SOLAR OBLATENESS AND GENERAL RELATIVITY*

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New measurements of the solar oblateness have given a value for the fractional difference of equatorial and polar radii of $(5.0 \pm 0.7) \times 10^{-5}$. A corresponding discrepancy of 8% of the Einstein value for the perihelion motion of Mercury is implied.

It is generally believed that the observed classical excess motion of the perihelion of Mercury agrees[1] with the Einstein value with an accuracy of 1%. This assumes negligible contribution from a possibly oblate sun. However, the solar quadrupole moment can account for an appreciable part[2] of the excess. The suggestion[3,4] that the whole of the excess might have this origin is in conflict with planetary observations,[3,5] but as much as a 20% contribution might be accommodated.[2] This is a serious uncertainty, for the precession of Mercury's orbit is presently the most interesting of the three classical tests of general relativity. In fact the gravitational red shift[6] is predicted by a wide range of theories, and the gravitational deflection of light is but poorly known.[7]

According to the scalar-tensor theories of gravitation,[8-10] the precession of Mercury's perihelion should be less than the Einstein value, by about 10% if several independent observations have been correctly interpreted.[11-13] We are reporting measurements of the solar oblateness which indicate that 8% of the precession may be due to a solar quadrupole moment. This implies an 8% discrepancy in the Einstein value, and lends support to the scalar-tensor theory.

The sun may have a sizable quadrupole moment due to an internal rotation with a period of 1-2 days, for the magnetic torque induced by the solar wind on the sun's surface is believed to be great enough to keep the convective surface layer rotating slowly, balancing the viscous torque caused by a rapidly spinning core.[1,14]

How is the quadrupole moment of the sun, due to the unseen interior, to be measured? It is determined to a good approximation by the shape of the observed surface of the sun, in a manner independent of the details of the interior.[2] With negligible shear stresses in these outer "seen layers" of the sun, the surfaces of constant P, T, ρ, and Φ all coincide.[15] Here $\Phi = \varphi - \frac{1}{2} r^2 \omega_S^2 \sin^2\theta$ is the gravitational potential supplemented by the centrifugal force term of surface rotation. The observed solar oblateness is essentially that of a surface of constant density, hence of constant Φ. The gravitational potential φ is thus determined over this surface and hence, through Laplace's equation, exterior to the sun. This exterior solution determines the quadrupole moment of the sun.

The surface layers of the sun do contain weak shear stresses due to turbulence velocities, differential rotation, and magnetic fields. These are observable, and their effects appear to be small. Details will be given elsewhere.

The observed oblateness of the sun is designated $\Delta = (r_{\text{eq.}} - r_{\text{pole.}})/r$. The oblateness of a surface of constant gravitational potential at the solar surfaces is

$$\Delta_\varphi = \Delta - \tfrac{1}{2}\omega_S^2 r/g_S = \Delta - 1.0 \times 10^{-5},$$

where g_S is the gravitational acceleration at the sun's surface. If $\Delta_\varphi = 5 \times 10^{-5}$, the quadruple moment would contribute 4.3″ arc/century, 10% of the general-relativity effect. An oblateness this small would have no other important effect on planetary orbits.[2,5]

We have characterized the shape of the solar image by two parameters, the orthogonal components

$$\Delta_v = \Delta_0 \cos 2\alpha$$

and

$$\Delta_d = \Delta_0 \sin 2\alpha,$$

where α is the orientation angle expressed as a counter-clockwise rotation of the minor axis from the north-south direction and Δ_0 is the fractional difference between the major and minor axes. Δ_v and Δ_d are designated as "vertical" and "diagonal" components.

The observed oblateness of the solar image must be corrected for the effect of atmospheric refraction. To a good approximation the correction can be computed as due to a laminar optical structure which bends a light ray by an amount which depends only on the zenith angle and the index of refraction of the air at the instrument. The magnitude of this "refractive oblateness" is $\Delta_R \sim 2.5\times10^{-4}\tan^2 Z$ where Z is the zenith angle of the sun. This correction is an odd function of the time of day (about noon) for the diagonal component and an even function for the vertical component.

After subtraction of the refractive effect, the residual oblateness should be along the rotation axis of the sun. α is then replaced by β, the orientation angle of the solar axis. This angle is known from other observations, and β is a known function of time. Thus, after this correction for atmospheric distortion, the true solar oblateness Δ can be obtained from either the vertical or the diagonal component.

