for many helpful discussions.

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EVIDENCE FOR THERMODYNAMIC FLUCTUATIONS IN A SUPERCONDUCTOR*

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The subject of thermodynamic fluctuations in superconductors is one which has received scant attention. A reason for this is the prevalent notion that the effect of such fluctuations on superconducting properties is immeasureably small. In this Letter we show that a particularly favorable regime in which to study thermodynamic fluctuations is a superconducting microstrip in the presence of current flow. We sketch a qualitative model which predicts that thermodynamic fluctuations lead to an anomalous temperature dependence of the critical current near the superconducting transition temperature T_c, which is marked in a superconducting strip of small but readily obtainable dimensions. Preliminary experimental results on Sn microstrips are in qualitative agreement with the model.

The starting point in the following discussion is the assumption, which is standardly made, that in a homogeneous system the probability that a part or all of the system will be in a nonequilibrium state of free energy density f' is given by

$$P \sim \exp[-(f'-f)V/kT], \tag{1}$$

where f is the equilibrium value of the free energy density and V is the volume of the system which is in the state f'. Consider now a superconducting strip, as shown in Fig. 1, at a temperature T and in the presence of a current density $J=0.80J_c$, where J_c is the critical current density. If the shaded square whose dimensions are $t\times 0.20w\times 0.20w$, where t is the thickness and w the width of the strip, suffers a fluctuation to the normal state, then the supercurrent will divert to avoid that region. This causes the current density to increase to its critical value in the region bounded by the dashed lines, which results in the region reverting to the normal state. Applying Eq. (1) to this situation, we may write for the time-averaged resistance of the region bounded by the dashed lines, allowing only the shaded square to fluctuate,

$$R' \sim R_N' \exp[(f_s-f_n)(0.20w)^2t/kT], \tag{2}$$

where f_n and f_s are the free energy densities of the normal and superconducting states, respectively, and R_N' is the normal resistance of the region bounded by the dashed lines. Generalizing the problem to allow for "square-patch" fluctuations of arbitrary size and location in the strip, one obtains for the time-averaged resistance of a strip of arbitrary length

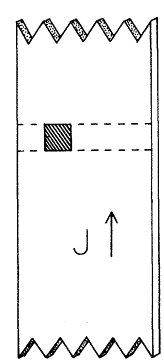

FIG. 1. Section of superconducting microstrip containing a "square-patch" fluctuation (see text).

the expression

$$R \sim \frac{R_N}{(1-J/J_c)} \exp\left[\frac{(f_s-f_n)(1-J/J_c)^2 w^2 t}{kT}\right], \qquad (3)$$

where R_N is the normal resistance of the strip. The factor $(1-J/J_c)^{-1}$ in front of the exponential results from considering the multiplicity of nonoverlapping fluctuation configurations which lead to the appearance of resistance in the same region of the sample.

The "square-patch" fluctuations considered for simplicity above are unrealistic since the most probable shape for a fluctuating region is circular. Correcting Eq. (3) in view of this, which introduces some problems concerning overlapping configurations, would lead to a numerical correction of as much as 50% in the exponent. However, since the above treatment is at best a qualitative model, Eq. (3) is adequate for our purposes. A proper theory must treat in a consistent way all possible kinds of resistance-producing fluctuations. One type of fluctuation which is less important than the type mentioned above, but which nevertheless must be included in a proper theory, is one where the order parameter ψ is depressed from its equilibrium value (but is still finite) in a region of the sample. Since the phase transition produced by forced current in a simply connected superconductor (in a regime where the current flow is uniform, such as a microstrip) is first order (occurring at finite ψ),[1] such a fluctuation can also produce resistance. A proper theory must also consider time effects. In the above model which leads to Eq. (3), it is assumed that the fluctuations occur slowly enough so that inductive effects associated with rapidly changing current configurations can be ignored. Since, in general, the characteristic times associated with fluctuations near a second-order phase transition become infinite in the limit $T \rightarrow T_c$, it is expected that there exists near T_c a temperature region where our model is applicable. Finally, a complete theory must take into account coherence-length effects, which would be particularly important for systems with large coherence lengths.

