We wish to thank the staff and crew of the PPA for their able and continuing support of this experiment. Mr. John Liu, Mr. Richard Bower, and Mr. Melvin Shochet assisted in the early stages of preparation of the experiment. Professor G. T. Reynolds gave many helpful suggestions during the design stage. We are grateful for the excellent analysis programs written by Mr. James Pilcher. Thanks go to the staff of the Elementary Particles Laboratory for their part in construction of the apparatus, and to our able scanners. Finally, we would like to acknowledge many valuable discussions with Professors Val L. Fitch, Sam B. Treiman, and Pierre Piroué.

†Work supported by the U. S. Office of Naval Research, Contract No. Nonr-1858(06).

*This work made use of computer facilities supported in part by a National Science Foundation Grant No. NSF-GP 579.

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SUM RULES FOR BARYON RESONANCE WIDTHS

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(Received 14 November 1966)

Recently sum rules[1] for strong interactions have been derived on the basis of a dispersion-theoretic approach. In such derivations, one makes assumptions[2] concerning the high-energy behavior of scattering amplitudes, which specify the convergence properties of the relevant dispersion relations. If the dispersion integral is approximated by a sum consisting of intermediate resonant states, the masses and widths of different resonances are related. Thus AFRF consider $\rho\pi$ forward scattering and show that

$$g_{\varphi\rho\pi}^{\;2}=0;\quad g_{\omega\rho\pi}^{\;2}=4g_{\rho}^{\;2}\pi\pi/m_{\rho}^{\;2}. \tag{1}$$

The interesting feature of relations (1) is that they are the well-known coupling-constant relations[3] that follow from the considerations of higher symmetries. This suggests the possibility that some of the results of higher symmetries can be derived from certain dynamical requirements. With this point of view, we consider meson-baryon scattering within the framework of SU(3) symmetry and discuss the relations between masses and widths of baryonic resonances with different total angular momentum J.

The invariant amplitudes $A(\nu,t)$ and $B(\nu,t)$ in meson-baryon scattering have different asymptotic behavior.[4] Thus, if $A(\nu,t)\xrightarrow[\nu\to\infty]{}\nu^{\alpha(t)}$, $B(\nu,t)\xrightarrow[\nu\to\infty]{}\nu^{\alpha(t)-1}$. In a Regge-pole model, $\alpha(t)$ refers to the dominant Regge trajectory in the crossed $P+\overline{P}\to B+\overline{B}$ t channel. Now, from our present knowledge of the meson mass spectrum, it is reasonable to assume that $\alpha(t)<0$ $(t\leq 0)$ for 27, 10, 10* trajectories. Consequently one is led to three sum rules:

$$\sum C_{RR'}\int_{-\infty}^{\infty} \mathrm{Im}B^{R'}(\nu,t)d\nu=0$$

$$(t\leq 0;\ R=\underline{27},\underline{10},\underline{10}^*),\qquad (2)$$

where $C_{RR'}$ are the appropriate elements of the crossing matrix[5] and $B^{R'}$ are the SU(3) eigenamplitudes in the $P+B\rightarrow P+B$ s channel. From crossing symmetry, the sum rules in the case of $\underline{10}$, $\underline{10}^*$ representations in the t channel are trivially satisfied since $\mathrm{Im}B(\nu,t)=-\mathrm{Im}B(-\nu,t)$. Therefore, just one nontrivial sum rule,[6] due to $\underline{27}$ crossing, is obtained,[7] viz.,

$$\int_0^{\infty}\left[\frac{7}{40}\mathrm{Im}B^{27}(\nu,t)-\frac{1}{12}\mathrm{Im}B^{10}(\nu,t)-\frac{1}{12}\mathrm{Im}B^{10^*}(\nu,t)+\frac{1}{5}\mathrm{Im}B^{8S}(\nu,t)-\frac{1}{3}\mathrm{Im}B^{8A}(\nu,t)+\frac{1}{8}\mathrm{Im}B^{1}(\nu,t)\right]d\nu=0.\qquad (3)$$

For the further discussion of the sum rule (3), we approximate the integral by a sum of direct-channel resonances. In the narrow-width approximation,

$$\mathrm{Im}B(\nu,t)=\sum_{l_\pm} R_{l_\pm}(t)\Gamma_{l_\pm}^{\mathrm{el}}\delta(s-M_{l_\pm}^{2}),$$

where

$$R_{l_\pm}(t)=\pm(4\pi/q_{l_\pm}^{3})[\{(M_{l_\pm}-M)^2-\mu^2\}\{P'_{l\pm1}(Z_{l_\pm})-P_l'(Z_{l_\pm})\}-4M_{l_\pm}MP_l'(Z_{l_\pm})],\qquad (4)$$

and $s=M^2+\mu^2+2\nu-\frac{1}{2}t$. M (μ) denotes the external baryon (meson) mass. $\Gamma_{l_\pm}{}^{\mathrm{el}}$ and $M_{l_\pm}$ are the total elastic width and mass of the resonance of total angular momentum $J=l+\frac{1}{2}$. $q_{l_\pm}$ is the center-of-mass momentum given by

$$q_{l_\pm}=[(M_{l_\pm}+M)^2-\mu^2]^{1/2}[(M_{l_\pm}-M)^2-\mu^2]^{1/2}/2M_{l_\pm},$$

and

$$Z_{l_\pm}=1+t/2q_{l_\pm}^{2}.\qquad (5)$$

Now, if we consider the forward scattering amplitude and retain only the $J^P=\frac{3}{2}^+$ decuplet and $J^P=\frac{1}{2}^+$ baryon states, we obtain the following relation between the 33 resonance [$\Delta(1238)$] width Γ_Δ, the parameter f,[8] and the $NN\pi$ coupling constant g:

