Table I. The known spectrum of Sc^{42} as compared with the Sc^{42} spectrum calculated from a particle-hole transformation of assumed Sc^{48} spectrum.

Assumed Sc^{48} spectrum E (MeV)	J	Calculated Sc^{42} spectrum E (MeV)	J	Known Sc^{42} spectrum E (MeV)	J
g.s.	6	g.s.	0	g.s.	0
0.131	5	0.420	7	0.6 ±0.1	7
0.230	4	0.704	1	0.620	1
0.580	2	1.540	5		
0.610	3	1.556	3		
1.170	7	1.607	2	1.590[a]	2
2.700	1	2.835	4	2.750	4
7.150	0	3.127	6	3.191	6

[a]It is possible, of course, that the Sc^{42} $(f_{7/2})^2$ 2^+ state is split, as in Ca^{42}, into two levels with a centroid at about 2.0 MeV. However, no such splitting has as yet been identified in Sc^{42}.

eracy would resolve the conflicting results of recent Sc^{42} studies, one of which indicates that a state of spin 3 lies at about 1.5 MeV,[4] and one of which indicates that a state of spin 5 lies at about 1.5 MeV.[3] In any event, if the 0.131-, 0.230-, and 0.610-MeV Sc^{48} states have spins of 5, 4, and 3, respectively, as suggested by Chasman, Jones, and Ristinen, then the measurements reported here do not support suggestions[1,3] that either the 3^+ or the 5^+ $(f_{7/2})^2$ state of Sc^{42} lies above 2.0 MeV.

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ENERGY DEPENDENCE OF THE FORM FACTOR IN K_{e3}^0 DECAY*

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A sample of 762 decays $K_L \rightarrow \pi + e + \nu$ in the 80-inch hydrogen bubble chamber have been identified using electron detection by shower production in a tantalum plate. The results are consistent with vector interaction and constant form factor.

The $|\Delta I| = \frac{1}{2}$ rule for the semileptonic weak decay predicts that the dependence of the vector form factor on pion energy be the same in the K_{e3}^0 and K_{e3}^+ decays. Experiments on the K_{e3}^+ decay show no evidence for an energy dependence of the form factor.[1] Of the two previous experiments on the K_{e3}^0 decay, one allows for an energy dependence within large errors,[2] and the other indicates a large energy dependence in the form factor.[3] The experimental data appear to be in good agreement with the $|\Delta I| = \frac{1}{2}$ rule for the decay rates, i.e., $\Gamma(K_L \rightarrow \pi + e + \nu) = 2\Gamma(K^+ \rightarrow \pi + e + \nu)$.

We report here results of an experiment on K_L decays, in which the K mesons are produced from a 7-BeV/c π^- beam incident on an aluminum target and are detected with the Brookhaven 80-inch liquid-hydrogen bubble chamber.[4]

We present here results on the $K_{e3}{}^0$ decay only. We extract from all the visible decays a sample of $K_{e3}{}^0$ in which the electron or positron is identified by its having produced a shower in a 1.49-radiation-length tantalum plate. Electron-positron pairs are removed by a cut at 30 MeV on the effective mass of the two visible tracks when interpreted as e^+e^-. A total of 835 decays remained in which one track is clearly identified as an electron or positron.

We have studied the probability for an electron which strikes the plate to produce a shower, as a function of electron energy, in a calibration experiment on known electron-positron pairs. The result are shown in Fig. 1. We also estimate the shower detection efficiency from the calculations of Crawford and Messel.[5] The result of this theoretical calculation has the correct energy dependence but is lower than the result of the calibration experiment by 5%. In all subsequent calculations we use the theoretical estimate, multiplied by the normalization constant 1.05. The result is shown as the smooth curve in Fig. 1.

The events are computed on an IBM 7094 computer using the YAP reconstruction program and the YACK kinematic fitting program.[6] Since in the decay $K_L \to \pi + e + \nu$ the neutrino is not observed and the magnitude of the momentum of the K_L meson is unknown, the decay is of the zero-constraint type. Of the 835 decays, 92 have negative discriminant in the zero-constranit solution for the beam momentum. For these events we set the discriminant equal to zero to obtain the beam momentum. Of the 92 events with negative discriminant, 36 then have center-of-mass variables outside the kinematic limits and are rejected. Monte Carlo calculations indicate that the measurement errors and the scattering of the K_L beam in the lead γ-ray filter located between the target and the bubble chamber account for the number of rejected events. The majority of rejected events are due to K_L mesons which have scattered in the lead. These events tend to remain outside the kinematic limits even when the discriminant is set to zero. The net effect of setting the discriminant to zero is a slight (<1%) loss of good events near the boundary of the Dalitz plot. This effect was taken into account by the simulation technique used in the efficiency calculation which is discussed below.

