TOTAL PION-PION CROSS SECTIONS FOR THE 2-GeV DI-PION MASS REGION*

N. N. Biswas, N. M. Cason, I. Derado, V. P. Kenney, J. A. Poirier, and W. D. Shephard
University of Notre Dame, Notre Dame, Indiana
(Received 31 October 1966)

Assuming single-pion exchange in the reaction $\pi+p\to\pi+\pi+B$ where B is the final-state baryon, we have calculated the total $\pi^+\pi^-$, $\pi^-\pi^0$, and $\pi^-\pi^-$ cross sections from the elastic scattering amplitude in the forward direction.

We have studied the three-body reactions

$$\pi^- + p \to n + \pi^+ + \pi^-, \tag{1}$$

$$\to p + \pi^- + \pi^0, \tag{2}$$

$$\to N^*(1236)\pi^- + \pi^- \tag{3}$$

in a hydrogen bubble chamber experiment at an incident pion momentum of 8 GeV/c. For high $M_{\pi\pi}$ we observe a concentration of events with small t_{NN}, the four-momentum transfer from the target proton to the final-state baryon, as well as with small $t_{\pi^-\pi^-}$, the four-momentum transfer from the incident to the final state π^-. We present arguments that these events may occur via the mechanism of one-pion exchange with $\pi\pi$ diffraction scattering. We have extrapolated the differential cross section over a small interval in $\cos\theta_{\pi^-\pi^-}$ to yield the forward $\pi\pi$ elastic scattering cross section. From this we calculate the total $\pi\pi$ cross section via the optical theorem, neglecting the real part of the forward scattering amplitude. We report the total $\pi\pi$ cross sections for the systems $\pi^+\pi^-$, $\pi^-\pi^0$, and $\pi^-\pi^-$, and from these we deduce the total cross sections in the three isospin states.

A total of 40 000 photographs of π^-p interactions were taken in the Brookhaven 80-inch hydrogen bubble chamber in a beam of 8.05 ± 0.04-GeV/c momentum. Some 20 500 two-prong and 10 000 four-prong events were measured and processed. Events corresponding to Reactions (1) and (2), and to the reaction

$$\pi^- + p \to p + \pi^+ + \pi^- + \pi^-, \tag{4}$$

were selected on the basis of GRIND fits, requiring in addition (a) that there be no alternative elastic fits for events of types (1) and (2), (b) that the ionization of the positive particles be consistent with the fitted momenta, and (c) that the square of the missing mass be in the interval 0.30 to 1.50 GeV^2 for Reaction (1), -0.08 to $+0.08$ GeV^2 for Reaction (2), and -0.02 to $+0.02$ GeV^2 for Reaction (4).

In Reactions (2) and (4), about 90% of the events could be uniquely identified from ionization. Of the 1318 events of Reaction (1), the positive particle was uniquely identified by ionization in 684 cases, and for 485 others the GRIND fit was unique. An additional 149 events had acceptable GRIND fits for both Reactions (1) and (2) but were included in the sample for Reaction (1) on the basis of effective-mass and momentum-transfer distributions. (Details of this analysis will be given in a future publication.) Our sample also includes 708 events for Reaction (2) and 1800 events of Reaction (4). Of the latter, 377 events have $M_{p\pi^+}$ in the N^*(1236) region (1.125 to 1.325 GeV) and have been classified as Reaction (3). We estimate the background in the N^* region to be approximately 10%.

