THEORETICAL DISCUSSIONS CONCERNING η'-δ DEGENERACY

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Recently, evidence has been found for the existence of a δ meson[1,2] with $I=1$ and mass around 963 MeV, but of unknown spin and parity. The pair δ and the better-known η' (mass 959 MeV, $I=0$, and $J^{PG}=0^{-+}$)[3,4] is one of the first examples[5] in particle physics where members of different isospin multiplets have mass differences of the order of a few MeV, comparable with the usual electromagnetic mass differences.

In spite of the paucity of experimental information concerning δ (which prevents unambiguous conclusions to be drawn at this stage), it is the purpose of the present note to investigate the consequences of the proposition that the mass degeneracy of δ and η' is <u>not accidental</u>. We restrict our analysis to those internal symmetry groups which commute with Lorentz transformation. It follows that δ and η' must have the same spin parity, namely 0^-. Depending on the G parity of δ, there are two distinct cases: (a) $J^{PG}=0^{-+}$, where δ and η' belong to one $SU(2)\otimes SU(2)$ multiplet,[6] but not the same SU(3) multiplet.[7] In this case, δ^0 must have the particle-antiparticle conjugation assignment $C=-1$; thus δ has no place in any of the usual SU(3) multiplets. To the extent that $SU(2)\otimes SU(2)$ is good in strong interactions, both the mass degeneracy of δ and η' and the abnormally small production cross section of δ from present experiments[1,2] can be understood. The search for δ, which decays predominantly into $\omega+\pi$, in the reaction $K^-+n\to\Lambda+\delta^-$,[8] $K^-+n\to\Sigma^-+\delta^0$, and $K^-+p\to\Sigma^\pm+\delta^\mp$ is interesting since these amplitudes are connected by $SU(2)\otimes SU(2)$. (b) $J^{PG}=0^{--}$ for δ, where δ and η' belong to the same SU(3), but not $SU(2)\otimes SU(2)$, multiplet. This multiplet is a new 0^- octet or nonet. The dominant decay modes are $\rho\pi$ and 3π; in the case of the charged δ, the decay product contains one charged particle (1c) or three charged particles (3c) with comparable probability, consistent with preliminary data.[1] The explanation of mass degeneracy and the observed small production cross section becomes, however, less transparent. In this case, it would be most interesting to look for the other members of the multiplet.

In what follows we discuss in detail these two cases after the necessary (theory-independent) phenomenology and mathematical preliminaries.

<u>The decay of δ.</u> – The charged δ have been observed through the reactions $\pi^-+p\to p+\delta^-$ by Focacci <u>et al.</u>[1] and $p+p\to d+\delta^+$ by Oostens <u>et al.</u>,[2] while δ^0 has not yet been seen. It is not known experimentally what the decay products are. Here we estimate the decay rates of δ for various assignments of J, P, and G; the assumptions involved are the usual conservation laws and that phase-space and barrier-factor considerations give reasonable approximations to various decay rates.

The results are summarized in Table I, where the main modes of decay and their widths are given. The results are obtained as follows. (a) The radius of interaction has been arbitrarily taken to be $(500\text{ MeV})^{-1}$. (b) For $J^{PG}=0^{++}$, the width for $\delta\to 2\pi+\gamma$ is estimated by

$$\Gamma(\delta\to 2\pi+\gamma)\sim e^2\times(\text{phase-space factor})\times(p/500\text{ MeV})^2,$$

where p is some typical momentum for the decay. If we take the phase-space factor to be 115 keV and $p^2+m(\pi)^2=[m(\delta)/3]^2$ together with $e^2=4\pi/137$, then $\Gamma(\delta\to 2\pi+\gamma)\sim 3.5$ keV.[9] In this case of $J^{PG}=0^{++}$, the partial width for $\delta\to 4\pi$

Table I. Dominant modes and decay widths of δ. All dimensionless strong coupling constants g have been set to unity.

