[15]M. Lusignoli, M. Restignoli, G. A. Snow, and G. Violini, Phys. Letters 21, 229 (1966).
[16]F. Bulos et al., Phys. Rev. Letters 13, 486 (1964).
[17]W. B. Richards et al., Phys. Rev. Letters 16, 1221 (1966).
[18]D. Berley et al., Phys. Rev. Letters 15, 641 (1965).
[19]D. Cline and M. Olsson, in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, California, 1966 (unpublished).
[20]E. Hyman et al., Nevis Report No. 155, Columbia University, 1966 (to be published).
[21]J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. 139, B1650 (1965); T. D. Lee, in Proceedings of the Oxford International Conference on Elementary Particles, Oxford, England, 1965 (Rutherford High Energy Laboratory, Chilton, Berkshire, England, 1966), p. 255.
[22]A. W. Martin, Nuovo Cimento 32, 1645 (1964).
[23]P. Bareyre et al., Phys. Letters 18, 342 (1965); A. Donnachie, A. T. Lea, and C. Lovelace, Phys. Letters 19, 146 (1965).
[24]T. D. Lee and C. N. Yang, Phys. Rev. 122, 1954 (1961). See Sec. 8, just before Theorem 4.
[25]A. J. Macfarlane and R. H. Socolow, Phys. Rev. 144, 1194 (1966).
[26]F. Gürsey, T. D. Lee, and M. Nauenberg, Phys. Rev. 135, B467 (1964).
[27]J. Schwinger, Phys. Rev. 135, B816 (1964).
[28]M. Gell-Mann, Phys. Rev. Letters 14, 334 (1965); R. H. Dalitz, in Proceedings of the Oxford International Conference on Elementary Particles, Oxford, England, 1965 (Rutherford High Energy Laboratory, Chilton, Berkshire, England, 1966).
[29]M. Gell-Mann, to be published.
[30]I. P. Gyuk and S. F. Tuan, Phys. Rev. 151, 1253 (1966).
[31]T. D. Lee, Nuovo Cimento 35, 933 (1965).
[32]S. Coleman, S. L. Glashow, and D. Kleitman, Phys. Rev. 135, B779 (1964).
[33]S. Coleman, thesis, California Institute of Technology, 1962 (unpublished).
[34]J. Schwinger, Phys. Rev. 104, 1164 (1956); Ann. Phys. (Paris) 2, 407 (1957).
[35]M. Gell-Mann, Phys. Rev. 106, 1296 (1957). See also a related study by R. P. Feynman, "A Model of Strong and Weak Couplings," 1958 (unpublished).

LOCALITY AND THE ISOSPIN OF SELF-CONJUGATE BOSONS*

P. Carruthers
Laboratory of Nuclear Studies and Laboratory of Atomic and Solid State Physics,
Cornell University, Ithaca, New York
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It is shown that the isospin operator is nonlocal for zero-spin bosons of half-integral isospin.

The observed mesons fall into two classes. For self-conjugate mesons, particles and their anitparticles lie in the same isospin multiplet. Secondly, there exist "pair-conjugate" particles, for which the "particles" comprise an isospin multiplet distinct from that of the "antiparticles." Familiar examples of these cases are the self-conjugate pions π^+, π^0, π^-) and the pair-conjugate K mesons (K^+, K^0); $(\bar{K}^0, K^-)$.

Do the quantization rules and locality and causality requirements place any restriction on the allowed representations of the isospin group SU(2)? One is accustomed to the interconnection between charge conjugation and Lorentz invariance. Here we show that self-conjugate bosons would lead to violation of locality and causality if their isospin were $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, $\cdots$. In this note we restrict our attention to spin-zero bosons.

To describe the spin-zero meson multiplet we introduce[1,2] $2T+1$ operators $a_\alpha(k)$, where k is the four-momentum and the variable α runs from $-T$ to $+T$, describing the eigenvalues of T_3. We require the operators $a_\alpha(k)$ to obey the standard commutation rules

$$[a_\alpha(k), a_\beta^*(k')] = \delta_{\alpha\beta}\delta_{kk'},$$
$$[a_\alpha(k), a_\beta(k')] = 0 \qquad (1)$$

and to be normalized so as to create an isospin multiplet with the Condon-Shortley phase relation[3] $(T_\pm = T_1 \pm iT_2)$

$$|T\alpha\rangle = a_\alpha^*|0\rangle,$$
$$T_3|T\alpha\rangle = \alpha|T\alpha\rangle,$$
$$T_\pm|T\alpha\rangle = [(T\mp\alpha)(T\pm\alpha+1)]^{1/2}|T\alpha\pm1\rangle. \qquad (2)$$

