HOLE STATES IN NUCLEI—^{41}Ca AND ^{47}Ca†

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The wave functions and energy levels of low-lying positive-parity states of ^{41}Ca and ^{47}Ca are calculated by assuming a configuration consisting of one hole in the $2s$-$1d$ shell and the valence nucleons in the $1f$-$2p$ shell.

A large amount of work has been done on particle-hole excitations in closed shell nuclei, e.g., the negative-parity states of ^{16}O and ^{40}Ca. In this work we consider particle-hole excitations in nuclei which have valence nucleons, in particular, ^{41}Ca and ^{47}Ca. We thus attempt to describe, by configurations consisting of one hole in the $2s$-$1d$ shell and two particles in the $2p$-$1f$ shell, states in, say, ^{41}Ca which have parities opposite to the parity of the ground state. Our calculation for ^{41}Ca is analogous to one done in ^{17}O by Margolis and de Takacsy.[1]

In ^{41}Ca the hole was restricted to be in either the $2s_{1/2}$ or $1d_{3/2}$ state; the particles could be in the $f_{7/2}$, $p_{3/2}$, $f_{5/2}$, and $p_{1/2}$ states. The corresponding single-particle energies, obtained from experiment, are taken as −9.76, −7.26, 0, 1.9, 6.4, and 4.0 MeV. We used the two-body matrix elements calculated by Kuo, who obtained an effective interaction from the Hamada-Johnston potential. This effective interaction includes core-polarization corrections in the particle-particle matrix elements and screening corrections in the particle-hole matrix elements.[2] A Hamiltonian matrix was constructed using these matrix elements, and wave functions and energy levels were obtained by diagonalizing this matrix. Some selected wave functions and energies are shown in Table I. The basis wave function is written as $[j_1j_2]J_0T_0j_3$

Table I. Selected States of ^{41}Ca.

J	T	E (MeV)	Wave Function $[j_1,j_2]J_0T_0j_3$
3/2	1/2	4.50	−0.86 [7/2,7/2]01 3/2+0.26 [7/2,7/2]21 3/2 −0.19 [5/2,5/2]01 3/2−0.18 [3/2,3/2]01 3/2 +0.17 [7/2,7/2]10 3/2−0.16 [7/2,5/2]10 3/2 +•••
3/2	1/2	6.59	−0.53 [7/2,7/2]10 3/2−0.42 [7/2,7/2]21 3/2 −0.41 [7/2,5/2]10 3/2−0.37 [7/2,7/2]30 3/2 −0.23 [3/2,1/2]10 3/2+0.19 [3/2,3/2]10 3/2 −0.16 [7/2,7/2]10 1/2+•••
3/2	3/2	6.12	−0.81 [7/2,7/2]01 3/2+0.43 [7/2,7/2]21 3/2 +0.21 [7/2,7/2]21 1/2−0.19 [3/2,3/2]01 3/2 −0.17 [5/2,5/2]01 3/2+0.13 [7/2,3/2]21 3/2 +•••
1/2	1/2	6.05	0.75 [7/2,7/2]21 3/2+0.40 [7/2,7/2]01 1/2 −0.41 [7/2,7/2]10 3/2+0.17 [7/2,5/2]10 3/2 +•••
1/2	1/2	6.40	−0.62 [7/2,7/2] 1/2−0.53 [7/2,7/2]10 3/2 −0.41 [7/2,7/2]10 1/2+0.20 [7/2,5/2]10 3/2 +0.17 [7/2,5/2]10 1/2+•••
9/2	1/2	6.62	0.79 [7/2,7/2]41 3/2+0.24 [7/2,7/2]50 1/2 −0.23 [7/2,7/2]30 3/2+0.23 [7/2,3/2]41 3/2 −0.21 [7/2,3/2]50 3/2−0.20 [7/2,7/2]61 3/2 −0.20 [7/2,7/2]50 3/2+•••

where j_1 and j_2 are the angular momenta of the two particles, J_0 and T_0 are the spin and isospin to which the two particles couple, and j_3 is the angular momentum of the hole.

The following interesting points are present in Table I:

(1) In the lowest $J=\frac{3}{2}$, $T=\frac{1}{2}$ state the two particles couple mostly to $T_0=1$. This is a consequence of the fact that the interaction between a hole and several particles is lowest when the particles have the largest possible isospin (consistent with a given total isospin).

