DYNAMICAL MODEL FOR NEGATIVE-PARITY REGGE TRAJECTORIES IN THE PION-NUCLEON SYSTEM*

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The Cook-Lee model for the "second" pion-nucleon resonance $(1512, \frac{3}{2}^-)$ has been generalized to arbitrary initial and final baryons. It is shown that the virtual production of S-wave ρ mesons included in $\pi+S^+ \to \rho+S^+$, where S^+ is the set of all positive-parity resonances originating in the nucleon and the 3-3 resonance, generates the pair of trajectories S^- comprised of a series with $(T, J^P)=(\frac{1}{2},\frac{3}{2}^-),(\frac{1}{2},\frac{7}{2}^-),\cdots$ and $(\frac{1}{2},\frac{5}{2}^-),(\frac{1}{2},\frac{9}{2}^-),\cdots$. In addition, subsidiary trajectories $(\frac{1}{2},\frac{1}{2}^-),(\frac{1}{2},\frac{5}{2}^-),\cdots$ and $(\frac{3}{2},\frac{1}{2}^-),(\frac{3}{2},\frac{5}{2}^-),\cdots$ are obtained.

The most prominent pion-nucleon resonances[1] can be grouped into two "principal series" S^+ and S^-. The group of objects S^+ is made up of the members of two positive-parity Regge trajectories N_i^+ and Δ_i^+. The subscript values $i=1,2,\cdots$ correspond to states of isospin-spin parity (T, J^P) of $(\frac{1}{2},\frac{1}{2}^+),(\frac{1}{2},\frac{5}{2}^+),\cdots$ for $N_1^+, N_2^+,\cdots$ and $(\frac{3}{2},\frac{3}{2}^+),(\frac{3}{2},\frac{7}{2}^+),\cdots$ for Δ_1^+, $\Delta_2^+,\cdots$. A partial understanding of the dynamical framework underlying S^+ has been obtained previously.[2,3]

In the present Letter we show that the series S^- can be generated from S^+ by a systematic generalization of the model developed by Cook and Lee[4] for the 1512-MeV pion-nucleon resonance. S^- is comprised of the trajectories N_i^- and $N_i'^-$, where $N_1^-, N_2^-,\cdots$ have (T, J^P) of $(\frac{1}{2},\frac{3}{2}^-),(\frac{1}{2},\frac{7}{2}^-),\cdots$ and $N_1'^-, N_2'^-$, $\cdots$ have (T, J^P) of $(\frac{1}{2},\frac{5}{2}^-),(\frac{1}{2},\frac{9}{2}^-),\cdots$. As yet there is no firm evidence for $N_2'^-$ but the other members of S^- appear to be established.[5] In addition, somewhat weaker forces occur which give another trajectory $N_i''^-$: $(\frac{1}{2},\frac{1}{2}^-)$, $(\frac{1}{2},\frac{5}{2}^-),\cdots$. Finally, there is a trajectory Δ_i^-: $(\frac{3}{2},\frac{1}{2}^-),(\frac{3}{2},\frac{5}{2}^-),\cdots$.

In our model the series S^- arises from the attractive forces due to virtual transitions from the πS^+ configuration into closed ρS^+ (s-wave) channels. The individual transitions contained in

$$\pi+S^+ \to \rho+S^+ \tag{1}$$

are assumed to be dominated by one-pion exchange.

We have found it useful to represent the "constellation" of resonances built from the four trajectories by a parallelepiped (Fig. 1). In this figure we have omitted the isospin but labeled the baryons with an additional quantum number,[6] the "γ parity," where $\gamma=(-1)^{J+L+\frac{1}{2}}$, L being the orbital angular momentum of the coupled pion in $\pi+N\to B_i$. The structure of the vertex $\pi B_i B_j$ is determined[6] by the relative γ parity of the two arbitrary-spin baryons B_i and B_j.