In order to check the above model, i.e., check Eq. (3), we have made critical-current measurements on Sn microstrips with typical dimensions of 1000 Å×1 μ×100 μ. These were prepared by coating a glass slide with a thin layer of collodion which was diluted with amyl acetate, scratching the organic film with a sharp razor blade, evaporating a Sn film onto the resulting substrate, and finally dissolving away the organic film which leaves behind a Sn microstrip at the location of the scratch. Conventional four-probe techniques were used to monitor the resistance of the microstrips in the presence of current flow. Measurements were made on five Sn samples whose dimensions are tabulated in Table I.

Table I. Sample data: w is the width and t the thickness of the Sn strips.

Sample	t (Å)	w (μ)
A	580	1.2
B	825	1.5
C	580	2.1
D	1900	2.5
E	1900	10

The results for two representative samples (C and E) are shown in Fig. 2. The straight line satisfies the relation $J \propto (\Delta T)^{3/2}$, where $\Delta T = T_c - T$, which is the correct critical current-temperature relation in the absence of fluctuation effects.[1,2] Sample E, for which fluctuation effects should be ignorably small according to Eq. (3), accurately follows the $(\Delta T)^{3/2}$ law (if we use the criterion $R/R_N = 0.01$ to define the critical current). Sample C, which has a much smaller cross section, departs appreciably from the $(\Delta T)^{3/2}$ dependence for $\Delta T \lesssim 0.030°$K. Experimental results and curves calculated from Eq. (3) are given for sample C for $R/R_N = 0.01$ and $R/R_N = 0.1$. The values of T_c and therefore ΔT were determined by allowing T_c to be an adjustable parameter which gave the best fit to the relation $J = J_c \propto (\Delta T)^{3/2}$ for large values of ΔT. The theoretical curve was calculated using Eq. (3), the Ginzburg-Landau expression for $(f_n - f_s)$, and standard values for the parameters of Sn.

There is poor quantitative agreement between experiment and theory, which is not surprising because of the crudeness of our model; the qualitative agreement is gratifying. A distinctive feature of the experimental results is a more rapid falling off (a plummeting in fact) of the curve of J vs ΔT than that predicted by the model. The reason for this might be traced to a kind of cooperative effect taking place among fluctuation regions, which becomes increasingly important as the density of fluctuations increases (as ΔT decreases). This effect is manifested in the following way. If the fluctuating regions are relatively large and closely spaced there is produced a tortuous twisting of the current streamlines which elevates the kinetic energy density and is deleterious to the superconducting state.

According to the Ginzburg-Landau theory[3] the quantity $(f_n - f_s)$ is given by

$$f_n - f_s = 5.6k^2 N(0)(\Delta T)^2, \tag{4}$$

where $N(0)$ is the density of states at the Fermi surface. From Eqs. (3) and (4) we see that

$$(\Delta T_c)^2 w^2 t = \text{const}, \tag{5}$$

where ΔT_c is the value of ΔT corresponding to $J/J_c = 0$. In Fig. 3 we have plotted the dependence of $(\Delta T_c)^2$ on $(w^2 t)^{-1}$ for $R/R_N = 0.1$ for the five samples listed in Table I. The equation of the straight line is $(\Delta T_c)^2 \propto (1/w^2 t)^b$, where $b = -1.1 \pm 0.2$, which is in good agreement with Eq. (5).

In summary, we have constructed a simple model which predicts an anomalous temperature dependence of the critical current of a superconducting microstrip near the transition temperature due to fluctuations in the superconducting order parameter. Our experimental results on Sn microstrips qualitatively confirm this model. The demonstration that thermodynamic fluctuations can lead to measurable effects in superconductors establishes the need for a profound attack on the theory which we are not prepared to undertake.