$$[2M^*M-\{(M^*-M)^2-\mu^2\}]\Gamma_\Delta/q_\Delta^{3}$$
$$=8[3f^2-(1-f)^2]g^2/4\pi,\qquad (6)$$

where M^* is the mass of $\Delta(1238)$. With $\Gamma_\Delta=120$ MeV and $g^2/4\pi=15$, $f=0.42$ which is remarkably close to the value of $f=0.4$ corresponding to a D/F ratio of $\frac{3}{2}$. We can express (6) in a more familiar form in the limit $M^*=M$ and $f=0.4$:

$$\Gamma_\Delta=\frac{12}{25}\frac{q_\Delta^{3}}{M^2}\left(\frac{g^2}{4\pi}\right),\qquad (7)$$

which is essentially the SU(6) relation[9] between $\Delta(1238)$ width and $NN\pi$ coupling constant.

The approximation made in deriving (6) can be justified by the following argument. For low-mass resonances,

$$4M_{l_\pm}M\gg(M_{l_\pm}-M)^2-\mu^2.\qquad (8)$$

Hence from (4),

$$R_{l_\pm}\approx\mp\frac{16\pi}{q_{l_\pm}^{3}}M_{l_\pm}MP_l'(Z_{l_\pm}).\qquad (9)$$

$P_l'(Z_{l_\pm})$ is an oscillating function of t. Thus, in order that the sum rule (3) is satisfied for a finite range of t, <u>the sum rule should hold for each set of resonances which have the same orbital angular momentum l and nearly the same mass.</u>[10] Hence, an approximate sum rule for separate l can be obtained from (3) and (4) by setting $t=0$ and summing only over $J=l\pm\frac{1}{2}$ states. Empirically, resonances with higher l have larger mass, so that the approximation leading to (9) may not be valid. Therefore, we expect the sum rule for separate l to hold for small l.

To test the above arguments, we follow Barger and Cline[11] and assume their classification of baryon resonances in terms of Regge trajectories: the α octet, the δ decuplet, the γ

octet, and the γ singlet. Since the α octet and the δ decuplet have positive parity, there are pairs of resonances with the same l, i.e., $N_\alpha(938)$ and $\Delta_\delta(1238)$, $N_\alpha(1688)$ and $\Delta_\delta(1924)$, etc.; likewise for the γ octet and the γ singlet, $N_\gamma(1512)$ and $\Lambda_\gamma(1520)$, $N_\gamma(2210)$ and $\Lambda_\gamma(2110)$, etc. From the sum rule (3), if we assume the same D/F ratio along an octet trajectory, we obtain

$$\frac{\Gamma_\Delta R_\Delta}{6g^2}=\frac{\Gamma_\Delta{}^{\mathrm{el}}R_\Delta(1924)}{\Gamma_N{}^{\mathrm{el}}R_N(1688)}=-\frac{8}{3}(2f_\alpha{}^2+2f_\alpha-1), \quad (10\mathrm{a})$$

$$\frac{\Gamma_\Lambda{}^{\mathrm{el}}R_\Lambda(1520)}{\Gamma_N{}^{\mathrm{el}}R_N(1512)}=\frac{\Gamma_\Lambda{}^{\mathrm{el}}R_\Lambda(2110)}{\Gamma_N{}^{\mathrm{el}}R_N(2210)}=\frac{4}{3}(2f_\gamma{}^2+2f_\gamma-1), \quad (10\mathrm{b})$$

where the R's are computed from (4) for the specified resonances by setting $t=0$. $\Gamma_N{}^{\mathrm{el}}$ and $\Gamma_\Delta{}^{\mathrm{el}}$ are the elastic widths in πN channel. $\Gamma_\Lambda{}^{\mathrm{el}}$ is the elastic width in the $\Sigma\pi$ channel; f_α (f_γ) is the parameter f for the α-octet-BP (γ-octet-BP) interaction. If we use the quoted numbers[11] for $\Gamma_\Lambda{}^{\mathrm{el}}(1924)$ and $\Gamma_N{}^{\mathrm{el}}(1688)$, we obtain

$$\frac{\Gamma_\Delta{}^{\mathrm{el}}R_\Delta(1924)}{\Gamma_N{}^{\mathrm{el}}R_N(1688)}=0.49\sim0.69,$$

to be compared with

$$\Gamma_\Delta{}^{\mathrm{el}}R_\Delta(1238)/6g^2=0.51.$$

Thus the agreement of (10a) with experiment is fairly good. A similar comparison of (10b) with experiment cannot be made at the present time because the widths are not known with any accuracy.

Clearly any further quantitative comparison has to await further experimental information.[12] However, it is interesting to note that the sum rule (3) for separate l cannot be satisfied without having two or more SU(3) multiplets of baryon resonances with the same l (but possibly different J). Also, these resonances must have nearly the same mass. This indicates a close relationship between the sum rule and possible supermultiplets of baryons.

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*Work supported by the U. S. Atomic Energy Commission under Contract No. AT(11-1)-881.

†Work done under the auspices of the U. S. Atomic Energy Commission.

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[6]One of us (B.S.) has been informed by Dr. M. Suzuki that he and his collaborators have considered essentially the same convergent dispersion relation as ours.

[7]F. E. Low, at the Berkeley Conference, has made a criticism of the AFRF[1] derivation of the relations (1), namely that they did not take into account all superconvergent sum rules which follow from their assumptions. This criticism does not apply in our case [as long as we assume α_{10} is not less than -1].

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