If the series S^+ continues indefinitely to higher spin states, our calculations show that S^- also will contain a corresponding number of particles, provided that the (presently unknown) $\pi B_i B_i$ couplings do not become too small. The exciting possibility of an infinite (or very large) number of excited states of the nucleon has been raised by the analysis of Barger and Cline.[7]

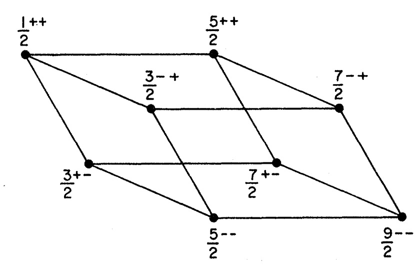

FIG. 1. The principal Regge trajectories of the pion-nucleon system are represented conveniently by a parallelepiped. The baryons have been labeled by $J^{P\gamma}$, where in addition to the usual J^P assignment we have included the "γ parity." (This quantity[6] determines the structure of the $\pi B_i B_j$ vertex.) The dynamical relations among these trajectories are considered in the text. The isospins are all $\frac{1}{2}$, except for the states $\frac{3}{2}^{+-}$, $\frac{7}{2}^{+-}$, which have isospin $\frac{3}{2}$.

According to our present dynamical understanding of the situation, these high-spin excitations will become more and more difficult to observe in ordinary πN experiments, since the πN channel becomes dynamically less significant as the energy is increased.

The general formulas required for the analysis have been presented elsewhere.[8] Multichannel N/D equations are solved in the pole approximation. Here we present only the most interesting results of an extensive numerical calculation.[9]

First consider the family of two-channel models in which the channel πB_i is coupled to ρB_i by transitions of the type $\pi + B_i \to \rho + B_i$, where B_i is one of the states in S^+, having spin S_i and mass M_i. The resulting odd-parity states have $J = S_i + 1$, S_i, and $S_i - 1$ reached from πB_i states with minimum L of 2, 0, and 2, respectively. The $L = 0$ state lacks a centrifugal barrier and is not considered further here. From the analysis of Ref. 8 it follows that the amplitude for production of a ρ polarized oppositely to the final baryon spin is suppressed relative to that for the parallel configuration. Thus the maximum spin $S_i + 1$ is the dominant state in this model. Isospin $\frac{1}{2}$ is also strongly preferred.[8] When $T_i = \frac{1}{2}$, the ratio of squared transition amplitudes is 4:1 in favor of isospin $\frac{1}{2}$ over isospin $\frac{3}{2}$. When $T_i = \frac{3}{2}$, the corresponding quantities are in the ratio 25:4:9 for $T = \frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$. The kinematical conditions in all models of this class are quite similar.[10]

We suggest that the preceding mechanism is the most important for the existence of S^-. The case $B_i = N_1^+ = N$, originally considered by Cook and Lee,[4] leads to N_1^-, while $B_i = \Delta_1^+$, discussed by Auvil and Brehm,[11] gives rise to $N_1'^-$. Taking $B_i = N_2^+$ and Δ_2^+, respectively, gives the recurrences N_2^- and $N_2'^-$. These states appear in the πN channel as resonances in the states D_{13}, D_{15}, G_{17}, and G_{19}, respectively.

In order to put the above results in perspective we have to discuss in addition the many configurations πB_i which couple to a given pair ρB_j. We first consider $M_i \leq M_j$. If $T_i = T_j$, isospin $\frac{1}{2}$ dominates, while if $T_i \neq T_j$, isospin $\frac{3}{2}$ is preferred by a ratio 5:2 ($T_i = \frac{1}{2}$ or $\frac{3}{2}$). The πB_i configuration having the greatest minimum L will dominate.[12] When several πB_i configurations couple, a detailed analysis is required to determine the most important. Lack of knowledge of many $\pi B_i B_j$ couplings is a burden, but some educated guesses can be made.[13]

The results for ρN_2^+ are typical. If we consider $\pi + B_i \to \rho + N_2^+$ for $B_i = N_1^+$, Δ_1^+, N_1^-, N_2^+, we find that only N_1^+ and N_2^+ are important. In fact, N_1^+ is too important[14] when the unmodified one-pion-exchange amplitude is used. This can be traced to the "unitarity disease" which afflicts all those Born amplitudes having large momentum transfers. To deal with this difficulty we modified the one-pion-exchange amplitudes by a universal factor F containing one parameter. Our choice was required to give good results for the N_i^- trajectory, i.e., the N_1^- resonance is required to remain near the experimental value, while for N_2^- we require sufficient suppression of $\pi + N_1^+ \to \rho + N_2^+$ ($t_0 = -50.0\mu^2$) relative to $\pi + N_2^+ \to \rho + N_2^+$ ($t_0 = -20.0\mu^2$) so that the latter dominates (the phase shift passes through zero at resonance[15]). The most satisfactory choice of F was found to be $\exp[-(t-\mu^2)^2/M^4]$, with M^2 equal to $20\mu^2$.

