the system. The proton is loosely bound and is located mainly in the surface region where its mean kinetic energy is small—comparable with the energy of the free proton when the system breaks up. The leakage rate of a particle through a barrier is proportional to its velocity. In this case the Coulomb barrier for the proton at a radius of 2.5 fm is 1.15 MeV, while its free energy in the center of mass of the Li^5 system is 1.57 MeV. The effect of the barrier is therefore small, and so the time taken for the p-wave proton to cross the potential well representing the alpha particle and for it to leak out of the well are comparable. We therefore suppose that the proton localization produced by the primary reaction should be evident in approximately half of the observed events. This implies that the Li^5 breakup occurs on a steep part of the curve describing the decay of the localization as a function of time.

The Li^5 ground state has a width close to 1 MeV. For the loosely bound proton an energy increase of a few hundred keV substantially increases the barrier penetrability effecting its escape. This results in the well-known asymmetric line shape seen for this state.[4] We therefore expect the more energetic protons in our spectra to represent shorter-lived intermediate systems within the natural width of Li^5. When the spectra are divided into corresponding subgroups, we find the asymmetry to be greater for higher proton energies. We observe an increase in the asymmetry factor of ~30% for a 500-keV change in proton energy (in the Li^5 center of mass). More detailed measurements with better counting statistics combined with the calculated variation of the p-α penetrability might yield a quantitative estimate of the decay rate of the proton localization.

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STATISTICAL THEORY OF INTERMEDIATE RESONANCES*

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It has been proposed, on the basis of detailed dynamical models of the process of "compound nucleus" formation, that the energy variations of nuclear-reaction cross sections should display, in addition to narrow fine-structure fluctuations, also broader "intermediate-structure" resonances.[1] The purpose of this note is to point out that in the presence of strongly absorbed channels, such intermediate resonance features are also predicted by statistical models of highly excited nuclear states. These models specify the frequency of occurrence and other average properties of the intermediate resonances and yield properties very similar to those predicted by the dynamically more detailed doorway-state models.[2]

What is involved here is not the accidental "lumping" of resonance contributions to the reaction amplitude which has been found to arise from conventional statistical assumptions.[3] Rather, we find that in strong-absorption cases a proper statistical theory of the reaction amplitude yields such a very broad distribution of resonance pole widths and strengths that individual pole terms giving rise to intermediate structure resonances in the cross section occur with appreciable probability.

R-matrix models.—The development of a proper statistical theory proceeds as follows: We note the existence of a very well-developed and well-tested theory of the statistical distributions of the discrete eigenstates of a large many-body system and of its eigenvectors.[4] We also note the well-known connection between such discrete eigenstates and eigenvectors and the parameters of the R matrix in nuclear-reaction theory.[5] This connection yields independent, normal distributions with zero means for the real channel surface overlap integrals γ_{ic},[6] as well as short- and long-range correlations of the R-matrix pole positions E_i that are often referred to as "level repulsion."[7]

Finally, we note that the R-matrix formalism[5] provides a connection between the elements of the R matrix and the reaction amplitudes. In particular, this formalism permits us to take the above mentioned distributions of the real R-matrix parameters γ_{ic} and E_i and deduce from them the distributions and correlations of the complex parameters $g_{\mu c}$ and $E_\mu - \frac{1}{2}i\Gamma_\mu$ that describe the resonance pole expansion of the S-matrix elements

$$S_{cc'} = \exp[i(\varphi_c+\varphi_{c'})]\left[W_{cc'}{}^0 - i\sum_\mu \frac{g_{\mu c}g_{\mu c'}}{E-E_\mu+\frac{1}{2}i\Gamma_\mu}\right]. \quad (1)$$

