ac JOSEPHSON EFFECT IN SUPERFLUID HELIUM*

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The concept of a complex order parameter is of central importance to a unified description[1] of the superfluid behavior of both superconductors and liquid helium II. The phase φ of this parameter and the total number of particles N in the system can be shown to be conjugate variables, so that the equation of motion for φ becomes

$$-i\hbar\dot{\varphi} = [H, \varphi] = -i\partial H/\partial N,$$

which can be written in terms of the chemical potential $\mu \equiv \partial E/\partial N$ as

$$\hbar d\varphi/dt = \mu. \tag{1}$$

One interpretation of Eq. (1) (which has been called the Josephson frequency equation) is gotten by taking the gradient on both sides and interpreting $(\hbar/m)\nabla\varphi$ as the superfluid velocity; this leads to frictionless acceleration of the superfluid due to an applied force. A physically more interesting situation is the one which has been described by Anderson as "phase slippage" due, for example, to the motion of quantized vortices across a hole connecting two volumes of superfluid helium at the same temperature and between which a gravatational potential difference mgz exists (m = mass of helium atom; g = acceleration due to gravity; z = level difference). Application of Eq. (1) to this situation leads to the result that the time-averaged flow between the two volumes becomes zero whenever the following equation is satisfied:

$$\frac{mgz}{h} = \frac{dn}{dt}, \tag{2}$$

where dn/dt is the number per second of singly quantized vortices crossing the hole.

An indication of this effect was obtained in the classic experiment of Richards and Anderson,[2] who synchronized the generation of vortices to a sound wave of frequency ν, which was applied to one of the volumes of superfluid helium. They found that the flow through the hole showed irregular behavior whenever the following condition, derivable from Eq. (2), was satisfied:

$$n_1 mgz = n_2 h\nu, \tag{3}$$

where n_1 and n_2 were integers. The reader is referred to the papers of Anderson[1] and Richards and Anderson[2] for a description and discussion of the experiment.

In view of the significance of the Richards-Anderson experiment to our understanding of superfluids, we have repeated it and have been able to show with convincing clarity the arrest of flow through the hole when Eq. (3) is satisfied (Fig. 1).

The experimental arrangement, while it was similar to that of Ref. 2, included some changes which may have contributed to the increased stability of the experimental conditions. The experimental volume of liquid helium II was contained in a completely enclosed chamber inside which was a beaker. The bottom of the beaker was formed of a nickel foil of thickness 4×10^{-3} cm with a hole of 8×10^{-4} cm diam in it. This hole provided a weak link, and also the only link in the liquid phase, between the volume of helium II within the beaker and that in the chamber. The beaker was in the form of a coaxial capacitor, whose capacitance was monitored to give the liquid flow through the hole. This chamber was immersed in an outer bath of liquid helium which was pumped below the λ point and whose temperature was electronically regulated.[3] This arrangement ensured that the level of helium inside the experimental chamber did not change perceptibly in the course of an entire run, and also provided a very stable thermal environment for the chamber. Extreme care was also taken to minimize the extraneous vorticity in either of the two baths.

A quartz transducer produced a compressional sound wave in the vicinity of the hole, which served to couple the rate of vortex formation to the sound frequency. With the sound turned off, the gravitational flow through the hole showed the expected weak dependence on the pressure head (Fig. 2) with a critical velocity of 28 cm/sec. When the sound was turned on after the level inside the beaker had reached equilibrium, the liquid began to be pumped out of the beaker and the arrest of the flow at certain liquid levels was observed. The results shown in Fig. 1 are typical of what we have observed