With the universal form factor we then find the following results.

N_i^- trajectory.—N_1^- (D_{13}) occurs at 1535 MeV. For $g^2(N_2^+, N_2^+)/4\pi = 3.75 g^2(N_1^+, N_1^+)$ the recurrence state[16] (G_{17}) occurs at 2190 MeV with absorption parameter $\eta = 0.71$ and resonant phase shift 0°. This very strong absorption is compatible with Ref. 15.

$N_i'^-$ trajectory.—For $g^2(\Delta_1^+, \Delta_1^+)/4\pi$ equal to 2.56 times the SU(6) value there is a D_{15} state at 1700 MeV ($\delta_{\rm res}$ is zero).[17] Evidence for this state has been given by Bareyre et al.[18] and Lovelace.[19] We also obtain a $\frac{9}{2}^-$ recurrence at about 2330 MeV. The mass prediction is uncertain since none of the coupling constants are known.[20] This resonance is weakly coupled to the πN system (G_{19}) and could be most easily discovered in its $\pi\Delta_1^+$ or $\pi\Delta_2^+$ decay modes.

$N_i''^-$ trajectory.—The same mechanism leading to $N_i'^-$ ($L = 2$ pions in $\pi\Delta_1^+$ and $\pi\Delta_2^+$ channels) gives substantial forces when $\vec{L}$ points oppositely to the baryon spin. We predict a "companion trajectory" $N_i''^-$ which couples to the πN states S_{11} ($W_r \approx 1950$ MeV) and yet another D_{15} object (≈ 2375 MeV) slightly higher than the predicted G_{19}.

Δ_i^- trajectory.—Among those transitions with $M_i > M_j$, the only cases which suggest resonance activity are $\pi + \Delta_1^+ \to \rho + N_1^+$ and $\pi + \Delta_2^+ \to \rho + N_2^+$. These reactions are kinematically rather different from those previously considered and may be less reliable. The $L = 2$ $\pi\Delta_1^+$ state gives rise to a $T = \frac{3}{2}$, $\frac{1}{2}^-$ object which would couple to the S_{31} πN state. The energy obtained (1420 MeV) is rather too low compared with the value 1692 MeV obtained by Donnachie, Lea, and Lovelace.[21] We also obtain a recurrence (coupled to D_{35}) at an energy of about 2150 MeV and a $T=\frac{3}{2}$, $\frac{3}{2}^-$ object around 2130 MeV. These energies are probably unreliable but the resonances are worth looking for.

A systematic calculation of all transitions $\pi+B_i\rightarrow\rho+B_j$ revealed no other resonances.[22]

Finally, we also reconsidered the mechanism $\pi+S^+\rightarrow f+S^+$, special cases of which have been proposed by Auvil and Brehm[23] for the higher spin members of S^+. These transitions characteristically involve much larger momentum transfers than $\pi+S^+\rightarrow\rho+S^+$. If the f production amplitudes are domesticated by means of the same function F used for the ρ mechanism, the forces are so weakened as to be unable to produce resonances. It appears, however, that the f mechanism provides a nonnegligible attraction.

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[1]Baryon data are taken from A. H. Rosenfeld, A. Barbaro-Galtieri, W. J. Podolsky, L. R. Price, Matts Roos, Paul Soding, W. J. Willis, and C. G. Wohl, to be published. We have followed these authors in denoting isospin $\frac{1}{2}$ ($\frac{3}{2}$) objects by N (Δ). Other identifying labels differ in detail. We do not discuss objects regarded as threshold effects.

[2]P. Carruthers, Phys. Rev. Letters 10, 540 (1963); 12, 259 (1964); Phys. Rev. 133, B49 (1964); Phys. Rev. (to be published). Further results and a summary of the situation will be found in Lectures in Theoretical Physics (University of Colorado Press, Boulder, Colorado, 1965), Vol. VIIb, p. 83, or in Perspectives in Modern Physics, edited by R. E. Marshak (John Wiley & Sons, Inc., New York, 1966), p. 159.

