Rev. Letters 8, 261 (1962).

[18]S. Fubini and G. Segre, Nuovo Cimento 45A, 641 (1966).

[19]These are the two superconvergence relations of Ref. 1; an additional superconvergence relation of the type $\int \nu \, \mathrm{Im} f(\nu) d\nu = 0$; two independent Adler-Weisberger sum rules; two independent forward dispersion relations for the $I=2$ spin-nonflip amplitude supplemented by the PCAC and current-algebra calculation of the s-wave scattering lengths; the Cabibbo-Radicati sum rule (4); and our Eq. (2) supplemented by ρ dominance of the electromagnetic current.

EVIDENCE FOR AN $I=1$ DI-PION RESONANCE AT 1.63 GeV*

David J. Crennell, Paul V. C. Hough, George R. Kalbfleisch, Kwan Wu Lai, J. Michael Scarr, Thomas G. Schumann, Ian O. Skillicorn, Richard C. Strand, and Medford S. Webster
Brookhaven National Laboratory, Upton, New York

and

Philip Baumel, Alvin H. Bachman, and Robert M. Lea
The City College of the City University of New York, New York, New York
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A neutral di-pion $\pi^+\pi^-$ enhancement with a mass of 1675 MeV was first reported by Goldberg et al.[1] in high-energy $\pi+N \rightarrow \pi^+ + \pi^- + N$ interactions. Subsequently, several experiments have confirmed its existence.[2] Because of large background and inadequate statistics, little progress has been made in understanding the nature of this enhancement. To date, information relevant to its isospin[3,4] comes from a study of the reaction $\pi^+ + p \rightarrow \pi^+ + \pi^0 + p$. The analysis of this reaction is complicated by strong $N^*(1238)$ formation in both the π^+p and π^0p systems. In particular, the structure in the high $\pi^+\pi^0$ mass region (>1.0 GeV) comes entirely from events in the $N^*(1238)$ region; consequently, the interpretation of this structure is difficult. In this paper we report evidence for an isospin one ($I=1$), positive G parity ($G=+$), negative spatial parity ($P=-$), and odd spin ($J=3$ is favored) $\pi\pi$ resonance at ~1630 MeV with a width of about 100 MeV.

The sample of events for this study comes from a 6-GeV/c $\pi^\pm p$ experiment in the Brookhaven National Laboratory (BNL) 80-inch liquid-hydrogen bubble chamber. About 30 000 two-prong events were analyzed in the π^-p exposure, and about 6000 two-prong events were analyzed in the π^+p exposure. About half the events were measured by conventional measuring machines and the other half by the BNL flying spot digitizer. A systematic comparison of the characteristics of the events measured by the two systems has been reported elsewhere.[5] Here we note that the two methods of measurement and analysis give results which are equivalent for the purposes of this study. The numbers of events obtained from this analysis are[6]

$$\pi^+ + p \rightarrow \pi^+ + \pi^+ + n, \quad 373 \text{ events}; \tag{1}$$

$$\pi^- + p \rightarrow \pi^- + \pi^0 + p, \quad 1308 \text{ events}; \tag{2}$$

$$\pi^- + p \rightarrow \pi^- + \pi^+ + n, \quad 2661 \text{ events}. \tag{3}$$

Figures 1(a)-1(c) are the Dalitz plots and Figs. 2(a)-2(c) are the corresponding di-pion mass distributions for the final states (1), (2), and (3), respectively. Although Fig. 1(a) shows that there is some N^* formation in Reaction (1), there is no significant structure in the $\pi^+\pi^+$ mass distribution shown in Fig. 2(a).

The $\pi^-\pi^0$ mass projection from Reaction (2), Fig. 2(b), shows the ρ^- and a peak at 1630 MeV about 100 MeV wide,[7] which is about four standard deviations above background. The correction for scanning bias[8] against short protons is indicated by the dotted area in Fig. 2(b). Some $N^*(1238)$ formation in the π^0p system is evident in Fig. 1(b). The hatched area in Fig. 2(b) is the histogram of events having neither π^0p nor π^-p effective mass in the region 1200-1280 MeV. This clearly demonstrates that the 1630 peak is associated with events which do not show $N^*(1238)$ production.

