B MESON AND THE DECAY $\rho \to \pi + \omega$†

Michael Parkinson
Department of Physics, University of California, Los Angeles, California
(Received 28 December 1966)

A consideration of the two-channel problem consisting of $\pi\pi$ and $\pi\omega$ in the $J^P = 1^-$, $T = 1$ state leads to interesting effects in the process $\pi + \pi \to \pi + \omega$.[1] One finds that a peak will occur in the $\pi\omega$ effective mass at 1200-1300 MeV. This is near the observed B-meson mass. There has been some doubt in the past as to whether the 1220-MeV $\pi\omega$ bump was a bona fide resonance, due to a variety of ways of producing such a bump kinematically.[2] This paper presents another "kinematic" effect leading to a $\pi\omega$ enhancement around 1220 MeV.

The starting point for the analysis presented here is a simple relativistic, analytic generalization of the Breit-Wigner resonance formula,[3] which has been derived from two different points of view, N/D effective range theory and K matrix theory. The formula is

$$T_{ij} = \tilde{C}_i \tilde{C}_j / \{(s-s_p)[1-R_1(s)\tilde{C}_1{}^2 - R_2(s)\tilde{C}_2{}^2]\}, \quad (1)$$

where $i,j = 1,2$, channel 1 is $\pi\pi$, and channel 2 is $\pi\omega$. In (1), we have $s = -(p_\pi + p_\omega)^2$,

$$R_i(s) = \frac{s-s_p}{\pi} \int_{t_i}^{\infty} \frac{\rho_i(s')ds'}{(s'-s_p)^2(s'-s-i\epsilon)},$$

$\rho_1(s) = |\vec{p}_\pi|^3/E_\pi$, $\rho_2(s) = |\vec{p}_\omega|^3 E_\omega$, $t_1 = 4m_\pi{}^2$, $t_2 = (m_\pi + m_\omega)^2$, $m_\pi = 1$, $m_\omega = 5.61$, $\tilde{C}_1 = kC_1$, $\tilde{C}_2 = kC_2$, $C_1{}^2 = g_{\rho\pi\pi}{}^2/12\pi$, $C_2{}^2 = g_{\omega\rho\pi}{}^2/12\pi$; where $s_p < 0$ and is chosen such that the denominator of (1) has a zero at $s = m_\rho{}^2 = 29.9$; and where k is chosen so that the resonance widths have the correct values. We have taken $g_{\rho\pi\pi}{}^2/4\pi = 2.4$ and $g_{\omega\rho\pi}{}^2/4\pi = 0.67 m_\pi{}^{-1}$.[4] The ρ functions chosen have been written as they appear in the center-of-mass system, and are suggested by the invariant couplings $g_{\rho\pi\pi}\epsilon_\rho \cdot (p_{\pi+} + p_{\pi-})$ and $g_{\omega\rho\pi}\epsilon_{\mu\nu\lambda\sigma}(\epsilon_\rho)_\mu (p_\rho)_\nu (\epsilon_\omega)_\lambda (p_\omega)_\sigma$.

By inspection, it will be seen that (1) is an exact analog of the Breit-Wigner resonance formula. It will also be seen that the left-hand cut of the amplitude is simulated by a pole at s_p and that if we could somehow remove this part of the amplitude, we could isolate the ρ propagator. Thus, we wish to remove the dependence on s_p in a suitable way. Let us write the denominator $D(s)$ of (1) more explicitly:

$$D(s) = (s-s_p)f(s) - i\tilde{\Gamma}(s), \quad (2)$$

where $\tilde{\Gamma}(s) = \tilde{C}_1{}^2\rho_1(s)\theta(s-t_1) + \tilde{C}_2{}^2\rho_2(s)\theta(s-t_2)$ and $f(s) = 1 - \mathrm{Re}[R_1(s)C_1{}^2 + R_2(s)C_2{}^2]$. $f(s)$ involves principal-value integrals;

