$=\sum_i q_i\vec{v}_i\delta(\vec{r}-\vec{r}_i)$, introducing the usual notations for the Dirac δ function, space coordinate vector $\vec{r}$, and particle position vector $\vec{r}_i$. The conservation relation, $\nabla\cdot\vec{J}+\dot{\rho}=0$, is satisfied; a dot over a variable here indicates partial differentiation with respect to time t.

The mechanical momentum of a particle is $\vec{P}=m\vec{v}(1-v^2/c^2)^{-1/2}=\vec{p}-q\vec{A}/c$, for canonical momentum $\vec{p}$. The Lagrangian yields, for the force on particle i,

$$d\vec{P}_i/dt=-q_i\dot{\vec{A}}_i/c-q_i\nabla_i\varphi_i+(q_i\vec{v}_i/c)\times(\nabla_i\times\vec{A}_i). \quad (3a)$$

We shall set this equal to

$$e_i\vec{E}_i+g_i\vec{B}_i+(e_i\vec{v}_i/c)\times\vec{B}_i-(g_i\vec{v}_i/c)\times\vec{E}_i, \quad (3b)$$

assuming that each particle has an electric charge e_i and a magnetic pole strength (or magnetic charge) g_i. $\vec{E}$ is the electric field vector, and subscript i indicates values taken for (or at) particle i. Equations (3) yield

$$e_i\vec{E}_i+g_i\vec{B}_i=-q_i\dot{\vec{A}}_i/c-q_i\nabla_i\varphi_i; \quad (4)$$
$$e_i\vec{B}_i-g_i\vec{E}_i=q_i\nabla_i\times\vec{A}_i.$$

A consistent description of the electromagnetic field requires the relation of the field vectors $\vec{E}$ and $\vec{B}$ to the potentials $\vec{A}$ and φ to be independent of the magnitude of charges acted upon in Eqs. (3) and (4). This condition is satisfied here if and only if the ratio e/g is the same for all particles under consideration (i.e., a constant of the system). Assuming this is true, it is convenient to let $q=(e^2+g^2)^{1/2}$, giving

$$\vec{B}=(e/q)\nabla\times\vec{A}-(g/q)(\dot{\vec{A}}/c+\nabla\varphi), \quad (5a)$$

$$\vec{E}=-(e/q)(\dot{\vec{A}}/c+\nabla\varphi)-(g/q)\nabla\times\vec{A}. \quad (5b)$$

$\vec{B}$ and $\vec{E}$ of Eqs. (5) satisfy Maxwell's equations, noting that the electric charge density is $e\rho/q$ and the magnetic charge density is $g\rho/q$ in the generalized equations.

The constancy of e/g assures that $q_iq_j=e_ie_j+g_ig_j$, as would be expected, assuming that the potential energies of electric and magnetic interaction are additive, thus leading to the correct form of the system Hamiltonian in Coulomb gauge. The restriction to constant e/g is very severe, but allows continuous transition from the extreme of pure electric charges to the extreme of pure magnetic poles. Letting $g/e=K$, the quantum condition $eg=\hbar cn$, for integer n, yields

$$e=(\hbar cn/K)^{1/2}; \quad g=(\hbar cnK)^{1/2}. \quad (6)$$

K must be assumed constant for all particles of the system, but n can be a different integer for different particles.

This generalization goes farther than the obvious reversal of roles between $\vec{B}$ and $-\vec{E}$, in Eqs. (5) or the generalized Maxwell equations, when electric charge is replaced by magnetic charge, but it does not appear to provide a clue to an entirely satisfactory treatment of the problem of the potentials in quantum mechanics.