[3]In Refs. 2 it is shown that the combined effect of the elastic forces due to exchange of the ρ meson and the constituents of S^+ is to give strong elastic forces in just those states comprising S^+. The other states have repulsive or only weakly attractive forces (disregarding the s waves).

[4]L. F. Cook, Jr., and B. W. Lee, Phys. Rev. 127, 283, 297 (1962).

[5]The following values for the masses (in MeV) were used for the particles N_1^+, N_2^+, Δ_1^+, Δ_2^+, N_1^-, N_2^-, Δ_1^-: 938, 1688, 1238, 1920, 1512, 2190, 1670. Reference 1 gives N_1^- as 1525 MeV.

[6]P. Carruthers, Phys. Rev. (to be published). The definition of γ in the present paper is opposite in sign to that in this reference.

[7]V. Barger and D. Cline, Phys. Rev. Letters 16, 913 (1966).

[8]P. Carruthers, Phys. Rev. (to be published).

[9]P. Carruthers and M. M. Nieto, to be published.

[10]The most important quantity, the invariant momentum transfer t_0, is not sensitive to i when $B_i=B_j$. For $B_i=N_1^+$, Δ_1^+, N_1^-, N_2^+, Δ_2^+, the value of $-t_0$ at the B_i threshold is, respectively, 15.5, 17.3, 18.6, 19.3, and 20.0 in units of the pion mass squared. When $B_i\neq B_j$ this quantity varies widely.

[11]P. R. Auvil and J. J. Brehm, Phys. Rev. 145, 1243 (1966).

[12]This was not assumed, but was checked in every case by explicit calculation.

[13]Some experimental information exists which can be used to derive πB_iB_j couplings for the pairs $(B_i,B_j)=(N_1^+,B_i)$, any i, (Δ_1^+,N_1^-), (Δ_1^+,N_2^+). The formulas of Ref. 6 were used. For (Δ_1^+,Δ_1^+) we can compare with the SU(6) value (see Ref. 12). We assumed that the coupling $\pi N_1^+N_2^+$ was the same as $\pi\Delta_1^+\Delta_2^+$. $\pi N_2^+N_1^-$ was assumed to be the same as $\pi N_1^+N_1^-$. All "diagonal" couplings πB_iB_i were assumed to be of the same order of magnitude. Except for $\pi\Delta_1^+\Delta_1^+$ we adjusted them to fit experiment. Further details on these questions will be published elsewhere.

[14]The resonance G_{17} in the model with coupling $\pi+N_1^+\rightarrow\rho+N_2^+$ appears at 1550 MeV.

[15]A. Yokosawa, S. Suwa, R. E. Hill, R. J. Esterling, and N. E. Booth, Phys. Rev. Letters 16, 714 (1966).

[16]This result is for the three-channel model in which πN_1^+, πN_2^+, and ρN_2^+ are coupled by the one-pion-exchange force.

[17]This result is for a three-channel model: πN_1^+, $\pi\Delta_1^+$, and $\rho\Delta_1^+$. πN_1^+ is only weakly coupled to $\rho\Delta_1^+$ for isospin $\frac{1}{2}$ and so influences the mass only slightly.

[18]P. Bareyre, C. Brickman, A. V. Stirling, and G. Villet, Phys. Letters 18, 342 (1965).

[19]C. Lovelace, to be published.

[20]The quoted mass was obtained for a three-channel model ($\pi\Delta_1^+$, $\pi\Delta_2^+$, $\rho\Delta_2^+$) with the following coupling constants: $g^2(\Delta_1^+,\Delta_2^+)/4\pi=1.96\times10^{-2}$, $g^2(\Delta_2^+,\Delta_2^+)=g^2(N_1^+,N_1^+)$.

[21]A. Donnachie, A. T. Lea, and C. Lovelace, Phys. Letters 19, 146 (1965). Reference 1 lists the $(\frac{3}{2},\frac{1}{2}^-)$ Δ_1^- at 1670.

[22]We only considered $B_j=N_1^-$ among the negative-parity objects. No resonance ensued in the most favorable state (F_{15}) although one could occur if $g^2(N_1^-,N_1^-)$ were substantially larger than $g^2(N_1^+,N_1^+)$.

[23]P. R. Auvil and J. J. Brehm, Phys. Rev. 140, B135 (1965); Ann. Phys. (N.Y.) 34, 505 (1965).