$$\theta(s) = 1, \quad s > 0;$$
$$= 0, \quad s < 0.$$

The first term of (2) may be rewritten as follows:

$$Q(s) = (s-s_p)f(s) = (m_\rho{}^2 - s_p)f(s) + (s-m_\rho{}^2)f(s). \quad (3)$$

It will now be seen that the first term on the right of (3) is the only one that contributes to $Q'(s)$ at $s = m_\rho{}^2$, since the second term vanishes as $(s-m_\rho{}^2)^2$; it is $Q'(s)$ which determines the resonance widths. The second term, however, is necessary to produce the pole at $s = s_p$. Therefore, we will set the second term equal to zero, in order to isolate the resonance part of $D(s)$. If we then divide top and bottom of (1) by k^2, and remove the $\rho\pi\pi$ and $\rho\omega\pi$ coupling constants from the numerator, we obtain the following expression for the propagator of the ρ meson:

$$P_{\mu\nu}{}^{(\rho)}(s) = \{[(m_\rho{}^2 - s_p)/k^2]f(s) - i\Gamma(s)\}^{-1} \times \left(\delta_{\mu\nu} + \frac{(p_\rho)_\mu (p_\rho)_\nu}{m_\rho{}^2}\right), \quad (4.1)$$

$$P_{\mu\nu}{}^{(\rho)}(s) \approx \frac{\delta_{\mu\nu} + p_\mu p_\nu / m_\rho{}^2}{m_\rho{}^2 - s - i\Gamma(s)}, \quad \text{for } s \approx m_p{}^2, \quad (4.2)$$

where

$$\Gamma(s) = \Gamma_1(s) + \Gamma_2(s)$$
$$= \rho_1(s)C_1{}^2\theta(s-t_1) + \rho_2(s)C_2{}^2\theta(s-t_2),$$

and we have inserted the tensor factors necessary for a propagator. The fact that (4.2) is the same kind of expression as that obtained by taking the Feynman propagator and letting $m_\rho \to m_\rho - \frac{1}{2}i\Gamma$ is an encouraging heuristic verification of (4.2).[5]

Let us now consider the production reactions $\pi + p \to \pi + \pi + N$ and $\pi + p \to \pi + \omega + N$. We will assume Fig. 1(b) describes the $\pi\omega$ production. Abolins et al.[6] have used the one-pion–exchange model in order to extract the $\pi + \pi \to \pi + \omega$ cross

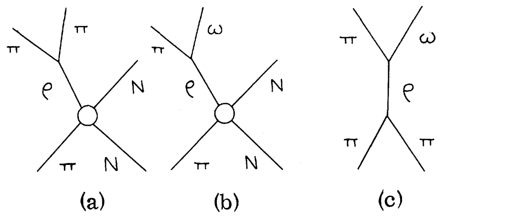

FIG. 1. (a) Resonant double-pion production. (b) $\omega\pi$ production, assuming ρ dominance. (c) $\pi+\pi\rightarrow\pi+\omega$ scattering, assuming ρ dominance.

section from their $\pi^+ + p \rightarrow \pi^+ + \omega + p$ data, and this affords us a simple comparison between our assumptions and experiment. We calculate the $\pi+\pi\rightarrow\pi+\omega$ cross section using the diagram of Fig. 1(c) and the ρ propagator of (4.2) and obtain:

$$\sigma_{\pi+\pi\rightarrow\pi+\omega}(s) = \frac{12\pi}{p\pi^2}\frac{\Gamma_1(s)\Gamma_2(s)}{(s-m_\rho^2)^2+[\Gamma(s)]^2}, \tag{5}$$