There are three enhancements in the $\pi^+\pi^-$ spectrum, Fig. 2(c), namely, ρ^0, f^0, and a broad $\pi^+\pi^-$ bump from 1560 to 1800 MeV. We see no obvious $N^*(1238)$ formation in the associated Dalitz plot, Fig. 1(c). The statistical significance of the 1560- to 1800-MeV bump is about four standard deviations above background. The difference in mass and width of this bump and the 1630 MeV $\pi^-\pi^0$ peak will be discussed below. It is also interesting to note

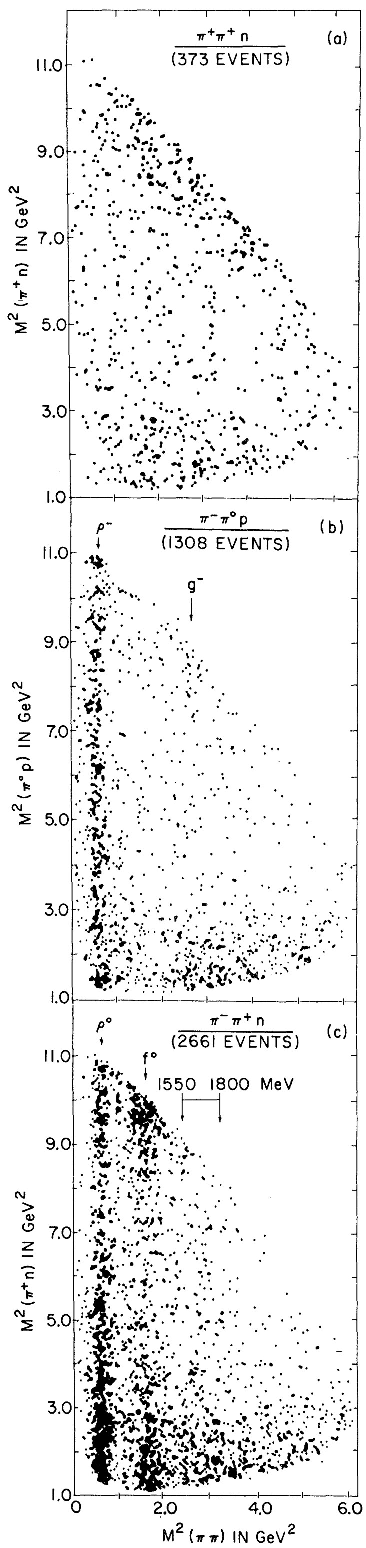

FIG. 1. Dalitz plots for the three final states as shown. Note that each event is plotted twice in Fig. 1(a) because the pions are indistinguishable.

