have assumed pure scalar, pure vector, and pure tensor interactions,[8] and have written the form factor as $f(q^2)=f(0)\,[1+\lambda q^2/M_\pi^2]$, in which q is the invariant mass of the leptons, or $q^2 = M_K^2+M_\pi^2-2M_KE_\pi$. Scalar and tensor interactions do not fit at all. The results are in good agreement with vector interaction and show $\lambda=-0.010\pm0.02$, where the error represents one standard deviation and includes an estimate of systematic errors.

If the energy dependence of f is due to a single intermediate state of mass M_{K^*}, the form factor will have the form $f(q^2)=f(0)\,[M_{K^*}{}^2/(M_{K^*}{}^2-q^2)]$. In this case, $M_{K^*}\geqslant 740$ MeV at the 99% confidence level and $M_{K^*}\geqslant 650$ MeV at the 99.9% confidence level.

The results of this experiment are in good agreement with the $|\Delta I|=\frac{1}{2}$ rule and the $K_{e3}{}^+$ data.[1] A more complete account of the K_L decay experiment will be published elsewhere.

We wish to thank Dr. Christopher J. B. Hawkins of Oxford University and Professor William J. Willis of Yale University for many helpful discussions. We also wish to thank the computing center staffs at Brookhaven National Laboratory, New York University, and Yale University for their assistance and cooperation.

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*Work performed under the auspices of the U. S. Atomic Energy Commission.

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QUARK DECOMPOSITION OF HADRON SCATTERING AMPLITUDES*

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The quark model of hadrons,[1] combined with the additivity postulate for the quark-quark scattering amplitudes,[2] has had remarkable success in the analysis of high-energy scattering data. In particular, many sum rules have been derived between meson-baryon total cross sections which are satisfied to within or near experimental limits.[3] However, in sum rules relating meson-baryon to baryon-baryon total cross sections, such as[4]

$$\sigma_T(\bar{p}p)+\sigma_T(pn)=2\sigma_T(\pi^-p)+\sigma_T(\pi^+p) \tag{1a}$$

or

$$\sigma_T(\bar{p}n)+\sigma_T(pp)=\sigma_T(\pi^-p)+2\sigma_T(\pi^+p), \tag{1b}$$

there exists a large discrepancy (~15-20%) with experiment. This is particularly unfortunate since the sum rules concerned depend only on additivity, involve only the sums of cross sections (and, therefore, have relatively small experimental errors), and altogether provide a particularly "clean" test of the model. We show here how this discrepancy is removed with careful formulation and application of the additivity postulate.

"Additivity" is a decomposition of the amplitude for the scattering of composite systems A and B into a sum of amplitudes which represent the scattering of the components A_i of A on the components B_j of B. This decomposition is to be made under the following conditions: (i) The internal velocities of A_i in A and B_j in B are negligible in comparison with the relative velocity of A and B, i.e., the Fermi momenta of the components A_i and B_j can be neglected. (ii) The bindings of the A_i into A or B_j into B do not appreciably distort the subsystems A_i or B_j. (iii) Such a decomposition will only be valid when it is possible to neglect double scattering, i.e., terms which involve more than one A_i and one B_j. Examples of situations where conditions (i)-(iii) might be expected to hold are scattering of high-energy pions on deuterons (composed of a proton and a neutron) or hadron-hadron scattering at high energies (the hadrons being composed of quarks).

Kokkedee and Van Hove[5] have given the most explicit statement of the additivity decomposition to date. They obtain the result

$$A_{AB}(s_{AB},t)=\sum_{ij} f_i^A(t) f_j^B(t) A_{ij}(s_{ij},t), \qquad (2)$$

where

$$f_i^A(0) f_j^B(0)=1,$$

and the amplitudes A are matrix elements of $S-1$ taken between states normalized to $\langle p|p'\rangle = \delta^3(p-p')$. Thus the amplitudes A are not Lorentz invariant.

We wish to discuss the kinematical conditions under which sum rules are to be compared with experiment, and while it is certainly possible to work with the noninvariant amplitudes A, it is extremely inconvenient to do so. We, therefore, first put the result (2) into a Lorentz-invariant form.