where p_π is the incident pion momentum in the center of mass and the Γ's are as in (4.2). We have plotted Eq. (5) and the Chew-Low extrapolation of Ref. 4 in Fig. 2. The similarity is striking, although the theoretical curve certainly will not produce a good fit to the data. In particular, the theory does not fall off fast enough as s increases. In Fig. 2 we have also plotted the resulting curve when $\Gamma(s)$ is set equal to a constant in the denominator of (5). (We chose the constant to be $m_\rho \times 125$ MeV, which produces the correct $\pi\pi$ cross section at resonance.) One can see how crucial the energy dependence of the width is. Since there are channels ignored here (e.g., $K\bar{K}$) which would add to the width, one can perhaps improve the fit. However, in view of the approximate character of one-pion exchange and the neglect theoretically of other important effects,[2] it is hardly worth worrying about quantitative details at this point. We do see, however, in Fig. 2 that the decay $\rho\rightarrow\omega+\pi$ will produce an enhancement in the $\omega\pi$ mass spectrum. For the remainder of this paper we will refer to this enhancement as the "B" meson.

It is clear from the model we have set forth that we would expect a definite ratio between B-meson production and ρ-meson production. In fact, one would expect that most of the difference in their production rates would be contained in the factor $\Gamma_j(s)/|[m_\rho^2-s-i\Gamma(s)]|^2$ coming from the decay of the ρ-meson intermediate state. The difference in the production amplitude as one varies the mass of the rho meson produced is hard to specify. One-pion exchange would predict no variation, whereas one-omega exchange gives an amplitude which varies as $(-p_\rho^2)^{1/2}$; i.e., as the mass. [These estimates are based on the couplings given after Eq. (1).] Ignoring variations in the production amplitude, one obtains the following estimate for $N_{\text{“}B\text{”}}/N_\rho$ because of the difference in the ρ-decay amplitude for the decays into $\pi\pi$ and $\pi\omega$: $N_{\text{“}B\text{”}}/N_\rho \sim 1\,\%$, where $N_{\text{“}B\text{”}}$ is the number of "B" mesons produced (a "B" meson of mass 1220 MeV and width of 140 MeV has been assumed) and N_ρ is the number of ρ mesons produced in the same reaction at the same energy.

Examining available experimental evidence one finds:

Reaction	Incident momentum (GeV/c)	Reference	Cross section (mb)
$\pi^- + p \rightarrow \pi^- + \omega + p$	4	7	$\sigma_{B^-} \approx 0.03$
$\pi^- + p \rightarrow \pi^- + \pi^0 + p$	4	8	$\sigma_{\rho^-} \approx 0.54 \pm 0.08$
$\pi^+ + p \rightarrow \pi^+ + \omega + p$	3.5	6	$\sigma_{B^+} \approx 0.1$
$\pi^+ + p \rightarrow \pi^+ + \pi^0 + p$	2.9	9	$\sigma_{\rho^+} \approx 1.1 \pm 0.2$

Thus experimentally, one finds $N_{B^-}/N_{\rho^-} \approx 7\,\%$ and $N_{B^+}/N_{\rho^+} \approx 10\,\%$. Since we do not expect the ρ decay to be solely responsible for the B-meson enhancement, and because of the neglect of the variation with rho mass of the ρ-production amplitude, these numbers are not unreasonably far away from $1\,\%$. At the very least, one may conclude that there is a substantial 1^- component present in the B-meson enhancement.

To sum up, we would like to emphasize the following two points:

(1) The decay $\rho\rightarrow\omega+\pi$ is capable of produc-

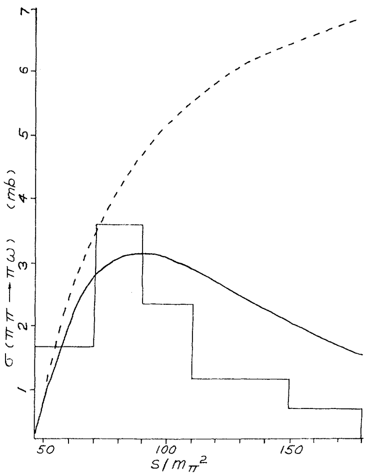

FIG. 2. The $\pi+\pi\rightarrow\pi+\omega$ cross section. The solid curve is from Eq. (5); the dashed curve gives the result with a constant width in the resonance denominator. The experimental histogram is from Ref. 6.

