[12]The experiments in which the decay $K_L^0 \to 2\mu$ has been searched for [see e.g., X. De Bouard, D. Dekkers, S. Jordan, R. Mermod, T. R. Willits, K. Winter, P. Scharff, L. Valentin, M. Vivargent, and M. Bott-Badenhausen, Phys. Letters 15, 58 (1965)] are not conclusive with respect to b.l. with $m_{b_\mu} \sim m_\pi$ because of the specific mass conditions.

[13]V. L. Fitch, R. F. Roth, J. S. Russ, and W. Vernon, Phys. Rev. Letters 15, 73 (1965).

[14]U. Camerini, D. Cline, C. Gidal, G. Kalmus, and A. Kerman, Nuovo Cimento 37, 1795 (1965).

[15]The upper limit of the branching ratio $(\pi \to b + \nu_b)/(\pi \to \mu + \nu)$ could again be considered a few percent, if the π_{e_2} experiment is taken into account; cf. however Ref. 8.

[16]H. Hulubei, E. M. Friedländer, R. Niţu, T. Visky, and J. S. Ausländer, Phys. Rev. 139, B729 (1965). In this experiment no filter was used and no asymmetry effect was found. Perhaps this could also explain some of the negative results of other experiments quoted in Ref. 9.

[17]With a path length of 700 cm and a Cu filter of 17 cm thickness as used in Ref. 9, a branching ratio $(\pi \to b + \nu_b)/(\pi \to \mu + \nu)$ of 1.5% could be sufficient to yield for a beam of 300-MeV pions an admixture of 2% b.l. (Ref. 10).

[18]E. M. Friedländer, private communication; and to be published.

[19]W. Z. Osborne, Nuovo Cimento 41A, 389 (1966).

[20]For the present the situation is rather confused. An analysis of this subject can be found in Ref. 9.

[21]Uncontrolled depolarization factors can cancel the asymmetry. This may partially or totally explain some contradictory results on this effect, in particular experiments (Ref. 5).

[22]A different interpretation of the π-μ effect is given in a paper by L. Banyai, N. Marinescu, V. Rittenberg, and R. M. Weiner, to be published.

DYNAMICS OF SUPERNOVA EXPLOSION RESULTING FROM PAIR FORMATION

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(Received 4 January 1967)

The supernova explosion of a star is usually believed to result from a catastrophic implosion which reverses into an explosion. A mechanism for an instability which could lead to such a catastrophic implosion has long ago been suggested and investigated.[1,2] This instability is assumed to occur when the temperature of an iron core of a star rises to $(5\text{-}6)\times10^9$ °K and the iron begins to dissociate into alpha particles and neutrons. Subsequently, it has been pointed out by Colgate[3] that the reversal of the motion of a strong implosion poses severe problems. In this paper we report on dynamical calculations of explosions triggered by an alternative mechanism, namely, the dynamical instability caused by pair formation. This instability occurs in heavy stars prior to the formation of any elements heavier than oxygen. The implosion caused by this mechanism is easily reversed by the oxygen burning. The explosion following the implosion disrupts the star, shedding out oxygen and its direct burning products (elements in the Mg-Si-S range) and all lighter elements which may be present in the envelope. No iron-peak elements are formed during this explosion.

The mechanism of pair formation has been considered by several authors[2,4] but was believed to be inadequate to result in any catastrophic event.

Evolutionary calculations have shown that stars heavier than $(20\text{-}30)M_\odot$ become dynamically unstable because of pair formation.[5] The instability sets in when the central temperature rises to $(1.5\text{-}2.2)\times10^9$ °K, the lower temperature corresponding to the heavier stars. The evolutionary calculations of several models have been continued beyond the dynamical instability by detailed calculations of the stellar hydrodynamics. During evolution the star contracts slowly preserving hydrostatic equilibrium. After the occurence of the instability the contraction is accelerated. Since in the heavier stars oxygen is burning very slowly at the moment they become dynamically unstable, the contraction gathers sufficient momentum so that, before the motion is reversed, the rate of nuclear reactions increases by many orders of magnitude and an energy corresponding to the consumption of several solar masses of oxygen is released. This is despite the fact that during the collapse the unbalance between the gravitational forces and the thermal pressure gradients is only about 1% and thus induces very small accelerations.