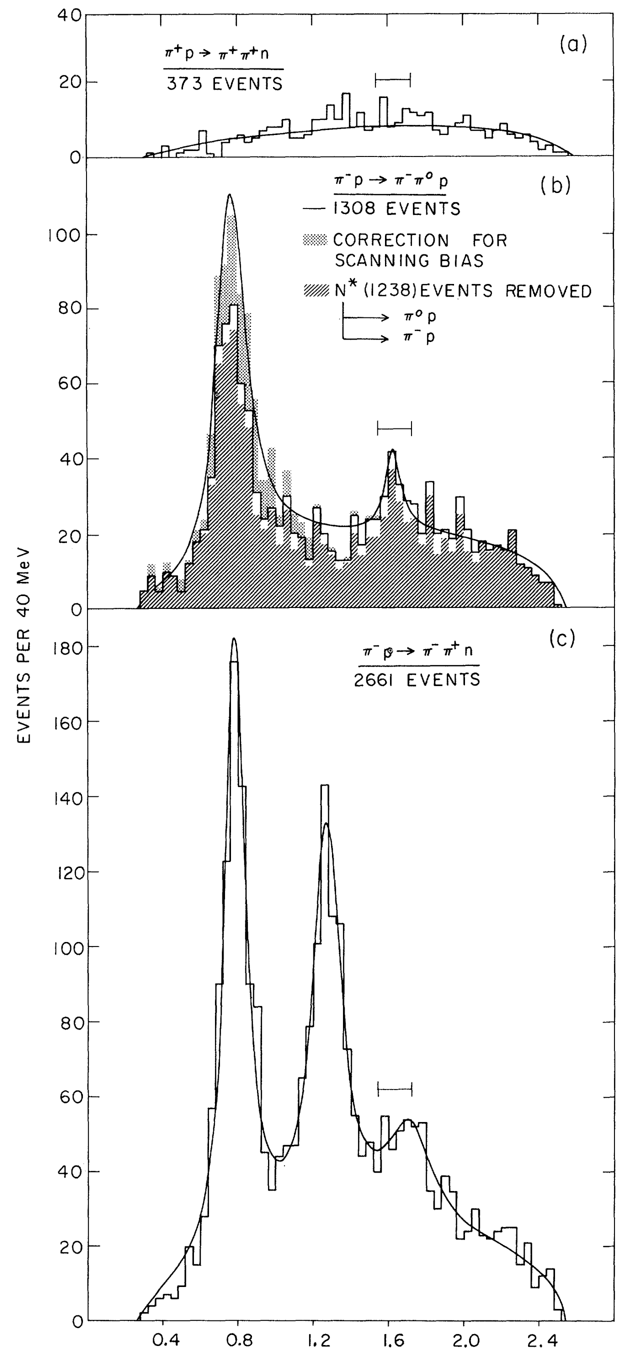

FIG. 2. Di-pion mass spectra for the three final states as shown.

the concentration of events at low π^+n mass and high $\pi^+\pi^-$ mass in Fig. 1(c) and a similar concentration in Fig. 1(b). These concentrations are probably a consequence of diffraction scattering in the $\pi\pi$ system at this high energy.

To further substantiate the evidence for the $\pi^-\pi^0$ enhancement at 1630 MeV, we present the $\pi^-\pi^0$ scattering angular distribution of the outgoing π^- relative to the incident π^- in the $\pi\pi$ rest frame in this mass region (Fig. 3). A backward peak is clearly seen and remains even with N^* events removed (hatched area). This peak does not exist in the adjacent $\pi^-\pi^0$ mass region [see Fig. 1(b)]. We note that there is very little backward peak in the neutral $\pi^+\pi^-$ system in the same mass region [see Fig. 1(c)], which may be due to a difference in the interference with background.

Our determination of the relevant quantum numbers (I, G, P, and J) for this enhancement may be summarized as follows.

G parity: The strong decay into two pions constitutes evidence for positive G parity.

Isospin: Observation of the $\pi^-\pi^0$ decay mode requires $I \geq 1$. For the $I=2$ assignment only the $I=\frac{3}{2}$ amplitude in the πp initial state can contribute and the relative intensities for $\pi^+\pi^+$, $\pi^-\pi^0$, and $\pi^+\pi^-$ would be 36, 9, and 2, respectively. Our data disagree with the $I=2$ assignment by about $3\frac{1}{2}$ standard deviations.[9] We

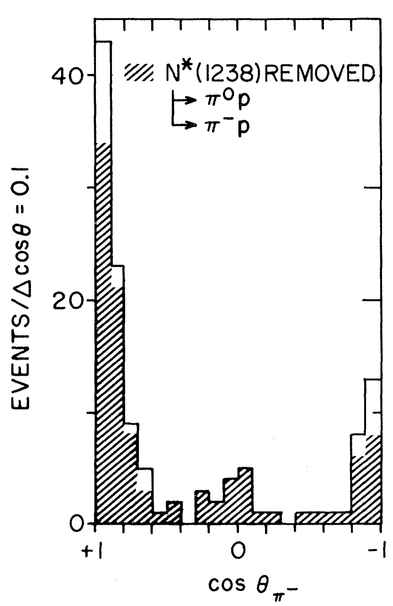

FIG. 3. The $\pi^-\pi^0$ scattering angular distribution in the $\pi\pi$ rest frame for the events from Reaction (2) with 1550 MeV $\leq M(\pi^-\pi^0) \leq$ 1700 MeV.

conclude that this is an $I=1$ di-pion state.

Parity: Bose-Einstein statistics requires that the total wave function for the diboson state be symmetrical. Hence, $I=1$ for this state implies odd parity and odd spin.