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REGGEIZED TADPOLE MODEL AND ELECTROMAGNETIC MASS DIFFERENCES*

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The problem of calculating the electromagnetic mass differences[1] within a given hadron multiplet has proved to be very difficult. Ordinarily, they are computed either by a straightforward perturbation method or some modification of such methods. These calculations usually involve difficulties with ultraviolet divergences, thus necessitating an introduction of a cutoff because of our ignorance of the high-energy behavior. We shall call such methods low-energy approaches, since the low-energy region rather than the high-energy one is expected to be more important. However, the appearance of divergences casts doubts upon their validity. Recently, Harari[2] has given a rather convincing argument on the basis of a dispersion theory showing that we can probably compute the mass difference due to $\Delta I=2$ electromagnetic effects by these conventional methods, but that it is not possible to do so for $\Delta I=1$ parts since then an additional unknown subtraction constant is needed in the dispersion integral. This suggests (at least for the $\Delta I=1$ part) that the contribution from the high-energy region is the most important for the electromagnetic mass differences of hadrons. We shall call such an assumption a high-energy approach in contrast to the previous low-energy approach. Although the truth may be somewhere in the middle, we shall assume the high-energy approach as our model in this paper. This approach will lead us to a Reggeized tadpole model. In this way, we can explain roughly all the mass differences among hadrons, including a curious, hitherto unnoticed fact[3] that a particle in a given isomultiplet with more electric charge has a smaller mass compared to others with less electric charges. Actually, an exception to this rule is the pion (and possibly the ρ meson), but we can also explain the reason for this apparent exception by our model.

Let us consider the case of baryons. The electromagnetic self-energy is given by[4]

$$\delta m=\frac{e^2}{2\pi}\cdot\frac{p_0}{m}\int d^4k\,(k^2-i\epsilon)^{-1}I(k,p),$$

$$I(k,p)=\int d^4x\, e^{ikx}\langle p\,|(j_\mu(\tfrac{1}{2}x)j_\mu(-\tfrac{1}{2}x))_+\,|p\rangle. \quad (1)$$

Note that $I(k,p)$ is proportional[4] to the scattering amplitude of a virtual photon with the momentum k by the baryon. The high-energy assumption implies that the above integral is dominated by contributions from large k. In this case, according to the Riemann-Lebesgue lemma on the Fourier integral, only the singular parts of the time-ordered product in $I(k,p)$ are important. It is possible to rewrite[5] $I(k,p)$ as an integral involving a retarded commutator instead of the time-ordered product. In that case, the singularity is concentrated on the light cone $x^2=0$. In a previous paper,[6] it was demonstrated that such a singularity represents an exchange of Regge trajectories. Indeed, in that way, we derived many sum rules like the Barger-Rubin relation.[7] At any rate, the most important contribution from the high-energy part in this model comes from exchanges of Reggeized particles between the photon and the hadron. Diagramatically, this is equivalent to considering Reggeized tadpole diagrams.[8] Now, for our case, only the R trajectory belonging to the 2^+ tensor nonet with the charge-conjugation parity +1 and with unit isospin can contribute to the electromagnetic mass splittings. Hence, we predict that the electromagnetic mass difference δm must be dominantly isovector, with its magnitude proportional to the coupling constant γ of the R trajectory with the hadron. Experimentally, γ has the same sign for all hadrons. This may be due to a possible existence of universal couplings of the 2^+ tensor nonet with all hadrons as has been conjectured by some authors.[9] Now we can explain the curious experimental fact[3] that a particle with less electric charge is heavier in a given isomultiplet. This rule applies even to isobars like N^* and $Y_1{}^*$. The pion, however, is an apparent exception to this rule. As has been noted by Coleman and Glashow,[8] this is not really serious but actually welcome since we have to compare the large $(K^0)^2-(K^+)^2$ with small $(\pi^0)^2-(\pi^+)^2$, where $K^{+,0}$ and $\pi^{0,+}$ represent masses of these particles. Also, the experimental violation of the resulting equal-mass spacing rule $\Sigma^--\Sigma^0=\Sigma^0-\Sigma^+$ is not so large and may be explainable as a correction due to the $\Delta I=2$ part of the electromagnetic interaction.