The behavior of a stellar model of 40 solar masses composed of oxygen is shown in Figs. 1 and 2. The evolution was started with an isen-

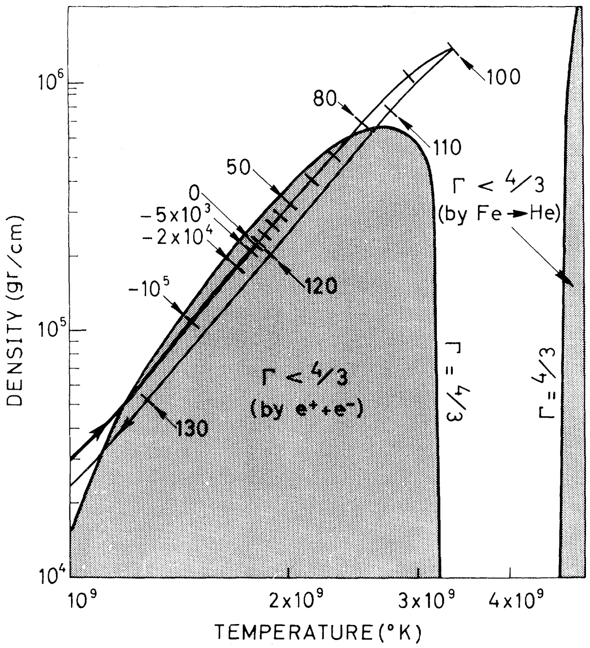

FIG. 1. Track in the temperature-density plane of the center of a $40M_\odot$ model. The evolution up to the dynamical instability is presented by a heavy line. The subsequent stages of implosion and explosion are presented by a thin line. The shaded areas show regions in which $\Gamma = (d\ln p/d\ln\rho)_{\text{adiabatic}} < \frac{4}{3}$. The numbers indicated on the evolutionary track give the time in seconds relative to the moment at which the dynamical instability appears.

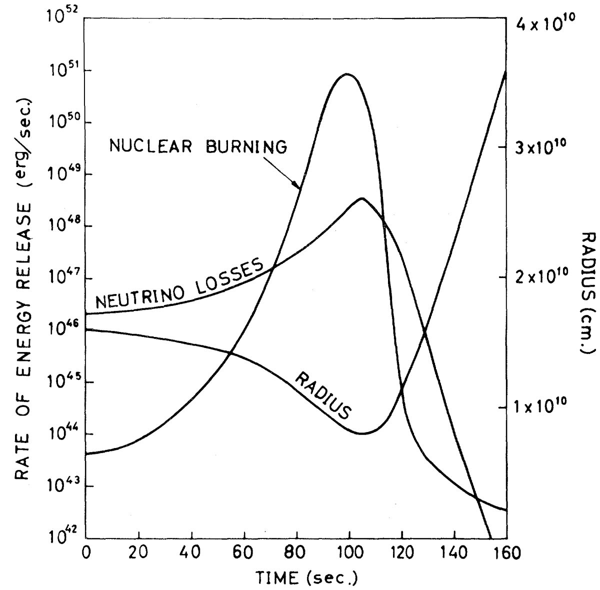

FIG. 2. Energy release and radius as function of time. The time is given in seconds relative to the moment at which the dynamical instability appears.

tropic model which had a central temperature of 10^8 °K. It is found that if neutrino losses are disregarded the evolution remains unchanged, but its rate becomes much slower. Admixture of carbon into the oxygen has no effect on the evolution after the central temperature T_{center} exceeds 10^9 °K.[6] Also, it should be stressed that rather massive dilute envelopes of H and He may surround the oxygen core. These envelopes have no appreciable effect on the evolution of the central regions of the stars. Therefore, it seems that a choice of a pure oxygen model should not invalidate our main conclusions. However our oxygen model may correspond in reality to a somewhat heavier star.

The $40M_\odot$ model becomes dynamically unstable at $T_{\text{center}} = 1.8\times10^9$ °K. The implosion heats up the center to 3.2×10^9 °K and about 4 masses of oxygen are burnt before the motion is reversed. In total, approximately 6 solar masses are burnt which release $\sim 6\times10^{51}$ erg, more than enough to disrupt the star. The above figures are obtained if one assumes that during the short time of the explosion (~10 sec) convection produces little mixing. If one assumes that the mixing by convection is fast enough to be considered as instantaneous, the explosion is somewhat stronger, burning $12M_\odot$ of oxygen.

The effect of outer layers of lighter nuclear fuels on the explosion has been checked qualitatively by adding a mantle of $5M_\odot$ of helium onto a core of $35M_\odot$ of oxygen. The over-all behavior remained as in the case of a pure oxygen star of $40M_\odot$.