(2) The second $J=\frac{3}{2}$, $T=\frac{1}{2}$ state is a complicated mixture of several basic states. The contribution of the $[f_{7/2},f_{5/2}]10d_{3/2}$ configuration is important. It should be noted that in ^{42}Sc the $J=1$ state also has a very large mixture of the configuration $[f_{7/2},f_{5/2}]10$.

(3) The energies of the calculated $J=\frac{3}{2}^+$ states lie higher than the experimental ones. States with $l_n=2$ strength at 2.017, 3.740, and 4.829 MeV were found by Belote, Sperduto, and Buechner[3] in a (d,p) experiment on ^{40}Ca. The lowest calculated $J=\frac{3}{2}$, $T=\frac{1}{2}$ state is at 4.50 MeV and the next one is at 6.59 MeV. The matrix elements that we used also give too high an energy for the particle-hole excitations in ^{40}Ca. This does not necessarily mean that the matrix elements are bad; it may well be that if other configurations are included, e.g., four-particle, three-hole, the energies of the positive-parity states would come out considerably lower. Furthermore, recent calculations[4] indicate that the $d_{3/2},f_{7/2}$ splitting should be taken somewhat smaller than 7.26 MeV, which is the value that we assumed.

(4) The lowest $J=\frac{1}{2}^+$ state has more $d_{3/2}$ hole component in it than $s_{1/2}$ hole. This is due to the large $s_{1/2}$, $d_{3/2}$ splitting that we assumed.

(5) A low-lying $J=\frac{9}{2}$ state is obtained at 6.62 MeV which is near the energy of the supposed single-particle $g_{9/2}$ state. This strongly suggests that the lowest $\frac{9}{2}^+$ state in ^{41}Ca is not a pure single-particle state but rather has in it a strong mixture of two-particle, one-hole component. In fact, Belote, Sperduto, and Buechner[3] found an $l_n=4$ state in ^{41}Ca at 4.983 MeV which had hardly any single-particle strength.

In a recent Letter by Belote et al.,[5] the results of the experiment ^{39}K(^{3}He,p)^{41}Ca were presented. This reaction should reach states of positive parity which have a proton-hole component in the $2s$-$1d$ shell. They found, among other things, that the lowest $J=\frac{3}{2}^+$, $T=\frac{1}{2}$ state had a cross section which was only about one-fifth as strong as the cross section to the lowest $J=\frac{3}{2}^+$, $T=\frac{3}{2}$ state. Looking at our results we see that the two wave functions, for $T=\frac{1}{2}$ and $T=\frac{3}{2}$, are very similar in their space-spin dependence. However, a $T=\frac{1}{2}$ state is a proton-hole state only $\frac{1}{3}$ of the time, whereas for the $T=\frac{3}{2}$ state the corresponding value is $\frac{2}{3}$. Thus our result suggests that the $T=\frac{3}{2}$ state should be reached twice as readily as the $T=\frac{1}{2}$ state. This is qualitatively in the right direction but it is not enough.

We next consider ^{47}Ca. In the shell model the ground state consists of a neutron hole in the $f_{7/2}$ shell. The states of positive parity consist of either neutron particle-hole excitations or proton particle-hole excitations from the s-d shell to the f-p shell. The following are examples of the basis wave functions we use: (a) $(d_{3/2})_n^{-1}$, neutron excited from $d_{3/2}$ shell into the $f_{7/2}$ shell; (b) $[(d_{3/2})_n^{-1}(f_{7/2})_n^{-1}]_{J_0}\times(p_{3/2})_n$, neutron excitation from $d_{3/2}$ shell to $p_{3/2}$ shell; and (c) $[(d_{3/2})_p^{-1}(f_{7/2})_n^{-1}]_{J_0}(f_{7/2})_p$, proton excited from $d_{3/2}$ to $f_{7/2}$, the proton hole and neutron hole coupling to angular momentum J_0.