We shall refer to the distributions obtained in this way as "R-matrix statistical models" of the S-matrix parameters.[8] In the absence of direct reactions, the only free parameters in these models are the values of the optical-model transmission coefficients in each of the open channels. In terms of the R-matrix parameters, these transmission coefficients are given by

$$T_c = [4\pi P_c\langle\gamma_{ic}{}^2\rangle_i/D]\times[1+\pi P_c\langle\gamma_{ic}{}^2\rangle_i/D]^{-2}, \quad (2)$$

where P_c is the penetration factor in channel c, D is the mean spacing of the E_i, and averages $\langle\ \rangle_i$ are with respect to the index i.[9,10]

S-matrix models.—On the other hand, the customary statistical models of Eq. (1) make the ad hoc assumptions that the real and imaginary parts of all the $g_{\mu c}$ are independent and normally distributed with zero means, that the Γ_μ are distributed according to a chi-squared distribution with a number of degrees of freedom of the order of the number of competing open channels, etc.[11] We shall call these the "S-matrix statistical models."

The S-matrix models are in general not unitary, as can be seen by the excessive number of free parameters which they specify independently, and their statistical hypotheses have no clear physical basis. Both of these objections are remedied by the R-matrix models. The difficulty of the R-matrix models, of course, is that, as in every proper theory, the computation of the S matrix is not simple, involving in general a matrix inversion.[5,8]

Unitarity.—We now proceed to investigate some of the consequences of the R-matrix models. Some effect of unitarity on the parameters of the pole expansion (1) of the S matrix were previously investigated by means of "statistical R-matrix" models having simple periodic arrangements of the parameters E_i and γ_{ic},[12] and the results have since been confirmed by means of numerical inversion of the matrices arising from more general models with finite numbers of poles. For the case of $R^0=0$ these consequences are

$$\tfrac{1}{2}\pi\langle\Gamma_{\mu c}\rangle_\mu/D = \tanh^{-1}(\pi P_c\langle\gamma_{ic}{}^2\rangle_i/D), \quad (3a)$$

$$\Gamma_\mu = \textstyle\sum_c \Gamma_{\mu c};$$

$$\pi\langle G_{\mu c}\rangle_\mu/D = \sinh(\pi\langle\Gamma_{\mu c}\rangle_\mu/D), \quad G_{\mu c} = |g_{\mu c}|^2. \quad (3b)$$

For small values of $\langle\gamma_{ic}\rangle^2{}_i/D$, corresponding to weakly absorbed channels ($T_c<0.5$), Eqs. (3) approximate well the linear relationships $\langle\Gamma_{\mu c}\rangle_\mu \approx \langle G_{\mu c}\rangle_\mu \approx 2P_c\langle\gamma_{ic}{}^2\rangle_i$ that are so familiar from the single-level approximation. However, what is important here is that for strong absorption ($T_c>0.5$), Eqs. (3) are highly nonlinear, yielding average S-matrix parameters $\langle G_{\mu c}\rangle_\mu$ and $\langle\Gamma_{\mu c}\rangle_\mu$ that increase progressively much faster than linearly with increasing $\langle\gamma_{ic}{}^2\rangle_i$. The periodic pole models[12] suggest that these relationships are even more nonlinear for the individual resonance parameters than for their averages. As a consequence we should expect that in strongly absorbed channels the R-matrix models predict that the parameters $G_{\mu c}$ and $\Gamma_{\mu c}$ have much more broadly distributed values than do the $\gamma_{ic}{}^2$. This in turn implies that the $G_{\mu c}$, the $\Gamma_{\mu c}$, and the Γ_μ are expected to have much more broadly distributed values than those specified in the above-mentioned S-matrix models.[11]

Statistics.—This expectation has been confirmed by numerical determination of S-matrix parameters from statistical models of R matrices with finite numbers of resonances.[13] Figure 1 shows the dispersions of the total widths Γ_μ and absolute resonance amplitudes $G_{\mu c}$ obtained in this way for several numbers of independent strongly absorbed channels ($T_c=0.89$) and compares them with the usual S-matrix model assumptions. The much larger R-matrix model dispersions of the widths and amplitudes approach those of the S-matrix models rapidly as transmission coefficients of all