The end result of the dynamical calculations is that the star is completely dispersed into space, the gases moving with velocities up to several thousand kilometers per second (the velocity of the outer layers being 5000-8000 km/sec).

Stars lighter than $30M_\odot$ evolve up to the Fe-He conversion zone without becoming dynamically unstable as a result of pair formation.[6] In the case of a $30M_\odot$ oxygen model, which is a kind of borderline case, some complications arise. The instability sets in at a higher temperature than in the $40M_\odot$ case (approximately at $T_{\text{center}} = 2.2\times10^9$ °K. At this higher temperature oxygen burns quite fast and the mild implosion following the dynamical instability is reversed after only about 1 solar mass of oxygen is burnt. This amount of energy is just enough to expand the star and produce oscillations around a dynamically stable configuration. The oscillations decay and the star evolves slowly once again towards a dynamically unstable state. The process is repeated again and again until oxygen is almost completely consumed from the central regions of the star. The investigation of these relaxation oscillations and the further evolution of $30M_\odot$ stars are in progress.

[1]E. M. Burbidge, G. R. Burbidge, W. E. Fowler, and F. Hoyle, Rev. Mod. Phys. 29, 547 (1957).

[2]W. E. Fowler and F. Hoyle, J. Appl. Phys. Suppl. 9, 201 (1964).

[3]S. A. Colgate and R. H. White, J. Appl. Phys. 143, 626 (1966).

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[5]G. Rakavy and G. Shaviv, to be published.

[6]G. Rakavy, G. Shaviv, and Z. Zinamon, unpublished.

SUPERCONVERGENCE RELATIONS IN THE PRESENCE OF REGGE CUTS*

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Recently several authors[1] have exploited superconvergence relations (SCR) to obtain relations between the masses and partial widths of the hadrons. The concept of superconvergence, which was first pointed out by de Alfaro, Fubini, Rossetti, and Furlan,[2] has as its basis the fact that an amplitude A which is odd under crossing symmetry and which satisfies

$$A(s,t) \lesssim s^{-1-\epsilon}, \quad \epsilon > 0, \tag{1}$$

and

$$A(s,t) = \frac{1}{\pi}\int \frac{A_s(s',t)ds'}{s'-s} + \frac{1}{\pi}\int \frac{A_u(s',t)ds'}{s'-u} \tag{2}$$

will then satisfy

$$\int A_s(s',t)ds' = 0. \tag{3}$$

In Eqs. (2) and (3), s, u, and t are the usual Mandelstam variables and $A_{s,u}(s,t)$ are the absorptive parts with respect to the s and u reactions. We have used the $s \leftrightarrow u$ crossing relation $A_s = -A_u$ in obtaining Eq. (3).

The asymptotic bound in Eq. (1) is obtained with the assumption of the Regge asymptotic behavior; the power of s, in Eq. (1), would be given by $\alpha - n$ (α is the Regge trajectory in question and n is determined by the helicities in the t reaction). In channels where the helicity is not sufficiently large, high isospin will be helpful in obtaining the asymptotic behavior in Eq. (1). Since there are no $I=2$ Regge trajectories with $\alpha_{I=2} > 0$, Eq. (1) is valid for a process where there is at least 1 unit of helicity flip in the t channel ($I=2$).

In this note we would like to point out that even though there is no evidence for $I=2$ Regge trajectories with $\alpha_{I=2} > 0$, there are Regge cuts with $I=2$ that will generally invalidate the $I=2$ superconvergence relations. It will be shown that there exists cuts with the property that $\alpha_{\text{cut}} > 0$.

Convincing arguments for the existence of cuts in the angular momentum plane have been advanced by Mandelstam.[3] The leading cut for an $I=2$ amplitude would arise from the exchange in the t channel of two $I=1$ Regge trajectories, Fig. 1. Of the known hadron trajectories the $I=1$ trajectory α_ρ with the quantum numbers of the ρ meson would be the highest. The Regge cut coming from the two-ρ intermediate state is necessary to mask an essential

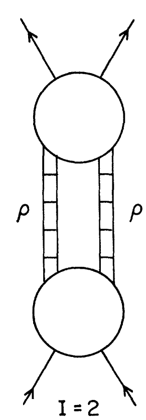

FIG. 1. Feynman diagram with the $I=2$ Regge cut.