Now, in our model, we have

$$\frac{n-p}{N}=\alpha\cdot\gamma_N(f+d),$$

$$\frac{\Sigma^--\Sigma^0}{\Sigma}=\frac{\Sigma^0-\Sigma^+}{\Sigma}=\alpha\gamma_N\cdot f,$$

$$\frac{\Xi^--\Xi^0}{\Xi}=\alpha\cdot\gamma_N(f-d), \quad (2)$$

where f (and d with an additional condition $f+d=1$) is a coefficient of the f-type coupling of the 2^+ tensor nonet to the baryon octet. Similarly, γ_N is a dimensionless residue of the coupling as is customarily defined in the ordinary Regge theory[9,10] and α is a dimensionless constant proportional to the coupling of the R trajectory with the photon. We have assumed[11] that the remaining multiplicative factor with the dimensions of mass for the expression in δm may be taken to be the individual baryon mass itself.

Analogously, for the pseudoscalar mesons, we now find

$$\frac{(K^0)^2-(K^+)^2}{n^2-p^2}=\frac{\gamma_M}{\gamma_N};\quad \frac{(\pi^0)^2-(\pi^+)^2}{(K^0)^2-(K^+)^2}=0, \quad (3)$$

where γ_m is the dimensionless residue of the coupling between the 2^+ tensor nonet and the pseudoscalar octet.

From the experimental analysis[10] of high-energy meson-baryon scattering data, it is known that $f/d = -(2.0 \pm 0.6)$. If we use the middle value $f/d = -2.0$ we obtain

$$\frac{n-p}{N} : \frac{\Sigma^- - \Sigma^+}{2\Sigma} : \frac{\Xi^- - \Xi^0}{\Xi} = 1:2:3. \tag{4}$$

On the basis of this equation and the experimental value of $n-p = 1.29$ MeV, we compute $\Sigma^- - \Sigma^+ = 6.79$ MeV, and $\Xi^- - \Xi^0 = 5.62$ MeV. These numbers must be compared to the experimental values[12] $\Sigma^- - \Sigma^+ = 7.90$ MeV and $\Xi^- - \Xi^0 = 7.0$ MeV. The reason why the particular combination $\Sigma^- - \Sigma^+$ is chosen is because it represents a pure $\Delta I = 1$ part. Note that the $\Delta I = 2$ interaction gives no effects to $n-p$ and $\Xi^- - \Xi^0$ mass differences. We emphasize here that our Reggeized tadpole model takes account of high-energy contributions from many Feynman diagrams, in contrast to the conventional tadpole model.[8] The experimentally nonvanishing $\Delta I = 2$ difference $\Sigma^+ + \Sigma^- - 2\Sigma^0 = 1.7$ MeV is small compared to $\Sigma^- - \Sigma^+ = 7.9$ MeV and must be attributed to the $\Delta I = 2$ interaction, together with the $\pi_0 - \pi_+$ mass difference.

As for the kaon-mass splitting, we have experimentally[10]

$$\gamma_M/\gamma_N = (2.7 \pm 1.5)/(1.7 \pm 1.5).$$

Unfortunately the error is very large, but if the middle value $\gamma_M/\gamma_N = 1.58$ is tentatively chosen, Eq. (3) yields $K_0 - K_+ = 4.3$ MeV which is to be compared to the experimental value of 3.9 MeV. On the other hand, if universality[9] is assumed to set $\gamma_M = \gamma_N$ we predict $K_0 - K_+ = 2.72$ MeV which is a bit smaller. It is desirable to have a more accurate experimental determination for this value, although we can roughly explain the sign and magnitude of $K_0 - K_+$ by the present data.