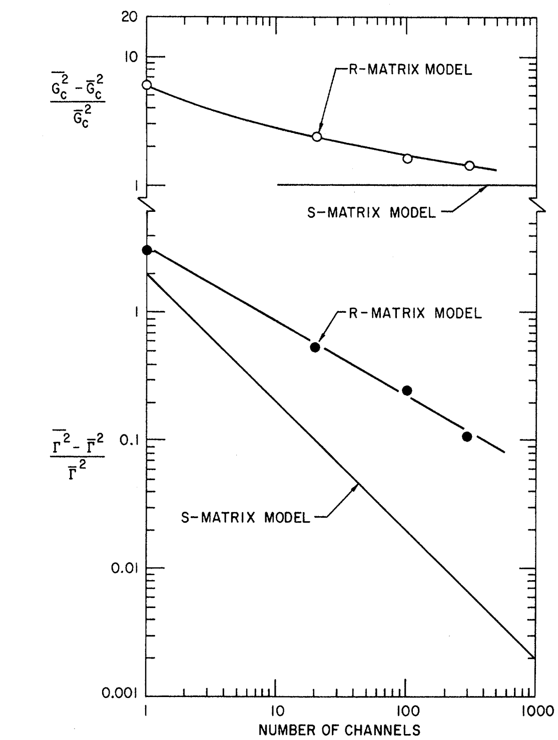

FIG. 1. The points give the calculated dispersions of the values of the S-matrix total widths Γ_μ and channel pole strengths $G_{\mu c}$ as obtained from R-matrix models with N independent competing channels each with transmission coefficient = 0.89, and for $N = 1$, 20, 100, 300. The labeled lines give the usual S-matrix model assumptions: exponential distribution for the $G_{\mu c}$ and chi-squared distribution with N degrees of freedom for Γ_μ.

competing open channels drop below 0.5. On the other hand, the difference between these two types of models becomes much greater than that shown in Fig. 1, when competing channels with greater transmission coefficients ($T_c = 0.9$ to 1.0) are present.

As a result, when strongly absorbed channels are open, we find a few total widths Γ_μ which are very much larger than their average values even when $\langle\Gamma\rangle/D \gg 1$. Similarly, we find very large values of $G_{\mu c}$ and these are found to occur preferentially in conjunction with large values of Γ_μ. For example, in the 20-channel situation shown in Fig. 1, 1 % of the Γ_μ exceed 6 times their median value and 1 % of the $G_{\mu c}$ exceed 20 times their median value. These numbers increase rapidly as transmission factors increase above 0.9.[14]

Intermediate resonances. – The contribution of any given pole μ to the energy averaged cross section $\langle\sigma_{cc'}\rangle$ is approximately $2\pi ƛ_c^2 N_\mu G_{\mu c} G_{\mu c'} / \sum_{c''} G_{\mu c''}$,[8,15] while the energy range over which a pole μ contributes to $\sigma_{cc'}$ is given by the width Γ_μ. Therefore, when for a given pole μ both Γ_μ and $G_{\mu c}$ are very much larger than the widths and strengths of nearby poles, then the pole μ will yield a dominant contribution to $\sigma_{cc'}$ over an energy range of order Γ_μ. The resulting feature, when observed in a measured cross section, will be recognized as an "intermediate resonance."

These statistical-model intermediate resonances share with the doorway-state models[2] the following properties[16]: Because of the broad distribution of the $G_{\mu c}$ only a very few partial waves c are strongly coupled to the same intermediate resonance μ. Hence such an intermediate resonance will be associated not only with a definite total angular momentum but also with definite orbital angular momentum in each channel. Each will therefore have a characteristic angular distribution, particularly in elastic scattering. Likewise a very strong intermediate resonance in the entrance channel of a reaction is likely to show up in several exit channels.