Figure 1 shows the distribution of $\cos\theta_{\pi^-\pi^-}$

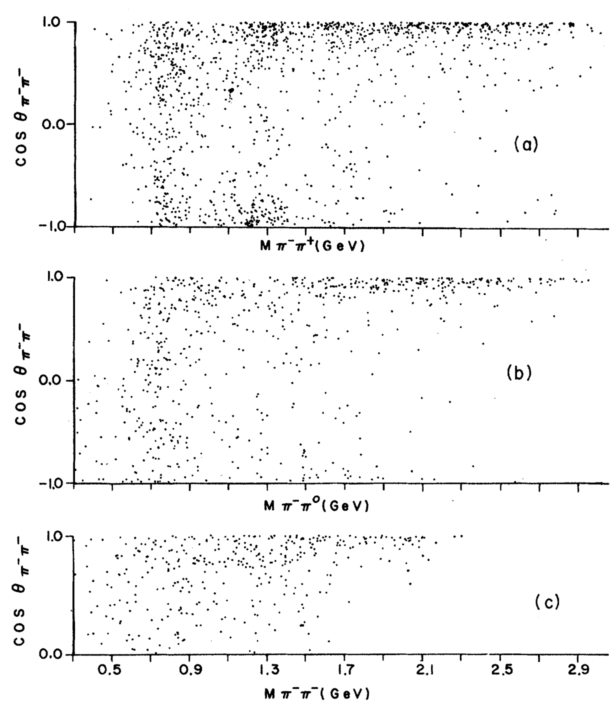

FIG. 1. The cosine of the angle between the incident and outgoing π^- in the di-pion center of mass as a function of the di-pion mass for the reactions (a) $\pi^-+p\to\pi^-+\pi^++n$, (b) $\pi^-+p\to\pi^-+\pi^0+p$, and (c) $\pi^-+p\to N^{*++}+\pi^-+\pi^-$. For (c), only the outgoing π^- with $\cos\theta_{\pi^-\pi^-}>0$ has been plotted.

(the angle between the incident and the outgoing π^- in the di-pion center of mass) as a function of $M_{\pi\pi}$ for Reactions (1)-(3). The events show a very strong concentration in the forward direction, becoming more pronounced as $M_{\pi\pi}$ increases.[1] It is interesting that the events in this forward peak occur with low t_{NN}; the distributions of t_{NN} show an exponential fall. The facts that the events with low t_{NN} appear with $\cos\theta_{\pi^-\pi^-}\approx 1$, and that the Trieman-Yang angular distributions are isotropic (see Table I), suggest that such events are produced by one-pion exchange and that we are observing pion-pion scattering[2] at energies above 1.5 GeV.

The one-pion-exchange process has been treated by a number of authors.[3,4] Detailed treatment of absorption effects is only possible in the limited case of quasi-two-body processes. In the present case, where the final state consists of three nonresonant particles, we rely on the fact that the Ferrari-Selleri form factors have been able to account for the gross features of the t_{NN} distribution in processes where one-pion exchange is applicable.

We use the Chew-Low formula as modified by Ferrari and Selleri[3]:

$$\frac{d^3\Sigma}{dt_{NN}dM_{\pi\pi}^2 d\cos\theta_{\pi\pi}} = G(t_{NN}, M_{\pi\pi})\frac{d\sigma_{\pi\pi}}{d\cos\theta_{\pi\pi}} \quad (5)$$

to relate the differential $\pi\pi$ elastic cross sections[5] to the observed differential cross sections for Reactions (1)-(3). For Reactions (1) and (2),

$$G(t_{NN}, M_{\pi\pi}) = \frac{f^2}{2\pi}\frac{m_N^2}{p^2 s}\frac{-t_{NN}}{m_\pi^2}\frac{F^2(t_{NN})}{(m_\pi^2-t_{NN})^2}M_{\pi\pi}q, \quad (6)$$

where $f^2 = 0.16$ and 0.08 for Reactions (1) and (2), respectively,

$$F(t_{NN}) = 0.28 + 0.72[1 + (m_\pi^2 - t_{NN})/(4.73 m_\pi^2)]^{-1},$$

s and p are the square of the total energy and the incoming nucleon momentum in the over-all center-of-mass system, and q is the pion momentum in the di-pion center-of-mass system.[6] For Reaction (3) the procedure is the same, except that instead of (5) above we use Eq. (1) in the first reference of footnote 1.