G \ J^P	0^+	0^-	1^+	1^-
+1	$2\pi\gamma$	$\omega\pi$	$\omega\pi$	2π
	$\lesssim 5$ keV	170 keV	4 MeV	16 MeV
−1	$\eta\pi$	$\rho\pi$, 3π	$\rho\pi$	$\eta\pi$
	13 MeV	430 keV	5 MeV	5 MeV

is believed to be of the order of a fraction of a keV. (c) For the case 0^{--}, $\delta\to 3\pi$ contributes appreciably, perhaps 20%, in addition to $\delta\to\rho+\pi$. With the exceptions of 0^{++} and 0^{--}, one mode dominates to within a few percent in all the other six cases.

From Table I, it is interesting to compare the decay of the charged δ into one charged particle plus neutrals (1c) versus three charged particles plus possible neutral (3c). (i) When $G=+1$, 1c dominates in the cases $J^{PG}=0^{++}$ and 1^{-+}, while 3c dominates for 0^{-+} and 1^{++}. (ii) When $G=-1$, $1c/3c=1$ for 1^{+-}; from the branching ratio[10] of η, $1c/3c\sim 2$ for 0^{+-} and 1^{--}; and for 0^{--}, because of the existence of two dominant modes, 1c/3c cannot be calculated but is of the order of 1. (iii) For δ^-, Focacci et al.[1] have obtained experimentally

$$1c/3c = 1.3^{+0.9}_{-0.7},$$

which favors $G=-1$.

The phenomenological analysis here of the decay of δ is quite general and not predicated upon the existence of a neighboring η'. However, if δ and η' are related by an internal symmetry group so that the spin parity of δ is 0^-, then it is clear from Table I that there are two possible cases, $G=+1$ and $G=-1$. These two cases can be distinguished by their dominant decay modes: $\omega\pi$ for $G=+1$, $\rho\pi$ and 3π for $G=-1$.

Minimal extension of $SU(2)\otimes SU(2)$.— Before we can apply $SU(2)\otimes SU(2)$ to the meson states, we must study the extension of this group by particle-antiparticle conjugation. For this purpose, we apply the results of Lee and Wick,[11] whose notation we shall follow. Let 1 denote the identity of SU(2), and u_0 the element which is $\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}$ in the Pauli representation. Then SU(2) has just three normal (or invariant) subgroups: N_0, which consists of the identity element 1 only; N_1, which consists of 1 and u_0; and $N_2=SU(2)$. Thus $SU(2)\otimes SU(2)$ has nine normal subgroups $N_i\times N_j$, with $i,j=0,1,2$. Under an automorphism of $SU(2)\otimes SU(2)$, the normal subgroups and their partial ordering are preserved. Since all automorphisms of SU(2) are inner, the group of outer automorphisms of $SU(2)\otimes SU(2)$ is of order 2. That is, if $(u,u')\to F(u,u')$ is an automorphism of $SU(2)\otimes SU(2)$, then there exists a (v,v') in $SU(2)\otimes SU(2)$ such that, for every (u,u') in $SU(2)\otimes SU(2)$,

$$F(u,u')=(vuv^{-1},v'u'v'^{-1})$$

or, for every (u,u'),

$$F(u,u')=P(vuv^{-1},v'u'v'^{-1})=(v'u'v'^{-1},vuv^{-1}),$$

where P exchanges the two SU(2)'s.

The F of Lee and Wick[11] can therefore be chosen to be either ϵ or P. In either case, by their Eq. (3.9), f belongs to $N_1\times N_1$, which is the center of the group $SU(2)\otimes SU(2)$. Accordingly, there are eight inequivalent minimal extensions of $SU(2)\otimes SU(2)$, specified explicitly by $F=\epsilon$ or $(P,f)=(1,1)$, $(1,u_0)$, $(u_0,1)$, or (u_0,u_0). This result will be used later.