The statistical model also predicts a broad spectrum of intermediate resonance widths and peak heights which merges without clear demarcation with the fine-structure widths and peak heights. Because of the importance of strong absorption in this model, reactions involving composite projectiles should exhibit this intermediate structure most strongly, particularly in elastic scattering.[17] With increasing excitation energy, as many hundreds of strongly absorbed channels become open, the distribution of both $G_{\mu c}$ and Γ_μ narrows (see Fig. 1). The resulting disappearance of the intermediate structure corresponds to the situation where all doorway states become unbound.

Calculations are now in progress to predict the detailed properties of these statistical-model intermediate-resonance poles and of the resulting cross sections in actual reactions of interest. The results of these will be reported later. For the present we give only a schematic representation in Fig. 2 of the properties of 50 R-matrix model statistical poles corresponding to the 20-independent-channel situation which was shown in Fig. 1 and discussed in more detail above. Also shown schematically in Fig. 2 are the contributions of the strongest 26 of these poles to one elastic cross sec-

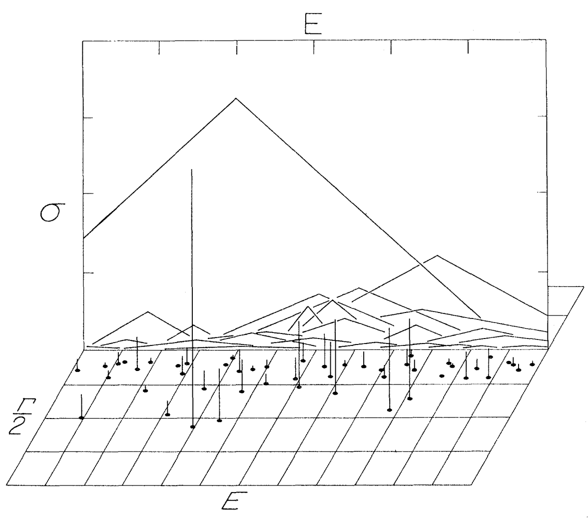

FIG. 2. The points on the horizontal complex energy plane give the positions $E_\mu - \frac{1}{2} i\Gamma_\mu$ of 50 neighboring poles of the S matrix as determined numerically from an R-matrix model corresponding to the 20-channel situation described in Fig. 1. The bars erected on each of these poles are proportional in length to the values of the $G_{\mu c}$ for one particular one of the 20 channels. The triangles drawn on the vertical cross-section plane has areas and peak heights proportional to the estimated contributions of each of the strongest 26 poles to the elastic scattering cross section in the same channel, calculated on the basis of single-level Breit-Wigner shapes.

tion. It is clear that this cross section is dominated by the contribution of one intermediate-resonance pole which will determine the shape of the cross section averaged over the fluctuations due to the remaining "fine-structure" poles.

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[14]These results affect the range of applicability of the analysis of Ref. 11 where it is assumed that $\langle\Gamma\rangle/D \gg 1$ and the Γ_μ are fairly constant. Figure 1, together with the results of P. A. Moldauer, Phys. Letters 8, 70 (1964), suggests that in general the formulas of Ref. 11 require $\pi\langle\Gamma\rangle/D \gtrsim 1000$.

[15]The factor N_μ (see Ref. 8) tends to enhance the contribution from poles with large values of Γ_μ.

[16]The number of parameters which specify the intermediate structure is effectively the same in the two types of models. In the absence of direct reactions, the doorway-state theory specifies for each channel the two parameters $\Gamma^\uparrow$ and $\Gamma^\downarrow$ while the statistical theory specifies T_c and $R_{cc}{}^0$. The latter has been assumed to vanish here for the sake of simplicity.

[17]One of the most convincing examples of intermediate structure has been found experimentally in the elastic scattering of α particles from Mg^{26} by P. P. Singh, B. A. Watson, J. J. Kroepfl, and T. P. Marvin, Phys. Rev. Letters 17, 968 (1966).