During the observation period of 1 June-23 September 1966, the strong time dependence of $\sin 2\beta$ and hence Δ_d, corrected for atmospheric refraction, crossing zero on 7 July, provides a convenient test for the validity of the results. By contrast the variation of $\cos 2\beta$ is slight, weakening the test of the vertical component. Furthermore, we had expected and found a positive systematic error in the vertical component, due to thermal spherical distortion of our main quartz mirror. This tracking mirror maintains a fixed orientation with the sun and because of symmetry this error could not affect the diagonal component. Another systematic error, affecting the vertical component only, was discovered in the data after the conclusion of the period of observations. Because measurements could not be repeated, nor new tests introduced, conclusions regarding its origin are uncertain. Nonetheless the effect is well characterized by the available data. The effect agrees quantitatively with the hypothesis that the error results from an anisotropic distortion of the "seeing disk." Calculations based on a model involving the mixing of warm and cool air at the optic entrance to the building is capable of accounting for the observations, but there is no other direct reason to believe that the model is significant.

The combination of these two systematic errors makes the vertical component of the data virtually worthless. But the analysis based on the diagonal component of the data appears to be of good quality. It passes all the tests

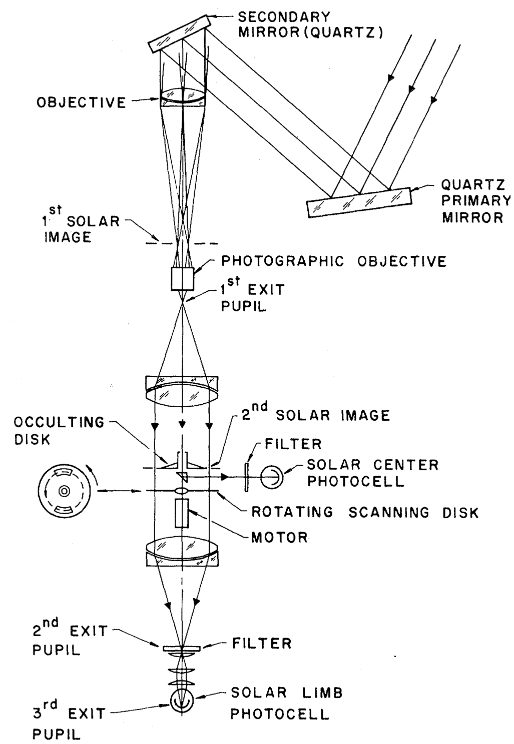

FIG. 1. Optical system; primary mirror tracks the sun.

for validity that we have devised. In particular, it appears to be free of the little-understood systematic error affecting the vertical component.

Previous measurements of the solar oblateness were based on observations of the limb of the sun.[16,17] By contrast, our measurements were based on integrations of the luminous flux from a radius well inside the limb of the sun outward beyond the limb. This technique is relatively insensitive to the effects of "anisotropic seeing." Consequently, the results were apparently not particularly adversely affected by the relatively poor atmospheric conditions at Princeton.

Referring to Fig. 1, the light from the sun is reflected from the primary quartz mirror, 16.5 cm in diameter, tracking the sun, to a fixed secondary quartz mirror, then down a vertical telescope of 90 cm focal length. The solar image, about 7 cm in diameter, is projected on an occulting disk. The light from the outer few percent of the sun's image is passed by the field stop, an annular gap of 44″ arc between the occulting disk and an outer iris. This light falls on a motor-driven scanning disk, rotating at a rate of 123 revolutions/sec and carrying two diametrically opposed apertures of slightly different size. Light passing the scanning wheel is detected by a photocell. After phase-sensitive detection the 123-cycle/sec components of the photocurrent provide two signal voltages used in a servo system to automatically orient the main mirror and center the solar image on the occulting disk.

Two quadrature components of the second harmonic signal are analyzed by phase-sensitive detection equipment to provide zero-frequency measures of two components of solar oblateness. 1-minute time averages of these voltages, together with measures of the light flux past the occulting disk and the central brightness of the sun's disk, are recorded. The instrument was calibrated frequently by replacing the occulting disk by a calibrator disk.

After each 1-minute integration, the whole of the optical system below the secondary mirror was rotated 90° about a vertical axis to eliminate errors associated with these components. The two quartz mirrors were periodically rotated 90° about axes normal to their faces to eliminate any errors due to astigmatism.

Another astigmatic effect, not corrected by mirror rotation, is that due to the primary mirror, slightly spherical and viewed off axis. This effect is proportional to the distance from the objective to the mirror and is small for the secondary mirror. This off-axis astigmatism, thermal in origin, was the source of one of the aforementioned systematic errors affecting the vertical component of the oblateness. As this primary mirror rotates with the sun, we had no direct way of determining this error or of eliminating it.