ing a 1200 to 1300-MeV $\omega\pi$ enhancement, but this effect alone probably does not explain the B meson. However, in conjunction with other "kinematic" effects,[2] one may be able to account for it. In this regard, however, it must be noted that the experimental data of Baltay et al.[10] on $\bar{p}+p\rightarrow\pi^++\pi^-+\pi^++\pi^-+\pi^0$ show a small bump around 1200 MeV in the $\omega\pi$ mass spectrum. To my knowledge, it has not yet been shown that such a bump could be a kinematic effect. It is certainly true that the effect discussed in this paper cannot produce a peak as narrow as the one they see.

(2) The energy dependence of the $\rho\rightarrow\omega+\pi$ resonance width is the most significant factor in producing the $\omega\pi$ enhancement as seen in Fig. 2, which indicates for this process the importance of properly taking into account the variation with energy of the widths appearing in a resonance denominator.[11]

I would like to acknowledge helpful discussions with Sharashchandra Patil, Nina Byers, and James Ball. My thanks to the Western Data Processing Center for access to their WDCOM system, which enabled the quick and easy performance of the numerical work necessary.

---

†Work supported by the National Science Foundation.

[1]The same two-channel problem was considered by W. R. Frazer, S. H. Patil, and N.-H. Xuong, Phys. Rev. Letters 12, 178 (1964); W. R. Frazer, S. H. Patil, and H. L. Watson, ibid. 11, 231 (1963); and Sharashchandra Patil, Phys. Rev. 136, B1102 (1964); but in a slightly different context. In these papers, there were always two poles on the second sheet—one for the ρ meson, and one for the B. Here, we assume there is only one pole—that for the ρ; we do not assume the B is a resonance.

[2]G. Goldhaber, in Proceedings of the Second Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, January, 1965, edited by B. Kurşunoğlu, A. Perlmutter, and I. Sakmar (W. H. Freeman & Company, San Francisco, California, 1965), p. 34; N. P. Chang, Phys. Rev. Letters 14, 806 (1965). Both these discuss effects due to symmetrization on identical particles. R. T. Deck, Phys. Rev. Letters 13, 169 (1964), discusses the diffraction scattering effect in the peripheral model.

[3]J. Ball and M. Parkinson, to be published.

[4]M. Parkinson, Phys. Rev. 143, 1389 (1966), Appendix II. The value of $g_{\rho\pi\pi}^2$ given in this appendix has been increased by 20 %, since the ρ width is now given as 125 MeV [A. H. Rosenfeld et al., Rev. Mod. Phys. 37, 633 (1965)].

[5]This prescription was used, for example, by S. Mandelstam, R. F. Peierls, and A. Q. Sarker, Ann. Phys. (N.Y.) 18, 198 (1962), in order to obtain the propagator for a resonance.

[6]M. Abolins et al., Phys. Rev. Letters 11, 381 (1963); N.-H. Xuong et al., Phys. Rev. Letters 11, 227 (1963).

[7]L. Bondar et al., Phys. Letters 5, 153, 209 (1963).

[8]I. Derado et al., Phys. Rev. Letters 13, 505 (1964).

[9]C. Alff-Steinberger et al., Phys. Rev. 145, 1072 (1966).

[10]C. Baltay et al., data presented at the Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, California, 1966 (unpublished).

[11]Another place where the energy dependence of the width is significant is in the sigma-meson model of the η decay. See L. M. Brown, in Proceedings of the Second Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, January, 1965, edited by B. Kurşunoğlu, A. Perlmutter, and I. Sakmar (W. H. Freeman & Company, San Francisco, California, 1965), p. 219. The sigma, by the way, is a (theoretical) spin-zero resonance, and thus has a very mild energy dependence to its width. Nevertheless, it is important for the η-decay problem.