The case $G=+1$.— We consider first the assignment $J^{PG}=0^{-+}$ for both η' and δ. We note, for this case, that (a) δ is not coupled to nucleon-antinucleon pair,[12] a fact which is related to the small production cross sections[1,2] of δ by $\pi^-+p\to p+\delta^-$ and $p+p\to d+\delta^+$; (b) under particle-antiparticle conjugation, η' is even but δ^0 is odd, and hence η' and δ cannot belong to the same SU(3) multiplet; and (c) if electromagnetic interactions do not violate charge-conjugation symmetry, η' and δ^0 cannot be mixed by virtual electromagnetic transitions.[13] Experimental data[3] on the decay of η' are consistent with (c).

In view of (b), we try to assign η' and δ to the $SU(2)\otimes SU(2)$ multiplet $(\frac12,\frac12)$. If $\vec{I}^{(1)}$ and $\vec{I}^{(2)}$ are the generators of the two SU(2) groups, then we identify, following Pais,[6] isospin with $\vec{I}^{(1)}+\vec{I}^{(2)}$; in other words, the isospin subgroup consists of elements of the form (u,u). Since, under the exchange of the two SU(2)'s, δ is even while η' is odd, we must choose a minimal extension with $F=\epsilon$ in order to get $G=+1$ for both η' and δ. Furthermore, since G is real, $f=(1,1)$ or (u_0,u_0), which are both the unit matrix in the $(\frac12,\frac12)$ representation.

If we use the assignments $\Lambda,\Sigma:(\frac12,\frac12)$, $N:(\frac12,0)$, and $K:(\frac12,0)$, then the reactions $\bar{K}+N\to(\Sigma,\Lambda)+(\delta,\eta')$ are described by two amplitudes T_0 and T_1. In particular,

$$\mathrm{amp}(K^-+p\to\Lambda+\eta')=-\mathrm{amp}(K^-+p\to\Sigma^0+\delta^0)=(2\sqrt2)^{-1}T_0,$$

$$\mathrm{amp}(K^-+p\to\Lambda+\delta^0)=-(2\sqrt2)^{-1}T_1,$$

$$\mathrm{amp}(K^-+p\to\Sigma^++\delta^-)=(2\sqrt2)^{-1}(T_0+T_1),$$

$$\mathrm{amp}(K^-+p\to\Sigma^-+\delta^+)=(2\sqrt2)^{-1}(T_0-T_1),$$

and

$$\mathrm{amp}(K^- + n \to \Lambda + \delta^-) = -\mathrm{amp}(K^- + n \to \Sigma^0 + \delta^-) = \mathrm{amp}(K^- + n \to \Sigma^- + \delta^0)$$
$$= -\mathrm{amp}(K^- + n \to \Sigma^- + \eta') = -\tfrac{1}{2}T_1. \qquad (1)$$

However, the resulting equality, for example,

$$2\times \mathrm{rate}(K^- + p \to \Lambda + \eta') + \mathrm{rate}(K^- + n \to \Lambda + \delta^-) = \mathrm{rate}(K^- + p \to \Sigma^+ + \delta^-) + \mathrm{rate}(K^- + p \to \Sigma^- + \delta^+), \qquad (2)$$

may have to be modified if there exists substantial asymmetry between the various meson-baryon couplings involved; note that such appears to be the case for $\pi\Sigma\Sigma$, $\pi\Sigma\Lambda$,[14] and KYN[15] couplings.

With the present choice of assignments for particles in $SU(2)\otimes SU(2)$, and requiring further π, η, and the deuteron d to have nondegenerate assignments $(1,0)$, $(0,0)$, and $(0,0)$, respectively, following reactions are allowed by $SU(2)\otimes SU(2)$:

$$\begin{aligned} &\pi^- + p \to \eta + n, \\ &K^- + p \to \eta' + \Lambda(\Sigma), \\ &K^- + p \to \delta + \Lambda(\Sigma), \end{aligned} \qquad (3)$$

while the following reactions are forbidden:

$$\begin{aligned} &p + p \to \delta^+ + d, \\ &\pi^- + p \to \delta + p(n), \\ &\pi^- + p \to \eta' + n, \\ &K^- + p \to \eta + \Lambda(\Sigma). \end{aligned} \qquad (4)$$