To determine the total $\pi\pi$ cross sections $\sigma_{\pi\pi}^{\text{tot}}$, we first fit the $\cos\theta_{\pi^-\pi^-}$ distributions of Reactions (1)-(3) with an expression

$$\frac{d\Sigma(\theta)}{d\cos\theta_{\pi^-\pi^-}} = \frac{d\Sigma(0)}{d\cos\theta_{\pi^-\pi^-}}\exp[\alpha t_{\pi^-\pi^-}] \quad (7)$$

for fixed $M_{\pi\pi}$ mass intervals and for events with $|t_{NN}| \le 20 m_\pi^2$. We have divided the events with $0.88 \le \cos\theta_{\pi^-\pi^-} \le 1.00$ into three equal intervals of $\cos\theta_{\pi^-\pi^-}$ and have performed a least-squares fit of the parameters $d\Sigma(0)/d\cos\theta_{\pi^-\pi^-}$ and with one constraint. The χ^2 values for the fits are all less than 1.8; the fits for three of the curves are shown in Fig. 2. The curves are quite consistent with an exponential behavior. All the values of α are within 1.5 standard deviations of the values for πp and pp elastic scattering.[7,8] It is important to note that the value for $d\Sigma(0)/d\cos\theta_{\pi^-\pi^-}$ is quite insensitive to α since we are extrapolating over a very small interval of $\cos\theta_{\pi^-\pi^-}$.

After converting to $d\sigma_{\pi\pi}(0)/d\cos\theta_{\pi^-\pi^-}$ using relations (5) and (6), we find the forward scat-

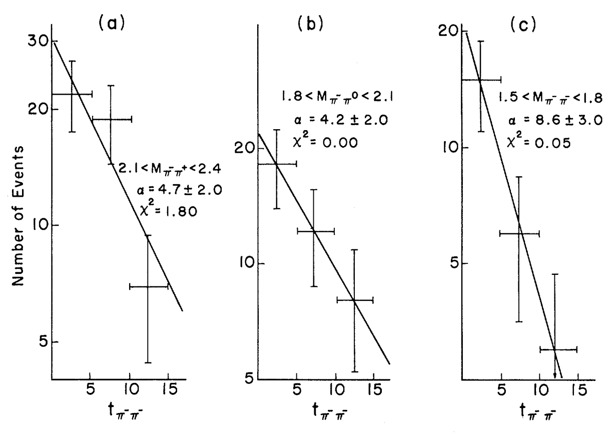

FIG. 2 Distributions of $t_{\pi^-\pi^-}$, the square of the four-momentum transfer between the incident and outgoing π^- measured in m_π^2 for (a), Reaction (1); (b), Reaction (2); and (c), Reaction (3). In each case the distribution shown corresponds to the highest $\pi\pi$ mass interval used in the analysis. The di-pion mass intervals (in GeV), the slopes of the fitted curves α (in GeV^{-2}), and the χ^2 values for the exponential fit (one degree of freedom) are also shown.

Table I. χ^2 values for isotropic fits to the Trieman-Yang angular distributions with five degrees of freedom.

Di-pion mass interval (GeV)	χ^2 Values for the Reactions $n\pi^+\pi^-$	$p\pi^-\pi^0$	$N^*\pi^-\pi^-$
1.2–1.5	1.0	9.3	7.4
1.5–1.8	7.9	4.6	4.2
1.8–2.1	6.6	2.2	···
2.1–2.4	5.2	···	···

tering amplitude from the relation

$$|f(0)|^2 = |f_I(0)|^2 + |f_R(0)|^2 = \frac{1}{2\pi}\frac{d\sigma_{\pi\pi}(0)}{d\cos\theta_{\pi^-\pi^-}}, \qquad (8)$$

where f_I and f_R denote the imaginary and real parts. Assuming $f(0)$ to be purely imaginary, we have

$$\sigma_{\pi\pi}^{\text{tot}} = (4\pi/q)f(0). \qquad (9)$$

It is known for the cases of πp, pp, and Kp scattering[9] that $|f_R(0)|^2$ is at most about 10% of $|f_I(0)|^2$, which suggests that $|f_R(0)|^2$ for $\pi\pi$ scattering is small. If $|f_R(0)|^2$ is nonzero, then our assumption will lead to an upper limit for the $\pi\pi$ total cross section. The results for $\sigma_{\pi\pi}^{\text{tot}}$ are shown in Fig. 3. For the 2-GeV region, $\sigma_{\pi^+\pi^-}^{\text{tot}}$ and $\sigma_{\pi^-\pi^0}^{\text{tot}}$ are about 20 mb and $\sigma_{\pi^-\pi^-}^{\text{tot}}$ is about 30 mb.[10]