Spin: From the parity argument, we consider the series $J^P = 1^-, 3^-, \cdots$ as the possible assignments for this state. With our present data, we cannot conclusively distinguish between spin parities of $J^P = 1^-$ and $J^P \geq 3^-$. However, the moments of the Legendre polynomials (P_n) of the $\pi^-\pi^0$ scattering angular distribution suggest $J^P = 3^-$. This follows from the presence of a bump in the P_4 moment and an interference oscillation in the P_6 moment at this mass, each of approximately three standard deviations.[10]

It was noted above that both the position and width of the charged and neutral $\pi\pi$ enhancements in the 1630 mass region differ significantly.[11] Since we have ruled out an $I=2$ enhancement in this mass region, these differences may indicate the existence of an isospin-zero $\pi\pi$ enhancement at a mass of ~1750 MeV.

We have searched in our film for the other possible decay modes with $I^G = 1^+$ and $J^P \geq 1^-$ in the reactions $\pi + p \rightarrow p + (K\bar{K})^-$, $p + (K\bar{K}\pi)^-$, and $p + (4\pi)^-$. Examination of our data yields relative branching ratios of $(K\bar{K})^-/(2\pi)^- \leq 0.10$ and $(\pi^-\pi^-\pi^+\pi^0)/(2\pi)^- \leq 1.5$ (with 90% confidence level). In the $(\bar{K}K\pi)^-$ system we do observe a broad enhancement at this mass region.[12] Because of its large width it is inconsistent with the production of only this resonance. We are not able to resolve structure in this peak with present data.

To speculate about the existence of a new meson multiplet in the context of the SU(3) symmetry, we might group this $I=1$ $\pi\pi$ resonance, which we call the g_1 meson,[13] with a recently discovered $I=\frac{1}{2}$ $K^*(1800)$.[14] From the Gell-Mann–Okubo mass formula one can predict an octet singlet ($I^G=0^-$) at ~1850 MeV. In the framework of the Regge-pole theory, the g_1 meson would be the recurrence of the ρ meson if the spin of the g_1 meson is indeed three.

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stages of the experiment.

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[1]M. Goldberg, F. Judd, G. Vegni, H. Winzeler, P. Fleury, J. Huc, R. Lestinne, G. DeRosny, R. Vanderhaghen, J. Allard, D. Drijard, J. Hennessy, R. Huson, J. Six, J. Veillet, A. Floret, P. Musset, G. Bellini, M. DiCorato, E. Fiorini, P. Negri, M. Rollier, J. Crussard, J. Ginestet, and A. H. Tran, Phys. Letters 17, 354 (1965).

[2]See the compilation of the $\pi^+\pi^-$ mass spectrum from $\pi+N\to\pi^++\pi^-+N$ reactions by G. Goldhaber, B. C. Shen, N. P. Samios, A. Astier, and K. W. Lai, in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, California, 1966 (unpublished).

[3]M. Deutschmann, R. Schulte, R. Steinberg, H. Weber, W. Woischnig, G. Grote, J. Klugow, W. Meyer, S. Nowak, S. Brandt, V. Cocconi, O. Czyzewski, P. F. Dalpiaz, E. Flaminio, G. Kellner, and D. Morrison, Phys. Letters 18, 351 (1965).

[4]Recent data on the mass spectrum of negative bosons (X^-) in the reaction $\pi^-+p\to p+X^-$ [X^- can decay into one, three, or five charged particles plus possible neutral(s)] also showed structure in this region, in particular, a peak at 1630 MeV called R_1^-. See M. N. Focacci, W. Kienzle, B. Levrat, B. C. Maglić, and M. Martin, Phys. Rev. Letters 17, 890 (1966). It is to be noted that isospin as well as G parity for these enhancements are undetermined.

[5]P. V. C. Hough, summary talk on bubble-chamber data processing developments, in Proceedings of the International Conference on High Energy Instrumentation, Stanford, California, 1966 (to be published).