Our expectation is that production cross sections which respect the symmetry will be enhanced over those which violate it. Experimentally, the cross section for the allowed process $\pi^- + p \to \eta + n$ remains large (of the order of $\frac{1}{2}$ mb) for several hundred MeV above threshold,[16,17] while the forbidden processes[18,19] $K^- + p \to \eta + \Lambda(\Sigma)$ have rapidly attenuated cross sections ($\lesssim 100$ μb) for energies shortly beyond the threshold "resonance" region. Likewise, there are indications[20] that the forbidden process $\pi^- + p \to \eta' + n$ has small cross section ($\lesssim 60$ μb). Thus there is some support for our particular assignments in $SU(2)\otimes SU(2)$.

We make the following remarks for the case $G = +1$: (i) To the extent that $SU(2)\otimes SU(2)$ symmetry is exact in strong interactions, the degeneracy of η' and δ up to mass differences of a few MeV is preserved. If our choice of assignments of particles within $SU(2)\otimes SU(2)$ is relevant, then the processes $\pi^- + p \to p + \delta^-$ and $p + p \to d + \delta^+$ are forbidden and hence we have further understanding for the small experimental cross sections (a few μb).[1,2] (ii) We are aware that, analogous to the situation in global symmetry,[6] several known reactions like $K^- + p \to \pi + \Lambda(\Sigma)$ are also forbidden in our scheme. This is perhaps not surprising since it is known[14] that the coupling constants associated at least with the pion-hyperon system follow closely the pattern dictated by SU(3) rather than $SU(2)\otimes SU(2)$. A convenient (though not wholly satisfactory) way to resolve the conflict is to have the K mesons assume a mixture of $(0,\frac{1}{2})$ and $(\frac{1}{2},0)$ assignments; such a procedure maintains the equalities (1) and (2). (iii) The existence of an $I = 1$, $J^{PG} = 0^{-+}$ δ meson whose neutral member has the particle-anitparticle conjugation property $C = -1$ is a novelty in the annals of hadron mass spectra. If one further speculates that the δ meson is relevant to recent discussions of C nonconservation in electromagnetism,[21] then η'-δ^0 mixing by virtual electromagnetic transitions[13] is pertinent, and examination of the neutral η' decay mode $\eta' \to \omega + \pi^0$ will be of interest. (iv) Our analysis can be easily extended to include other cases of mass degeneracy in particle spectrum, e.g., the pair[22] $Y_0^*(1670)$ and $Y_1^*(1660)$ with $J^P = \frac{3}{2}^-$ and the pair[23] $N_{1/2}^*(1700)$ and $N_{3/2}^*(1690)$ with $J^P = \frac{1}{2}^-$.

The case $G = -1$.—We now turn our attention to the situation where $J^{PG} = 0^{--}$ for δ. In this case, $SU(2)\otimes SU(2)$ is no longer relevant[24] for the following reason. If η' and δ belong to the same $SU(2)\otimes SU(2)$ multiplet but have opposite G parity, then we must use a minimal extension (by particle-antiparticle conjugation) with $F = P$. Accordingly, if the nucleon N belongs to the representation $(\frac{1}{2},0)$, the antinucleon $\bar{N}$ must belong to $(0,\frac{1}{2})$. Thus, for example, $N + \eta' \to N + \eta'$ but $N + \bar{N} \not\to 2\eta'$, which violates crossing symmetry. (In this note, we do not discuss the question of how compelling the experimental basis is for crossing symmetry.)