Having measured the total cross sections in the three charge states, one can calculate the cross sections for the three isospin states σ_0^{tot}, σ_1^{tot}, and σ_2^{tot} using the relations:

$$\sigma_0^{\text{tot}} = \sigma_{\pi^-\pi^-}^{\text{tot}} + 3(\sigma_{\pi^+\pi^-}^{\text{tot}} - \sigma_{\pi^-\pi^0}^{\text{tot}}), \qquad (10)$$

$$\sigma_1^{\text{tot}} = 2\sigma_{\pi^-\pi^0}^{\text{tot}} - \sigma_{\pi^-\pi^-}^{\text{tot}}, \qquad (11)$$

and

$$\sigma_2^{\text{tot}} = \sigma_{\pi^-\pi^-}^{\text{tot}}. \qquad (12)$$

At 2 GeV we find $\sigma_0^{\text{tot}} = 41 \pm 19$ mb, $\sigma_1^{\text{tot}} = 2 \pm 12$ mb, and $\sigma_2^{\text{tot}} = 32 \pm 7$ mb. Although the uncertainties are large, we conclude that scattering in the $I=0$ and $I=2$ channels dominates in this region.

We may finally mention that on the basis of the factorization of Regge-pole residues,[11] the following relation is expected to hold:

$$\sigma_{\pi\pi}^{\text{tot}} = (\sigma_{\pi N}^{\text{tot}})^2/\sigma_{NN}^{\text{tot}}. \qquad (13)$$

Using the asymptotic values of the cross sections[8] $\sigma_{pp}^{\text{tot}} = 37$ mb and $\sigma_{\pi N}^{\text{tot}} = 25$ mb, we find $\sigma_{\pi\pi}^{\text{tot}} = 17$ mb, which is in good agreement with our data.

This experiment was made possible by the help and cooperation of the Shutt bubble chamber group and the alternating gradient synchrotron personnel of Brookhaven National Laboratory. We wish to acknowledge the assistance

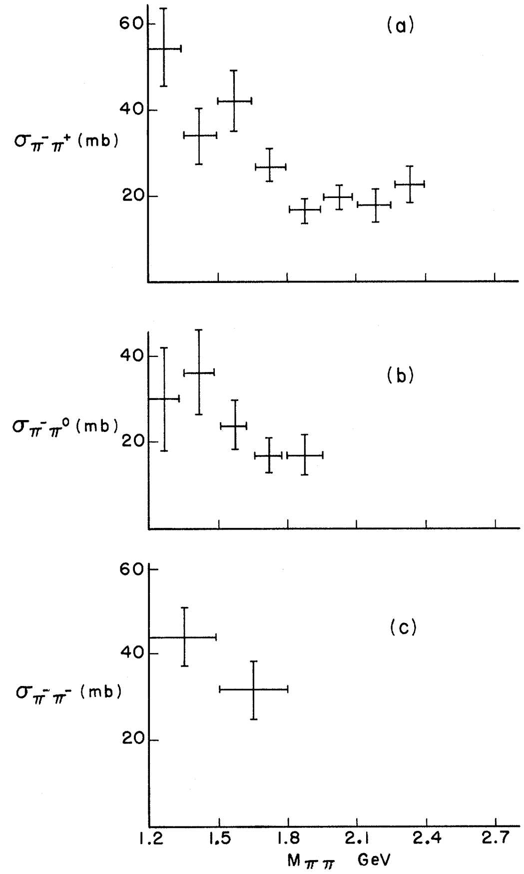

FIG. 3. Total $\pi\pi$ cross sections for (a) $\pi^-\pi^+$, (b) $\pi^-\pi^0$, and (c) $\pi^-\pi^-$ as a function of $M_{\pi\pi}$.

of Mr. John B. Annable with the data analysis. The cooperation of the Notre Dame Computing Center is appreciated.

