0)\lesssim0.60$. Even if $\alpha_\rho(0)$ is slightly less than $\frac{1}{2}$, the high-energy contribution to Eq. (3) will be large and it is doubtful that $I=2$ superconvergence relations can be saturated by a few low-lying resonances of the direct channel.

In conclusion we would like to point out that an overwhelming success of $I=2$ SCR would perhaps indicate that $\alpha_\rho(0)<\frac{1}{2}$ and that b is negligible.

The author would like to thank Professor F. E. Low, Professor C. E. Jones, and Professor V. L. Teplitz for conversations on the subject of superconvergence.

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*This work is supported in part througn funds provided by the Atomic Energy Commission under Contract No. AT(30-1)2098.

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