Thus the operators $a_\alpha{}^*$ transform under the

SU(2) transformation $\mathcal{O}(\vec{\lambda})=\exp(i\vec{\lambda}\cdot\vec{T})$ as

$$\mathcal{O}(\vec{\lambda})a_\alpha{}^*\mathcal{O}(\vec{\lambda})^{-1}=\sum_\beta a_\beta{}^*D_{\beta\alpha}{}^{(T)}(\vec{\lambda}), \qquad (3)$$

where $D_{\beta\alpha}{}^{(T)}$ is a standard representation matrix for the irreducible representation of SU(2) having dimension $2T+1$. (The notation of Edmonds is used.[4])

To describe the space-time behavior of the system we construct the field

$$\varphi^{(\alpha)}(x)=\sum_k[a_\alpha(k)f_k(x)+\eta_\alpha{}^T a_{-\alpha}{}^*(k)f_k{}^*(x)], \qquad (4)$$

where $f_k(x)=\exp(-ik\cdot x)/(2\omega)^{1/2}$. The operator $\varphi^{(\alpha)}$ decreases the isospin component by $T_3=\alpha$. The phase factor $\eta_\alpha{}^T$ is chosen to make the second part of (4) transform exactly as the first. Thus $\eta_\alpha{}^T$ is given by[5]

$$\eta_\alpha{}^T=\xi(-1)^{T+\alpha}, \qquad (5)$$

where ξ is independent of α and obeys $|\xi|=1$. Thus $\varphi^{(\alpha)*}$ transforms as $a_\alpha{}^*$ [cf. Eq. (3)].

In terms of the creation and destruction operators the isospin operator is

$$T_3=\sum_{\vec{k},\alpha}\alpha a_\alpha{}^*(k)a_\alpha(k),$$

$$T_\pm=\sum_{\vec{k},\alpha}[(T\mp\alpha)(T\pm\alpha+1)]^{1/2}a_\alpha{}^*(k)a_{\alpha\mp1}(k). \qquad (6)$$

In terms of the fields $\varphi^{(\alpha)}(x)$ the isospin operator is

$$\vec{T}=\sum_{\alpha,\beta}\frac{i}{2}\int d^3x\,\varphi^{(\alpha)}{}^*(x)\overleftrightarrow{\partial}_0\vec{t}_{\alpha\beta}\varphi^{(\beta)}(x), \qquad (7)$$

where $\vec{t}$ are the standard $(2T+1)$-dimensional isospin matrices. [In Ref. 1 the phase (5) was found by requiring (7) to give (6).]

From Eqs. (1) and (4) we derive the usual commutator

$$[\varphi^{(\alpha)}(x),\varphi^{(\beta)}{}^*(y)]=i\delta_{\alpha\beta}\Delta(x-y). \qquad (8)$$

However, the commutator

$$[\varphi^{(\alpha)}(x),\varphi^{(\beta)}(y)]$$
$$=\delta_{\alpha,-\beta}\eta_\beta{}^T\sum_k[f_k(x)f_k{}^*(y)$$
$$-(-1)^{2T}f_k{}^*(x)f_k(y)] \qquad (9)$$

is causal only for $2T$ an even integer:

$$[\varphi^{(\alpha)}(x),\varphi^{(\beta)}(y)]$$
$$=i\delta_{\alpha,-\beta}\eta_\beta{}^T\Delta(x-y),\quad T=0,1,2,\cdots; \qquad (10)$$
$$=\delta_{\alpha,-\beta}\eta_\beta{}^T\Delta^{(1)}(x-y),\quad T=\tfrac{1}{2},\tfrac{3}{2},\tfrac{5}{2},\cdots. \qquad (11)$$

The invariant function $\Delta^{(1)}$ does not vanish at space-like separations, in contrast to Δ.

To understand the implications of (11) we consider the relation of $\varphi^{(-\alpha)}$ to $\varphi^{(\alpha)*}$. These are not independent fields, and one may choose to describe the theory in terms of $\varphi^{(\alpha)}$ and $\varphi^{(\alpha)*}$ having non-negative α:

$$\varphi^{(-\alpha)}{}^*(x)=(\eta_{-\alpha}{}^T)^*\sum_k[a_\alpha(k)f_k(x)$$
$$+(-1)^{2T}\eta_\alpha{}^T a_{-\alpha}{}^*(k)f_k{}^*(x)]. \qquad (12)$$

We find

$$\varphi^{(-\alpha)}{}^*(x)=(\eta_{-\alpha}{}^T)^*\varphi^{(\alpha)}(x),\quad T=0,1,2,\cdots;$$
$$=-i(\eta_{-\alpha}{}^T)^*\int\varphi^{(\alpha)}(x')\overleftrightarrow{\partial}_0{}'\Delta^{(1)}(x'-x)d^3x',$$
$$T=\tfrac{1}{2},\tfrac{3}{2},\tfrac{5}{2},\cdots. \qquad (13)$$

Thus for integral-isospin bosons we can conveniently choose $\eta_\alpha{}^T=(-1)^\alpha$. But the second relation in (13) is definitely nonlocal; $\varphi^{(-\alpha)*}(x)$ receives contributions from $\varphi^{(\alpha)}(x')$ outside the light cone $(x-x')^2=0$.