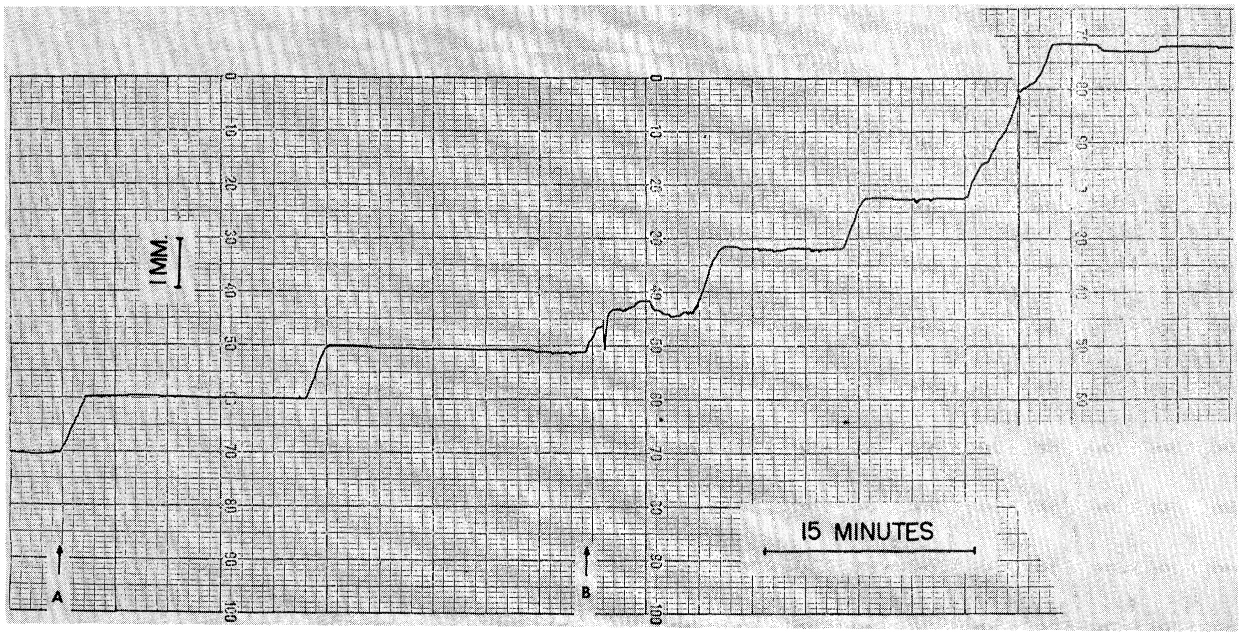

FIG. 1. Recorder trace showing the liquid-helium head as a function of time, due to the pumping action of the sound field, at 1.34°K. Time runs from left to right. The sound was switched on at point A, and the intensity was momentarily increased at point B in order to "kick" the level out of its second sticking position. The sharp spike immediately following B was a spurious external signal. The computed $z_0 \equiv h\nu/mg = 1.01$ mm. The transducer was excited at 99.722 kHz with an applied voltage of 1.5 V rms. An equivalent dc power input produced a head of <0.2 mm.

in over a dozen separate runs. The spacing between most of the steps observed was found to be in integral multiples of the unit $z_0 = h\nu/mg$, although in a few instances it was $(n_2/n_1)z_0$, where n_1 was an integer greater than unity. Steps with spacing of the latter kind were produced more often when the intensity of sound was raised.

The level of the liquid inside the beaker was observed to "stick" at one of these levels for a period of 1 min to as long as 45 min. Very often, when the liquid level appeared to have been arrested indefinitely in one of these steps, it could be kicked out of the step by a momentary increase in the sound intensity. An example of this is shown at the arrow marked B in Fig. 1. At other levels, an increase in the sound intensity (power input into the transducer) of as much as 100% was necessary to force the liquid out of that step. It was also noticed that