So far, experimental values for f/d and γ_M/γ_N ratios have been used. However, if we use the idea originated by Cabibbo, Horwitz, and Ne'eman,[13] these ratios can be computed theoretically. Translating their idea for our case, we postulate that the coupling between the R trajectory and the hadron is proportional to the matrix element of the scalar density operator $u_1^{\ 1} - u_2^{\ 2}$ between two hadron states. If we further assume that $u_3^{\ 3}$ is the medium-strong SU(3)-violating interaction, then we compute

$$f/d = -\frac{2}{3}\frac{\Xi - \Lambda}{\Sigma - \Lambda} \approx -2.1$$

and

$$\frac{\gamma_M}{\gamma_N} = \frac{3}{4}\frac{\eta - \pi}{\Xi - \Sigma}\frac{K}{N} \approx 1.33$$

as has been noted in the previous paper.[6] These values are quite near the ones we have used. A similar method is applicable to compute the coupling constant γ_Δ to obtain

$$\frac{N^{*-} - N^{*++}}{n-p} = \frac{Y_1{}^* - N^*}{\Xi - \Sigma}\frac{N^*}{N}.$$

This formula together with the conventional SU(3) results[14] leads to $\Xi^{*-} - \Xi^{*0} = N^{*+} - N^{*++} = Y_1{}^{*0} - Y_1{}^{*+} = 2.14$ MeV and $N^{*-} - N^{*++} = 6.42$ MeV, which are not incompatible with the present experimental values.[12] Similarly, we predict that $K^{*0} - K^{*+} = 1.5$ MeV and $\rho^\pm - \rho^0 = 0$.

As has been stated in the beginning, our model may be a little over-simplified in spite of its successes. Probably, we should still take account of contributions from a few conventional Feynman diagrams in addition to our Reggeized tadpole diagram. The situation is then very similar to the conventional tadpole model.[8] This problem will be treated elsewhere.

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π-p CHARGE EXCHANGE POLARIZATION AND THE POSSIBILITY OF A SECOND ρ MESON*

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The π-p charge exchange (CEX) reaction, $\pi^- + p \to \pi^0 + n$, at high energies is a particularly simple reaction from the standpoint of Regge-pole phenomenology because only the quantum numbers of the ρ may be exchanged in the crossed channel, t. The domination of the scattering amplitude by a single ρ Regge trajectory is verified by several analyses[1-3] of the differential cross section, $d\sigma/dt$.[4-6] If this domination were complete and only the ρ contributed, one would expect to observe zero polarization because the Regge flip and nonflip amplitudes have the same phase. The detection of a nonzero polarization by Bonamy et al.[7] shows that another term which went undetected in the measurement of $d\sigma/dt$ is also contributing to the scattering amplitude.

It was shown[8,9] that a qualitative explanation of the polarization can be obtained by assuming that this extra term is due to resonance exchange in the direct channel, s. Alternative explanations assume that the extra term arises either from a cut[10] or from a second ρ meson, the ρ'.[11] We shall extend our previous analysis[8] to the more recent 11.2-GeV data and show that the quantitative agreement is improved by the introduction of the ρ'. We shall also review the growing evidence for the existence of a ρ'.

It was shown[8] that the direct-channel resonances can affect the polarization at energies as high as 20 GeV. While there is some question as to the accuracy of the extrapolation of the Breit-Wigner formula to energies considerably above the resonances, we feel it is important to include the resonance contributions. We shall therefore consider the following three models: (I) ρ + resonances, (II) $\rho + \rho'$ + resonances, (III) $\rho + \rho'$, and compare their agreement with experiment. The third model is presented since there is some skepticism concerning the contribution of the direct-channel resonances at energies above 6 GeV. The prediction of the three models at different energies will also be presented, thus providing an experimental test for distinguishing them.

Before we turn to this task, however, we shall review the independent support (i.e., support from sources other than the πp CEX polarization) for the introduction of the ρ'. This support comes from a variety of sources. The first of these is a recent analysis of Högaasen and Fischer[11] of high-energy, nucleon-nucleon, charge-exchange scattering where they show the introduction of a ρ' is necessary to obtain a consistent Regge-pole description of the forward differential cross section and the total cross-section data. Evidence for the ρ' arises from two recent unitary symmetry analyses. Nelson[12] has constructed a mass operator for the SU(3) harmonics which gives the correct masses of the well-known 1^- nonet of mesons. This operator also predicts a 1^- meson with the quantum numbers of the ρ at 984 MeV. A