Of interest here is the relative amount of neutron-hole and proton-hole component in the hole states. Even though neutron excitations into the $f_{7/2}$ shell are almost blocked by the presence of seven valence neutrons, our results, in agreement with the prediction of French and Bansal and of Ripka,[4] are that the lowest $J=\frac{3}{2}^+$ state has mostly neutron-hole component. If we restrict ourselves to the $d_{3/2}$ and $f_{7/2}$ configurations only, then the wave function is

$$0.88(d_{3/2})_n^{-1}+0.45[(d_{3/2})_p^{-1}(f_{7/2})_n^{-1}]_2(f_{7/2})_p$$
$$-0.14[(d_{3/2})_p^{-1}(f_{7/2})_n^{-1}]_5(f_{7/2})_p$$
$$+\text{small terms.}$$

About 77% of the wave function is neutron hole. If other configurations are allowed to mix this goes down to about 70%.

We plan to write a more detailed paper on this subject which will include the theory and more results. We would like to thank Tom Kuo for providing us the matrix elements, without which this calculation would not be possible. Very useful conversations with A. M. Green are gratefully acknowledged.

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SU(3) SYMMETRY TESTS FOR REGGE RESIDUES*

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Six independent combinations of cross-section data involving meson-nucleon charge-exchange amplitudes at the forward direction are analyzed in terms of ρ ($J^P=1^-$, $I^G=1^+$) and A ($J^P=2^+$, $I^G=1^-$) Regge-exchange model. SU(3) tests for the ratios of the Regge-pole residues give $\gamma_{\rho\bar{K}K}/\gamma_{\rho\pi\pi}=1.12\pm0.12$ and $\gamma_{A\bar{K}K}/\gamma_{A\pi\eta}=1.07\pm0.15$, consistent with exact symmetry values of 1.

The validity of SU(3) symmetry for scattering amplitudes still remains an open question. Early attempts[1] to relate scattering amplitudes using exact SU(3) met with only limited success. It has become clear that the nature of the dynamics accounts in many instances for the observed deviations from the exact symmetry relations. For example, reactions mediated by strange-meson or -baryon exchanges characteristically show a more rapid decrease with energy than reactions which proceed via non-strange-meson or -baryon exchanges.[2] Hence, in order to avoid such intrinsic dynamical symmetry breaking, direct SU(3) comparisons of scattering amplitudes can be made only among reactions with similar exchanges. Even with this restriction, effects due to nondegenerate masses or direct-channel resonances can give rise in some cases to appreciable deviations from exact symmetry. Therefore, to make meaningful SU(3) comparisons of scattering amplitudes, it is necessary to employ a suitable dynamical framework. The purpose of this Letter is to test certain SU(3) predictions for scattering at high energy, using the Regge-pole model as the dynamical theory.

The usual spirit of application of SU(3) symmetry in the context of the Regge-pole model is to assume that the factored residues are approximately SU(3) symmetric.[3] Symmetry-breaking effects are taken into account by allowing nondegenerate masses for the external particles and nondegenerate trajectories for the Regge exchanges. A particularly suitable test of the symmetry assumption for the residue factors is provided by charge-exchange reactions at high energy. Here the baryon vertex $p\to n$ is fixed (hence no arbitrariness is introduced through F/D ratios), and the residual symmetry of the meson vertex can be evaluated. Further simplification results from the fact that there are only two candidates for exchanges in these reactions, namely, the trajectories associated with ρ meson ($J^P=1^-$, $I^G=1^+$) and the A meson ($J^P=2^+$, $I^G=1^-$). From the ρ and A Regge amplitudes at the forward direction, six independent cross-section combinations can be calculated and compared with experimental data at high energy.

Introducing, for energy-dependent factors,[4] the notation

$$R_E\equiv\frac{\gamma_{Ep\bar{n}}}{\sqrt{s}}\left[\left(s-\tfrac{1}{2}\sum_{i=1}^{4}m_i^2\right)\Big/s_E\right]^{\alpha_E}$$

three forward differential charge-exchange cross sections can be expressed as

$$\frac{d\sigma}{d\Omega}(\pi^-+p\to\pi^0+n)=2\gamma_{\rho\pi\pi}^{\;2}[1+\tan^2(\tfrac{1}{2}\pi\alpha_\rho)]R_\rho^{\;2},\quad\text{(i)}$$

$$\frac{d\sigma}{d\Omega}(\pi^-+p\to\eta+n)=\tfrac{2}{3}\gamma_{A\eta\pi}^{\;2}[1+\cot^2(\tfrac{1}{2}\pi\alpha_A)]R_A^{\;2},\quad\text{(ii)}$$