*Research supported in part by the National Science Foundation.

[1]This effect can be seen in other experiments. See, for example, Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I.C.)-Munchen Collaboration, Phys. Rev. 138, B897 (1965); and Nuovo Cimento 31, 729 (1964).

[2]Alternative explanations for the events in this region are that (a) they represent peripheral production of isobars, or (b) they represent diffraction dissociation of the target nucleon. [See, for example, M. L. Good and W. D. Walker, Phys. Rev. 120, 1857 (1960).] With regard to (a) we observe no evidence for $N^{*+}(1236)$ production for any value of $M_{\pi^+\pi^-}$ in Reaction (1) nor for events with $M_{\pi^-\pi^0} < 2$ GeV in Reaction (2). For $M_{\pi^-\pi^0} > 2$ GeV we see no clear evidence for $N^{*+}(1236)$ production, but it cannot be ruled out. These events are therefore excluded from the present analysis. No evidence is seen for the production of higher mass isobars in Reaction (3). With regard to (b), if we make the simplest assumption that the $n\pi^+$ or $p\pi^0$ system pos-

sesses the quantum numbers of the target nucleon, the angular distributions of the pion with respect to the target-proton direction in the $n\pi^+$ or $p\pi^0$ rest frame should be isotropic. They are observed to be anisotropic.

[3]E. Ferrari and F. Selleri, Nuovo Cimento 27, 1450 (1963).

[4]K. Gottfried and J. D. Jackson, Nuovo Cimento 34, 735 (1964).

[5]The $\pi\pi$ elastic cross section has been studied for a number of incident momenta. These data have been compiled by G. Wolf, Deutsches Elektronen Synchrotron Report No. DESY-65/13, 1965 (unpublished); and Phys. Letters 19, 328 (1965).

[6]A factor $(q_{\text{off}}/q)^{2l}$ in the Ferrari-Selleri form factor is neglected since there is no well-defined angular momentum state of the $\pi\pi$ system.

[7]CERN-Bologna-Liverpool-Michigan-California Collaboration, Nuovo Cimento 38, 60 (1965).

[8]K. Foley, R. Gilmore, S. Lindenbaum, W. Love, S. Ozaki, E. Willen, R. Yamada, and L. C. L. Yaun, Phys. Rev. Letters 15, 45 (1965).

[9]A. A. Nomofilov, I. M. Sitnik, L. A. Slepets, L. N. Strunov, and L. S. Zolin, Phys. Letters 22, 350 (1966); K. J. Foley, R. S. Gilmore, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. H. Willen, R. Yamada, and L. C. L. Yaun, Phys. Rev. Letters 14, 14, 862 (1965); J. Mott, R. Ammar, R. Davis, W. Kropac, A. Cooper, M. Derrick, T. Fields, L. Hyman, J. Loken, F. Schweingruber, and J. Simpson, Phys. Letters 23, 171 (1966).

[10]In the region of the ρ when the same procedure is used to calculate $\sigma_{\pi^+\pi^-}{}^{\text{tot}}$ and $\sigma_{\pi^-\pi^0}{}^{\text{tot}}$, the cross sections are 110 ± 30 mb and 85 ± 23 mb, respectively; these values are consistent with the geometrical cross section of 120 mb. Thus, the procedure gives reasonable values for the cross sections even though absorption effects are not explicitly considered.

[11]S. C. Frautschi, Regge Poles and S-Matrix Theory (W. A. Benjamin, Inc., New York, 1963), p. 172.

ERRATUM

BOSON PRODUCTION IN p-p COLLISIONS AT 12.3 BeV/c. H. L. Anderson, S. Fukui, D. Kessler, K. A. Klare, M. V. Sherbrook, H. J. Evans, R. L. Martin, E. P. Hincks, N. K. Sherman, and P. I. P. Kalmus [Phys. Rev. Letters 18, 89 (1967)].

Due to a misprint the momentum spread of the incoming beam was given as $\pm 1\%$ instead of $\pm 0.1\%$.