In view of the lack of reliable information concerning the mixing[25] of η' with the octet $(\pi, \eta, K, \bar{K})$, it is not compelling to assign η' as the singlet member in SU(3). We therefore consider the possibility that η' and δ belong to a new pseudoscalar octet. So far as the octet $(\pi, \eta, K, \bar{K})$ is concerned, either there is no ninth member[26] or the ninth member is not yet discovered (for example, the 1.5-BeV meson of Schwinger[27]). The existence of this second pseudoscalar octet complicates the quark model[28] and makes it necessary to introduce, for example, either a radial wave function[29] or pseudoquarks.[30] Alternatively, the fundamental structure may be associated with two integrally charged triplets.[31]

It is natural to call this $J^P = 0^-$ octet $(\pi', \eta', K', \bar{K}')$, where $\pi' = \delta$. We shall try to apply the mass formulas, even though their relevance is by no means clear.[32] The Gell-Mann–Okubo mass formula, with either m or m^2, predicts that $m(K') \sim 960$ MeV. The Schwinger[27] mass formula gives instead, with $m(\delta) = m(\eta')$,

$$[m(S')]^2 - 2[m(K')]^2 + [m(\eta')]^2 = 0,$$

where S' is the SU(3) singlet needed to complete the nonet. Note that, unlike the $(\pi, \eta, K, \bar{K})$ octet, the Schwinger mass formula does not reduce to the Gell-Mann–Okubo mass formula as $m(S') \to \infty$.

L'envoi. – It has been known for some time[24,33] that all groups which (i) have an eight-dimensional unitary representation (e.g., N, Λ, Σ, Ξ), (ii) contain a subgroup isomorphic to SU(2) $\otimes$U(1) (corresponding to isospin and hypercharge, respectively), and (iii) have the eight-dimensional representation reduce in the proper way (i.e., two isodoublets, a triplet, and a singlet) and have a subgroup which is locally isomorphic (viz. have the same Lie algebra) either to SU(3) or to the minimal global symmetry of Lee and Yang.[24] That SU(3) has had success[7] in hadron physics is well known. The question we ask here is whether these hadrons may, under suitable circumstances, manifest properties akin to global symmetry[6,34,35] as well. Experience with nuclear physics, where both rotational and vibrational levels are known to coexist, lends heuristic support to the notion that for broken symmetry such sharing is entirely reasonable. Note that the SU(2) $\otimes$SU(2) symmetry is not contained in SU(3) but is a subgroup of global symmetry of Lee and Yang.[24]

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LOCALITY AND THE ISOSPIN OF SELF-CONJUGATE BOSONS*

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It is shown that the isospin operator is nonlocal for zero-spin bosons of half-integral isospin.

The observed mesons fall into two classes. For self-conjugate mesons, particles and their anitparticles lie in the same isospin multiplet. Secondly, there exist "pair-conjugate" particles, for which the "particles" comprise an isospin multiplet distinct from that of the "antiparticles." Familiar examples of these cases are the self-conjugate pions π^+, π^0, π^-) and the pair-conjugate K mesons (K^+, K^0); $(\bar{K}^0, K^-)$.

Do the quantization rules and locality and causality requirements place any restriction on the allowed representations of the isospin group SU(2)? One is accustomed to the interconnection between charge conjugation and Lorentz invariance. Here we show that self-conjugate bosons would lead to violation of locality and causality if their isospin were $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \cdots$. In this note we restrict our attention to spin-zero bosons.

To describe the spin-zero meson multiplet we introduce[1,2] $2T+1$ operators $a_\alpha(k)$, where k is the four-momentum and the variable α runs from $-T$ to $+T$, describing the eigenvalues of T_3. We require the operators $a_\alpha(k)$ to obey the standard commutation rules

$$[a_\alpha(k), a_\beta^*(k')] = \delta_{\alpha\beta}\delta_{kk'},$$
$$[a_\alpha(k), a_\beta(k')] = 0 \quad (1)$$

and to be normalized so as to create an isospin multiplet with the Condon-Shortley phase relation[3] $(T_\pm = T_1 \pm iT_2)$

$$|T\alpha\rangle = a_\alpha^*|0\rangle,$$
$$T_3|T\alpha\rangle = \alpha|T\alpha\rangle,$$
$$T_\pm|T\alpha\rangle = [(T\mp\alpha)(T\pm\alpha+1)]^{1/2}|T\alpha\pm1\rangle. \quad (2)$$

Thus the operators a_α^* transform under the