We became interested in the present problem as a result of seeing a preprint by Little[4] which discusses resistance-producing fluctuations in the limit of zero current flow. We have ben-

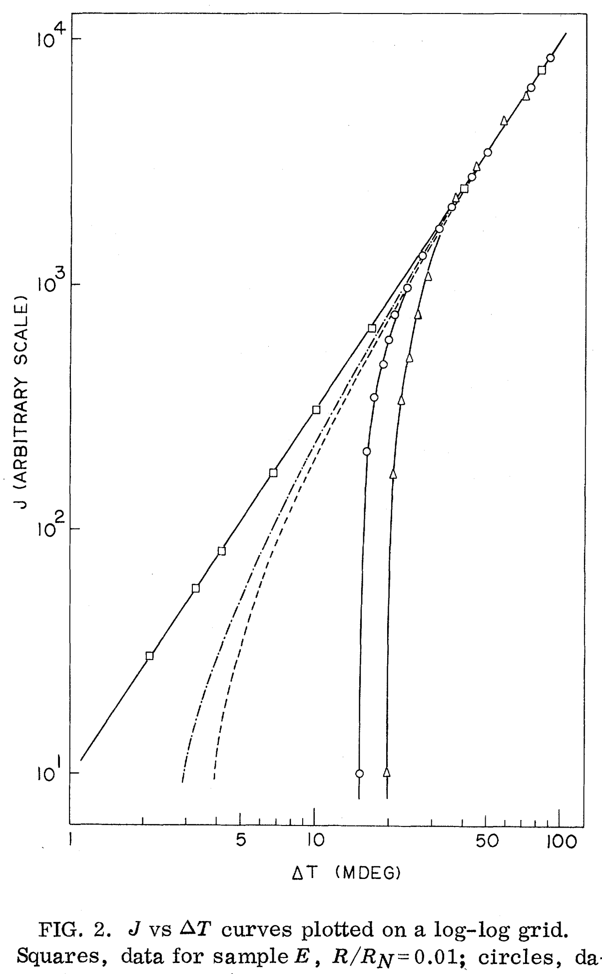

FIG. 2. J vs ΔT curves plotted on a log-log grid. Squares, data for sample E, $R/R_N = 0.01$; circles, data for sample C, $R/R_N = 0.01$; triangles, data for sample C, $R/R_N = 0.01$; dashed-dotted curve, curve of Eq. (3) for sample C, $R/R_N = 0.1$; dashed curve, curve of Eq. (3) for sample C, $R/R_N = 0.01$. The data are normalized so that all curves merge into the same one in the limit of large ΔT where fluctuation effects are unimportant.

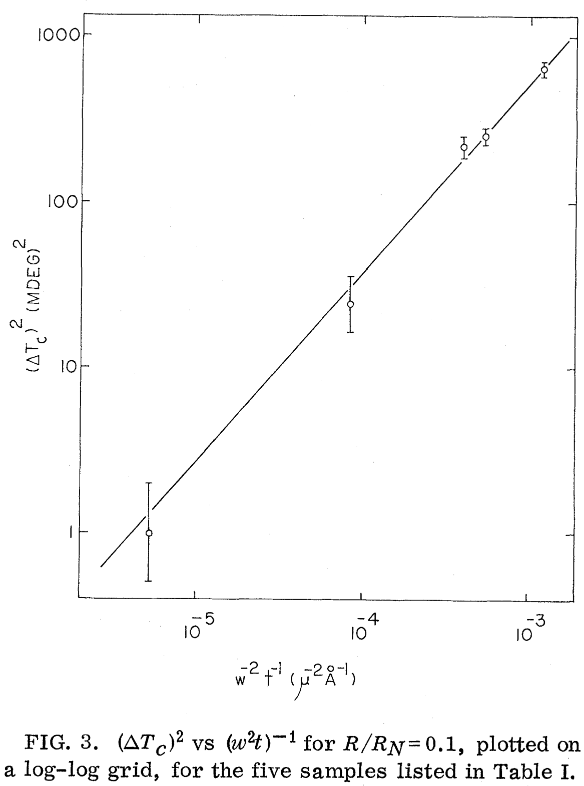

FIG. 3. $(\Delta T_c)^2$ vs $(w^2 t)^{-1}$ for $R/R_N = 0.1$, plotted on a log-log grid, for the five samples listed in Table I.