[6]The criteria used to obtain the sample of events for Reactions (1) and (3) were (a) 790 MeV < missing mass < 1090 MeV; and (b) consistency of observed or measured bubble density with that required by the kinematic fit. For Reaction (2), the criteria were (a) −300 MeV < missing mass < 450 MeV; (b) consistency of observed or measured bubble density with that required by the kinematic fit; (c) χ^2 probability >20%; and (d) error in missing mass < 500 MeV. The additional criteria for Reaction (2) were used in order to obtain a sample of well measured events for this study.

[7]The smooth curve in the figure is a maximum-likelihood fit to the data assuming phase space and the resonances. The resonances were fitted by Breit-Wigner distributions times phase space with the intensities, locations, and widths as variables. The values stated for the location and width of the reported $\pi^-\pi^0$ enhancement were obtained from this fit.

[8]A detection efficiency for short protons is obtained from the comparison of the slope of the diffraction peak deduced from our elastic events with the known slope from the existing data in this energy region [see, for example, K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters 11, 425 (1963)]. Little correction is necessary for events with $M(\pi^-\pi^0)$ greater than 1.0 GeV.

[9]There are 212 events on Fig. 2(b) in the seven bins from 1.48 to 1.76 GeV, and from the adjacent bins we estimate the background level as 19 ± 1 events per bin. This leaves 79 ± 20 events to be attributed to the peak, and correcting for the chi-squared cutoff of 20% (see Ref. 6) increases the estimate of events above background to 99 ± 25. From the adjacent bins in the $\pi^+\pi^+$ spectrum [Fig. 2(a)] we expect 10 ± 1 events per bin in the 1.48- to 1.76-GeV region. The hypothesis that the isospin is 2 requires 77 ± 19 events above background or a total of 147 ± 20 events. Only 73 ± 8.5 events are observed and these two numbers are inconsistent by 3.4 standard deviations.

[10]The moment analysis determined the coefficients a_n of a representation of the angular distribution $I(x) = \sum a_nP_n(x)$, where P_n is the nth Legendre polynomial and x is the cosine of the $\pi\pi$ scattering angle. The a_n are given by

$$a_n = (n+\tfrac{1}{2})[\sum P_n] \pm (n+\tfrac{1}{2})[\sum P_n^2]^{1/2},$$

where the sum is over the events. The coefficients $a_0, a_1, \cdots, a_{10}$ were plotted (not shown) versus the $\pi^-\pi^0$ mass in 80-MeV bins from 470 to 2470 MeV. The bin centered on 1630 MeV (1590-1670 MeV) shows a bump of approximately 3 standard deviations over the bins on either side in each of the a_0, a_2, and a_4 coefficients and an interference oscillation of similar size in a_6 in the bins 1510-1590 and 1590-1670 MeV. (Strong statistical correlations exist between the different a_n.) In general, the higher moments show a smooth behavior through the ρ^- and 1630-MeV regions. The odd coefficients $a_1, a_3, \cdots$ show smaller variations through the 1630-MeV region. A model with π and ω exchange and some background (due to N^* formation) can explain these results for both the ρ^- region and the 1630 region if the J^P of the 1630 enhancement is assumed to be 3^-. (We note that the nonisotropic Treiman-Yang angular distribution, not shown, of the ρ^- events requires more than pseudoscalar pion exchange.) Above 1700 MeV, the $\pi^-\pi^0$ scattering appears diffractionlike, although angular momenta higher than $l=3$ are not required by the a_n below 2100 MeV. A model with pure imaginary diffractionlike amplitudes for the background should give bumps in all a_n, even for a $J^P=1^-$ assignment for the 1630-MeV state. However, all a_n for even and odd n should show this behavior, which is not indicated by the data. Also, interference oscillations would not be expected. In summary, $J^P=3^-$ is suggested for the spin-parity assignment of this state. However, this suggestion cannot be considered conclusive in view of our lack of knowledge of the background.

[11]The maximum-likelihood fit gives about 3-standard-deviation differences for both their masses and their widths.

[12]The $K\bar{K}\pi$ mass spectrum from the $p(K\bar{K}\pi)$ final states in the samples containing two V^0 + two prongs and one V^0 + two prongs topologies showed a broad enhancement from 1.44 to 1.84 GeV.