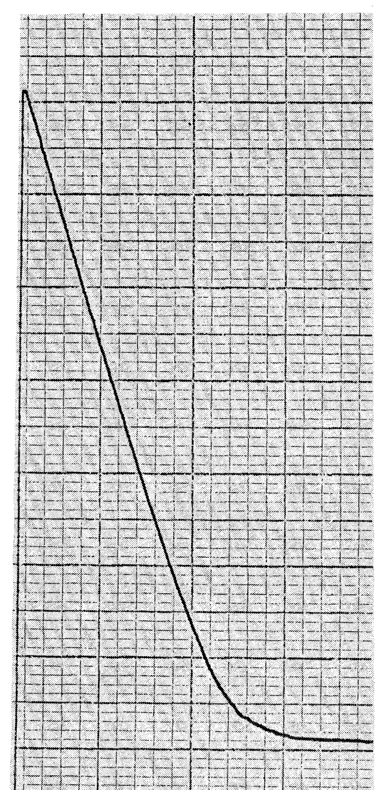

FIG. 2. Recorder trace showing the liquid-helium head as a function of time under isothermal gravitational flow into the beaker at 1.34°K. The time and distance scales are the same as in Fig. 1, and the flow occurred over the same region of the beaker as in Fig. 1.

the intensity of sound necessary to sustain the liquid at one of these steps was usually much lower than that required to pump to that level. These observations indicate the great stability of the phenomenon of "sticking" and provide convincing confirmation of the ac Josephson effect in helium II.

The significant improvements in the experimental conditions as demonstrated by these results now encourage us to try more complex interference experiments, e.g., the two-hole experiment,[1] in helium II.

Interesting discussions with P. W. Anderson, P. L. Richards, and P. R. Zilsel are gratefully acknowledged.

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[1]P. W. Anderson, Rev. Mod. Phys. 38, 298 (1966), reviews the problem and gives references to previous work.

[2]P. L. Richards and P. W. Anderson, Phys. Rev. Letters 14, 540 (1965).

[3]C. Blake and C. E. Chase, Rev. Sci. Instr. 34, 948 (1963).

CONFINEMENT OF CHARGED PARTICLES BY MULTIPLE-MIRROR SYSTEMS*†

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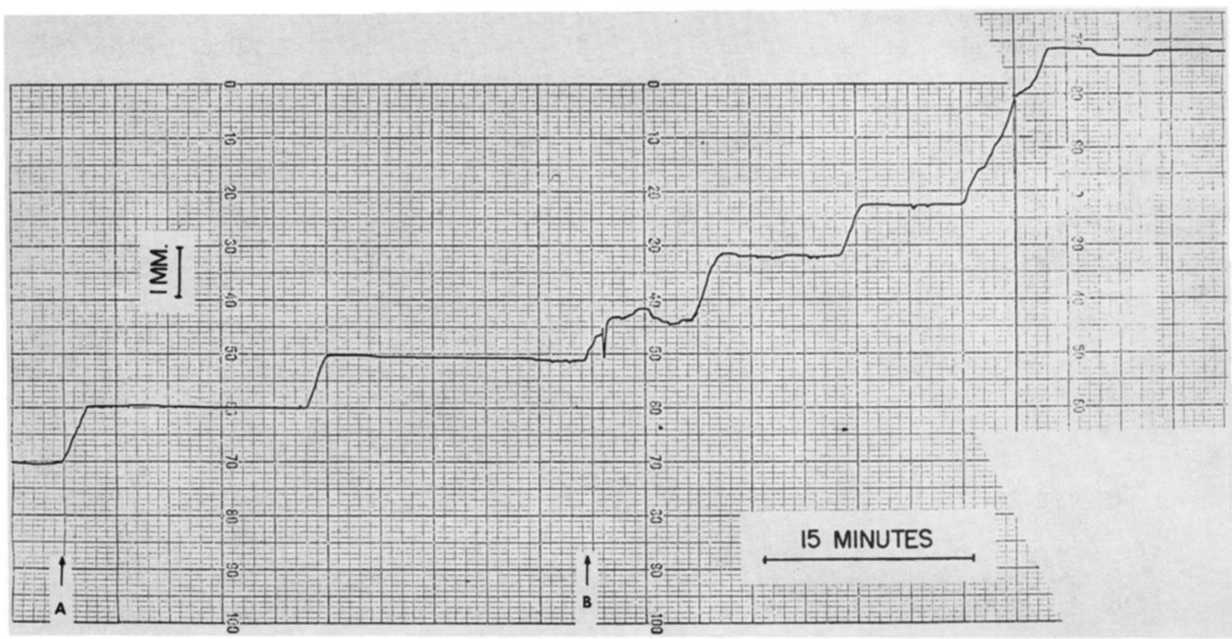
Monte Carlo techniques are used to evaluate particle confinement in mirror systems, including ones in which several mirrors are used in series. Predictions are compared with an experiment, and possible advantages of multiple-mirror systems with respect to plasma stability are described.