The relevant quantities are the amplitude T, the flux F, the two-particle density of states ρ_2, and $d\sigma/dt$, all of which are Lorentz invariant and are related by

$$\frac{d\sigma}{dt}=\frac{|T|^2\rho_2}{F}=\frac{1}{4\pi}\left|\frac{T}{F}\right|^2,$$

where we have used the fact for a two-particle interaction, $4\pi F\rho_2=1$. We now note that by supposition (i) above,

$$(|\vec{v}_A-\vec{v}_B||\vec{v}_A{}'-\vec{v}_B{}'|)^{1/2}=(|\vec{v}_{Ai}-\vec{v}_{Bj}||\vec{v}_{Ai}{}'-\vec{v}_{Bj}{}'|)^{1/2},$$

where the $\vec{v}$'s and $\vec{v}'$'s are, respectively, the velocities in any Lorentz frame before and after the collision. We introduce these factors on the left- and right-hand sides, respectively, of (2) and find

$$T_{AB}'(s_{AB},t)=\sum_{ij} f_i^A(t) f_j^B(t) T_{ij}'(s_{ij},t), \qquad (3)$$

where

$$T'(s,t)=T(s,t)/F(s)$$

and

$$\frac{d\sigma}{dt}=\frac{1}{4\pi}|T'(s,t)|^2.$$

The optical theorem reads

$$\sigma_T(s)=2\,\mathrm{Im}T_{\mathrm{el}}'(s,0). \qquad (4)$$

The amplitudes T' are Lorentz invariant and are independent of all state normalization factors.

Setting $t=0$ in (3) and using (4), we have immediately

$$\sigma_T(AB)=\sum_{ij}\sigma_T(A_iB_j),$$

i.e., total cross sections are additive. This is a satisfactory result, since the additivity decomposition is valid only when double scattering can be neglected, i.e., the components of A and B do not screen one another, and under precisely these conditions we expect total cross sections to be additive.

We now turn to the kinematical implications of Eq. (3). T_{AB}' is a function of s_{AB} while T_{ij}' is a function of s_{ij}. Now assuming only that $s_{AB} \gg m_A^2, m_B^2$, and that the internal motion of the quarks is nonrelativistic, it is straightforward to show that s_{AB} and s_{ij} are related by

$$\frac{s_{AB}}{2m_Am_B}=\frac{s_{ij}}{2m_im_j}.$$

This means that in comparing any sum rules with experiment, the various total cross sections entering the relation, e.g., $\sigma_T(AB)$ and $\sigma_T(CD)$, must be taken at different energies, such that

$$\frac{s_{AB}}{2m_A m_B} = \frac{s_{CD}}{2m_C m_D}. \tag{5}$$

The physical content of the statement is clear if we evaluate s_{AB} in the laboratory system with B at rest. Then

$$\frac{s_{AB}}{2m_A m_B} = \frac{p_A^{\ L}}{m_A} = \frac{v}{(1-v^2)^{1/2}},$$

where v is the laboratory velocity of one particle as seen from the other.[6] Thus (5) is the condition that the relative velocity of A and B is the same as that of C and D. It is indeed plausible that this simulates identical kinematical conditions for the quark-quark interactions.

Now let us specialize to the case of π or N scattering off a nucleon target. We require

$$\frac{p_\pi^{\ L}}{m_\pi} = \frac{p_N^{\ L}}{m_N} = \frac{v}{(1-v^2)^{1/2}},$$

i.e., the left- and right-hand sides of the relations (1a) and (1b) ought to be compared at lab momenta such that

$$p_N^{\ L}/p_\pi^{\ L} = m_N/m_\pi. \tag{6}$$

This is done in Fig. 1(a) and 1(b). Complete nucleon-nucleon (or antinucleon) data are available up to ~20 BeV/c, corresponding via (6) to pion momenta ~3 BeV/c.[7] However, below 6 BeV/c for the πN system we are getting into the resonance region where additivity is certainly no longer a valid approximation. However, it is clear that, within experimental error, the two full curves in each of 1(a) and 1(b) are extremely plausible extrapolations of one another. For comparison we show in the broken curves the N-N data plotted against the pion data at the same laboratory momentum (i.e., same s value).[8]

The experimental errors involved are appreciable. However, in the latter case (same s value) the discrepancy is large and several times the experimental error. In the former case (same relative velocity), though more data points with smaller errors are needed for a definitive test, the presently available data are consistent with the continuous curve predicted by the model.