[13]The authors in Ref. 1 proposed the name g for the

broad $\pi^+\pi^-$ enhancement which may contain more than one enhancement. Here we use g_1 for the $I=1$ di-pion resonance reported in this paper. We note that the masses of g_1 and R_1 (see Ref. 4) are the same.

[14]Aachen-Berlin-Cern-London (I.C.)-Vienna Collaboration, Phys. Letters 22, 357 (1966).

TOTAL MUON-CAPTURE RATE IN He^3 FROM CURRENT COMMUTATION RELATIONS*

C. W. Kim and Michael Ram
Department of Physics, The Johns Hopkins University, Baltimore, Maryland
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The total muon-capture rate in He^3 is calculated using the elementary-particle treatment of μ capture in complex nuclei and the current commutation relations introduced by Gell-Mann. Our calculation gives $\Gamma_{\text{theor}}(He^3) = 2020\ \text{sec}^{-1}$ to be compared with $\Gamma_{\text{exptl}}(He^3) = 2170^{+170}_{-430}\ \text{sec}^{-1}$.

We wish to present a calculation of the total muon-capture rate in He^3 in which we apply the elementary-particle treatment[1] of μ capture in complex nuclei and the current commutation relations introduced by Gell-Mann.[2] This calculation is meant to serve as additional evidence for the validity of the current commutation relations within the limits of the approximations introduced.

The total muon-capture rate in nuclei N_a is given by[3]

$$\Gamma(N_a) = \sum_b \Gamma(N_a \to N_b), \tag{1a}$$

where

$$\Gamma(N_a \to N_b) = \frac{G^2 m_\mu^{\ 5}}{(2\pi^2)} \frac{C(N_a)Z^3}{(137)^3} \eta_{ab}^{\ 2} \int \frac{d\Omega_\nu}{4\pi} \frac{1}{2(2J_a+1)} \sum_{\alpha=1}^{4} \sum_{\beta=1}^{4} \mathfrak{L}_{\alpha\beta} \mathfrak{M}_{\alpha\beta}(N_a \to N_b). \tag{1b}$$

In the above, we have set

$$\eta_{ab}^{\ 2} = \left(\frac{E_\nu}{m_\mu}\right)^2 \left(1 - \frac{E_\nu}{m_\mu + m_b}\right)\left(\frac{m_a}{m_\mu + m_a}\right)^3, \tag{2a}$$

$$\mathfrak{L}_{\alpha\beta} = (E_\mu E_\nu)^{-1}[(p_\nu)_\alpha (p_\mu)_\beta + (p_\nu)_\beta (p_\mu)_\alpha - \delta_{\alpha\beta}(p_\mu \cdot p_\nu) + \epsilon_{\alpha\beta\rho\sigma}(p_\nu)_\rho (p_\mu)_\sigma](-1)^{\delta_{\alpha 4}}, \tag{2b}$$

$$\mathfrak{M}_{\alpha\beta}(N_a \to N_b) = \sum_{M_a} \sum_{M_b} \langle N_a | \mathfrak{J}_\alpha^{(+)}(0) | N_b \rangle \langle N_b | \mathfrak{J}_\beta^{(-)}(0) | N_a \rangle (-1)^{\delta_{\alpha 4}}, \tag{2c}$$

$$\mathfrak{J}_\alpha^{(\pm)}(0) = V_\alpha^{(\pm)}(0) + A_\alpha^{(\pm)}(0), \tag{2d}$$

and

$$G = (1.00/m_p^{\ 2})10^{-5}. \tag{2e}$$

Natural units ($\hbar = c = 1$) are used throughout this paper. The proton mass is m_p, G the β-decay coupling constant, and Z the number of protons in the N_a nucleus. The muon and neutrino four-momentum are p_μ and p_ν, respectively, and E_μ and E_ν their energies. The masses of the muon and N_a and N_b nuclei are denoted by m_μ, m_a, and m_b, respectively. The nuclei N_a and N_b have spins J_a and J_b, respectively, and the summations in Eq. (2c) extend over the third components of the nuclear spins, M_a and M_b. The quantity $C(N_a)$ is a correction factor arising from the effect of the nonpoint character of the charge distribution of N_a. The sum over b in Eq. (1a) is over all energetically acces-