efited from discussing the problem of thermodynamic fluctuations with E. Abrahams, W. A. Little, P. C. Martin, and E. Montroll.

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SINGLE-PARTICLE STATES BUILT ON THE SECOND 0^+ STATE IN ^{90}Zr†

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The proton excitation function of the first excited (0^+, 1.75 MeV) level in ^{90}Zr has been measured. Three prominent analog resonances are found, which are single-particle excitations built on this excited state similar to those built on the ground state.

The excitation function of protons scattered inelastically by ^{90}Zr has been measured from $E_p = 4.5$ to 10.0 MeV at various angles. This region in the compound nucleus ^{91}Nb contains the analog of the low-lying states in ^{91}Zr. The results of these measurements clearly indicate that certain excited states of the nucleus ^{91}Zr consist of single-particle excitations of a neutron and a core which is the first excited state of ^{90}Zr (0^+, 1.75 MeV).

Inelastic analog resonance scattering has been previously identified[1] and its ability to give nuclear structure information pointed out.[1,2] Inelastic resonance scattering has been used to identify the particle-hole character of the residual state involved.[3] The present method differs from the latter in that it concerns states in the parent analog nucleus. Moreover, it involves states which are weakly excited by the reaction ^{90}Zr$(d,p)^{91}$Zr. Because of the low yield in the (d,p) reaction, which implies a small spectroscopic factor, resonant effects of the analogs of these states are expected to be small in the elastic excitation function. This was indeed observed to be the case.

The elastic excitation curves, in general, exhibit three resonances. These correspond to the parent analog states in ^{91}Zr with large spectroscopic factors: $d_{5/2}$, g.s.; $s_{1/2}$, 1.21 MeV; and $d_{3/2}$, 2.06 MeV.[4] The analog of the $g_{7/2}$, 2.21-MeV state is seen only at far backward angles and its weak intensity, as well as the reduced strength of the $d_{5/2}$, g.s., is explained by the small penetrability of the potential barrier.

The inelastic excitation curves to the first excited state of ^{90}Zr (0^+, 1.75 MeV are shown in Fig. 1. Again there are three prominent resonances. These resonances have about the same spacing as the $d_{5/2}$, $s_{1/2}$, and $d_{3/2}$ analog states seen in the elastic excitation curve. However, they are shifted upwards by an amount nearly equal to the excitation energy of the first excited state in ^{90}Zr. The decay properties of these resonances show that to a considerable extent, these states have a structure represented by a single particle built on the first excited state of ^{90}Zr.

The parent analogs of these states have been reported in the (d,p) work of Cohen.[4] They have weak intensities and his published results are the following: $d_{3/2}$, 1.48 MeV; $s_{1/2}$, 2.58 MeV; and $d_{3/2}$, 3.70 MeV. The 1.48-MeV level has an angular distribution characterized by $l_n = 2$ and is assigned $d_{3/2}$. With this same experimental evidence, however, Ramavataram[5] concluded that this level was $d_{5/2}$. With the identification $d_{5/2}$, $s_{1/2}$, and $d_{3/2}$, these levels have the same particle configuration as the three single-particle levels built on the ground state. This shell model interpretation is the simplest example of weak coupling which can be presented in the framework introduced by de Shalit and Talmi.[6] This is not presenting a surprising result, since the weak coupling model should predict the data as found. The situation found