Consider now the isospin density $\vec{t}(x)$ in Eq. (7), defined by $\vec{T}=\int\vec{t}(x)d^3x$. For half-integral isospin, $\vec{t}(x)$ does not commute with $\varphi^{(\alpha)}(x')$ even when $x-x'$ is a spacelike vector. From Eqs. (7)-(11) we find

$$[\vec{t}(x),\varphi^{(\lambda)}(x')]$$
$$=\tfrac{1}{2}\vec{t}_{\lambda\nu}\varphi^{(\nu)}(x)\overleftrightarrow{\partial}_0\Delta(x-x')$$
$$+\tfrac{1}{2}\eta_\lambda{}^T\vec{t}_{\nu,-\lambda}\varphi^{(\nu)}{}^*(x)\overleftrightarrow{\partial}_0\Delta^{(1)}(x-x'). \qquad (14)$$

Hence we have constructed the isospin operator for half-integral isospins at the expense of introducing nonlocal, noncausal effects.[7] In particular, the commutator of the isospin operator T with $\varphi^{(\alpha)}(x)$ receives contributions from points not coincident with x. For all these reasons we reject such theories.

We conclude that zero-spin bosons of half-integral isospin must carry another quantum number. (This is hypercharge in the real world.) Our considerations are consistent with the universal validity of the Gell-Mann–Nishijima formula. For pair conjugate bosons (replace $a_{-\alpha}{}^*$ in Eq. (4) by $b_{-\alpha}{}^*$, where $b_\alpha{}^*$ creates the antiparticle multiplet) the restrictions are not so great since the operators entering into the field are dynamically independent.

An interesting consequency of the present result is the clarification of an apparent paradox which arises when one studies the crossing properties of self-conjugate bosons. Because of the structure of (4), one can express $a_\alpha(k)$ in terms of either $\varphi^{(\alpha)}$ or $\varphi^{(-\alpha)*}$. Taking advantage of these apparently equivalent forms, one finds a consistent crossing phase[1] only when the isospin of the crossed meson is integral. From the present analysis one sees that the hypothetical case of half-integral isospin would lead to a noncovariant S matrix, because Eq. (11) would render the T product noncovariant. For the allowed case of integral isospin, all the expressions of Ref. 1 for the crossing matrix are identical, allowing a considerable simplification in phase questions.

Consideration of higher spins and other internal symmetry groups will be given elsewhere.

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[1]P. Carruthers and J. P. Krisch, Ann. Phys. (N.Y.) 33, 1 (1965).

[2]P. Carruthers, Introduction to Unitary Symmetry (Interscience Publishers, Inc., New York, 1966), Chap. 1.

[3]E. U. Condon and G. H. Shortley, The Theory of Atomic Spectra (Cambridge University Press, Cambridge, England, 1957), Chap. 3.

[4]A. R. Edmonds, Angular Momentum in Quantum Mechanics (Princeton University Press, Princeton, New Jersey, 1960).

[5]E. P. Wigner, Group Theory (Academic Press, Inc., New York, 1959), p. 288.

[6]Integration of Eq. (14) over space leads to the expected commutation rule of $\vec{T}$ with $\varphi^{(\alpha)}(x)$.

[7]The maximal internal symmetry group for a system described by two real fields is well known to be U(1) for local theories.

MESON + HYPERON FINAL STATES IN K^-p INTERACTIONS AT 4.1 AND 5.5 GeV/c*

J. Mott,† R. Ammar, R. E. P. Davis, W. Kropac, and F. Schweingruber‡
Northwestern University, Evanston, Illinois

and

M. Derrick, T. Fields,§ L. G. Hyman, J. Loken, and J. Simpson
Argonne National Laboratory, Argonne, Illinois

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Differential cross sections for reactions of the type $K^-+p\to$ neutral hyperon + neutral meson have been measured. The cross sections for peripheral production of vector mesons satisfy certain relations yielded by the independent quark model and other symmetry schemes. The cross section for meson production in the backward hemisphere provides some information on the presence of various baryon-exchange processes.

In order to study hypercharge-exchange processes yielding quasi-two-body final states in high-energy K^-p interactions, we have analyzed ~5000 bubble-chamber events with two prongs plus a visible Λ decay. This sample represented 0.67 events/μb at 4.1 GeV/c and 1.5 events/μb at 5.5 Gev/c (corrected for undetected Λ decays) in the 30-inch hydrogen bubble chamber, using the high-purity separated beam at the Argonne zero gradient synchrotron (ZGS).

Our results are based on events which yielded kinematic fits to these final-state hypotheses: $\Lambda\pi^+\pi^-$, $\Lambda\pi^+\pi^-\pi^0$, $\Lambda\pi^+\pi^-\eta$, ΛK^+K^-, $\Sigma^0\pi^+\pi^-$, and $\Sigma^0K^+K^-$. The most serious ambiguity which was encountered in kinematic fit-