The aim of this Letter is to explore, computationally, the properties of both ordinary and multiple-mirror systems in regimes where mirroring and stochastic processes are of comparable importance. Not only are the scaling laws for confinement in this regime qualitatively different from those in the more familiar "long mean-free-path" (mfp) regime, but the distribution functions of the confined particles become less anisotropic, and thus less subject to instabilities than are the "loss-cone" distributions usually encountered in mirror confinement. Assuming stability, the computations indicate the possibility of achieving substantial gains in effectiveness of confinement, relative to the long mfp case, through scaling and choice of regime.

Mirror confinement depends on the constancy of orbital magnetic moments, $\mu=\frac{1}{2}Mv_\perp{}^2/B$. Collisions, plasma instabilities, or other nonadiabatic processes tend to destroy this constancy, leading to deflection of the orbital pitch angles into the loss-cone angle, θ_C, followed by escape of the particle. When the mfp for such processes is large compared to the spacing between mirrors, confinement time is independent of spacing. As the mfp is reduced, however, end losses take on a different character. This occurs when the mfp for deflection through an angle equal to θ_C becomes comparable to the distance between mirrors. In such a regime the particle confinement time of a multicell mirror system will be longer than that characteristic of any cell taken by itself. This is because escape of particles from interior cells may be followed by recapture into adjacent ones. Furthermore, since stochastic processes in a plasma normally produce "diffusion" of the pitch angles rather than discontinuous changes, particles trapped between two unequal mirrors will escape preferentially through the weaker mirror. Thus the transition probabilities for diffusion through a series of mirrors of progressively increasing height are smaller going "uphill" than "downhill." Even when only one cell is employed, in the "intermediate" mfp regime encounters with the mirrors at each end tend to become statistically independent. As long as the mfp is long compared to the mirror regions, mirroring occurs, and confinement time increases with mirror separation. However, if the mfp becomes small compared to the length of the mirror regions mirroring is destroyed. In this limit rapid plasma escape would be expected,[1] through hydrodynamic free flow, regardless of the number of mirrors employed.

Theories[2-6] of mirror losses in the long mfp



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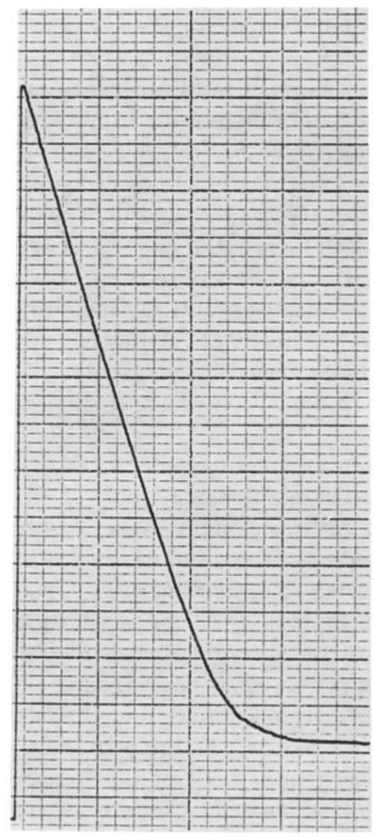

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