A systematic brightening of the sun in the equatorial or the polar regions could be a source of systematic error in the measured oblateness. The possible brightness variation about the limb of the sun is determined by measuring the effective oblateness with different amounts of the sun protruding beyond the occulting disk. By combining the observations, separate measures are obtained for both the oblateness and the brightness variation. Measurements were made at three different magnifications of the solar image, for which the amounts of exposed limb were 6.5, 12.8, and 19.1 seconds of arc.

An analysis of the diagonal component averaged over the period of 1 June to 14 August 1966 showed the sun to be remarkably uniform in its brightness. The brightness difference between pole and equator was too small to be reliably measured. Expressed in terms of a simple temperature difference, it would not exceed 3°K. Previous measurements by others[18-20] had given larger values or limits.

Other possible sources of systematic error are the effects of an oblate corona, prominences on the limb, an oblate chromosphere, and shear stresses induced by velocity and magnetic fields. As will be discussed elsewhere, these various effects were estimated and they appear to be negligible.

In Fig. 2 the plotted points represent the diagonal component of the visual oblateness Δ at the three magnifications. With the exception of the 15 July points, each plotted point represents approximately 10 days of observation. All data taken after 26 September (a few poor days) were omitted. The curve is computed from the known time dependence of β, i.e., the known position of the solar axis, assuming $\Delta = (5 \pm 0.7)\times{}^{-5}$. This value was chosen for a best fit and the error is based on the internal consistency of the measurements, without allowance for undiscovered systematic effects.

Any doubt about the reliability of the correction for atmospheric refraction might be resolved by plotting the data separately for different periods of the day. A graph similar to Fig. 2 but plotted for data averaged over the hours 9-11, 11-13, and 13-15, shows no systematic dependence of the residual oblate-

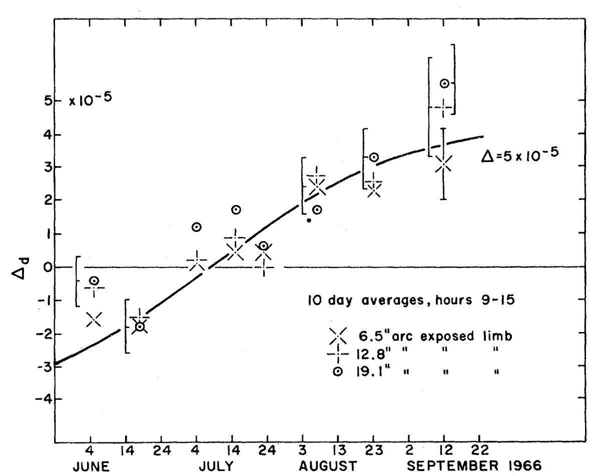

FIG. 2. Diagonal component of the solar oblateness.

ness on the time of day.

The observed solar oblateness implies a perihelion rotation of 3.4″ arc/century. After this is subtracted from the observed residual there remains as an effect, presumably relativistic, of 39.6″ arc/century. This is 8% less than Einstein's value, in reasonably good agreement with expectations under the scalar-tensor theory.

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SUM RULES FOR PHOTON-HADRON SCATTERING

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Here we consider the forward Compton scattering amplitude for photons incident on particles of charge e and mass M. When averaged over the spin of the incident photon and target particle the Compton amplitude is specified by a single analytic function $f(\nu)=f^*(-\nu)$ of the photon energy ν which satisfies the low-energy Thomson theorem, $4\pi f(0)=-e^2/M$. The optical theorem informs us that the absorptive part is specified by the total cross section $\mathrm{Im}f(\nu)=(\nu/4\pi)\sigma_{\mathrm{tot}}(\nu)$. These requirements on $f(\nu)$ along with the assumption that $f(\nu)/\nu^2 \to 0$ as $\nu\to\infty$ have as a rigorous consequence the dispersion relation

$$f(\nu)=-\frac{e^2}{4\pi M}+\frac{\nu^2}{2\pi^2}\int_0^\infty \frac{d\nu'\sigma_{\mathrm{tot}}(\nu')}{\nu'^2-\nu^2}.$$

As is well known the stronger asymptotic assumption that $f(\nu)\to 0$ as $\nu\to\infty$ leads to the contradiction $-e^2/M=2/\pi\int d\nu\sigma_{\mathrm{tot}}(\nu)>0$. Hence the dispersion relation for $f(\nu)$ must have at least one subtraction, a conclusion independent of any detailed theory of dynamics at high energy.

One may still extract information from this