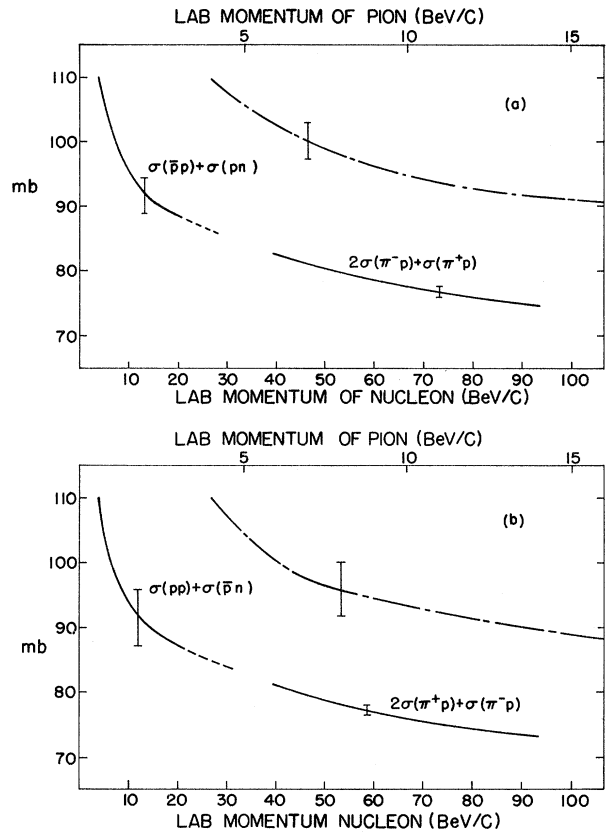

FIG. 1. The sum-rules (1a) and (1b), respectively, are plotted in parts (a) and (b) of this figure. The solid curves show the experimental data for the left- (nucleon-nucleon) and right-hand (pion-nucleon) sides of the sum-rules. The pion-nucleon data are plotted on the upper scale, and the nucleon-nucleon data on the lower, where the two scales differ by a scaling factor of m_N/m_π, as prescribed in the text. It is predicted that the experimental data lie on a single curve in each of (a) and (b). For comparison, we also show (broken curve) the nucleon-nucleon data plotted at the same laboratory momentum as the pion-nucleon data. A typical error bar for each curve is shown.

Finally, Freund[9] has shown that quark-model predictions for high-energy scattering are totally equivalent to the predictions of universality. At first sight it would appear that our relative-velocity prescription has destroyed this equivalence. This is not so.

The universality theory has to be supplemented by a dynamical assumption in order to make predictions, and it is usually assumed that reactions at high energy are dominated by Regge pole terms in the crossed channel. Such a term is of the form

$$\beta_i (s/s_0)^{\alpha_i(t)},$$

where s_0 is a scaling factor which is undetermined in the Regge theory. The two common choices of s_0 are $s_0=\text{constant}$[10] or $s_0=2m_Am_B$.[11] The former choice combined with universality leads to sum rules in which all cross sections are taken at the same s value. The latter choice of s_0 (plus universality) is precisely equivalent to our relative-velocity prescription. It implies that the Regge trajectories are associated primarily with the quark-antiquark systems rather than the hadron-antihadron systems. Of course, the value of s_0 is irrelevant unless one has a theory (quark decomposition or universality) which relates the residues β_i.

We are very indebted to Dr. H. J. Lipkin for advice and encouragement in this work, and are also grateful to Dr. R. Schult, Dr. R. K. Logan, and Dr. R. C. Arnold for helpful discussions.

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*Work supported by the National Science Foundation under Contract No. NSF GP-5622 and the U. S. Office of Naval Research under Contract No. 00014-66-C00010-A05.

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ANGULAR MOMENTUM UNCERTAINTY RELATION AND THE THREE-DIMENSIONAL OSCILLATOR IN THE COHERENT STATES

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An angular momentum uncertainty relation is obtained in terms of sine and cosine operators that have a meaning even in a second-quantization formalism. For a three-dimensional oscillator in coherent states the new uncertainty product is a minimum for large ΔL_z. Even for small ΔL_z the uncertainty product is very small.

It has long been known[1] that the interpretation of the commonly accepted uncertainty relation between angular momentum and angle,

$$\Delta L_z \Delta\varphi \geq \tfrac{1}{2}\hbar, \tag{1}$$

is not precise. The relation lacks meaning for small values of ΔL_z since the angle φ is restricted to values of $(0, 2\pi)$. In recent studies, Judge[2] and Susskind and Glogower[3] have independently suggested that the angle variable ψ defined by

$$\psi \equiv \varphi(\mathrm{mod}\,2\pi) \tag{2}$$

be used so that

$$[\psi, L_z] = i\hbar\left[1-2\pi\sum_{n=-\infty}^{\infty}\delta(\varphi-[2n+1]\pi)\right]. \tag{3}$$

Although an uncertainty relation can be defined by the commutator of L_z with ψ,[2,4] ψ lacks a well-defined operator definition and continuous eigenspectrum.