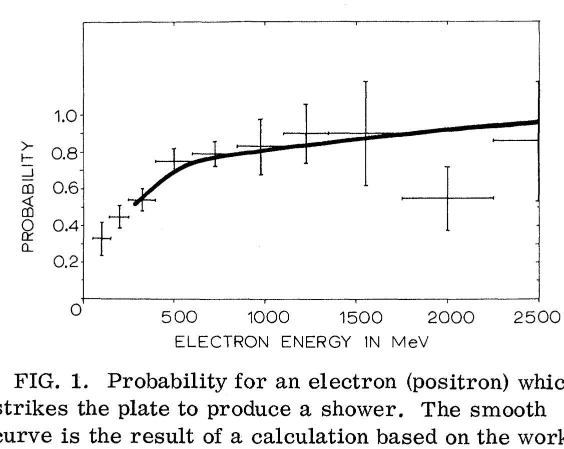

FIG. 1. Probability for an electron (positron) which strikes the plate to produce a shower. The smooth curve is the result of a calculation based on the work of Ref. 5 normalized to the experimental data.

Of the remaining 799 events, 309 are unambiguous in that one solution for the beam momentum is kinematically impossible, 195 events have two solutions less than 20 MeV apart on the Dalitz plot, and 295 events are ambiguous. The zero-constraint ambiguity is treated by choosing for each event the more probable of the two solutions. The ratio of the probabilities for the two solutions is the ratio of the product of (1) the a priori beam-momentum probability, known from the shape of the incident-momentum spectrum,[7] with (2) the Jacobian for the transformation from the known laboratory configuration to the center-of-mass configuration, calculated numerically for each event on a CDC 6600 computer, and with (3) the a priori probability for the Dalitz-plot position, computed with the assumption of a vector interaction with constant form factor. We note that the use of this interaction in the resolution of the zero-constraint ambiguity does not bias the data, but simply represents a known facet of the experimental resolution. For the 295 ambiguous events the sum of the probabilities of the chosen solutions is 215.35, and therefore, the fraction of significantly wrong choices is

$$1-\frac{309+195+215.35}{309+195+295}, \text{ or } 10\%.$$

Figure 2 shows the Dalitz plot for all the $K_{e3}{}^0$ decays, where we plot for each event only the more probable of the two solutions.

The detection efficiency as a function of position on the Dalitz plot is calculated by Monte

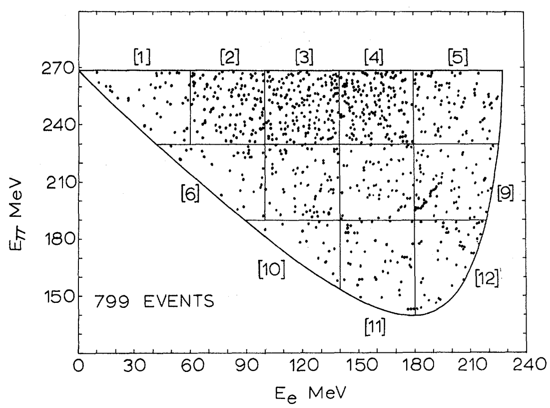

FIG. 2. Dalitz plot for the decay $K_L \rightarrow \pi + e + \nu$.

Carlo methods. We superimpose on the Dalitz plot a rectangular grid with intervals 20 MeV wide in both electron and pion energies. We construct an efficiency matrix, M, where M_{ij} is the probability of an event actually in the ith box appearing in our sample in the jth box. M_{ij} is calculated as follows: Events are generated uniformly in the ith box, the orientation of the decay in space is chosen at random, and a beam momentum is chosen according to the known shape of the incident-momentum spectrum.[4] The four-momenta of the two charged particles are then transformed to the laboratory, and a decay position is chosen at random in the fiducial volume. Using the known distribution of the magnetic field in the bubble chamber, we calculate the electron trajectory. An event in which the electron misses the plate is rejected. An accepted Monte Carlo event is then weighted according to the probability of the electron of the particular momentum producing a visible shower. The laboratory variables of the two charged particles are then varied according to the known formulas for measurement errors, assuming that the azimuth, dip, and curvature follow Gaussian distributions, and including the effects of correlations between the azimuth and curvature. The effects of the scattering of the K_L beam in the lead γ-ray filter are also included.[4] The Monte Carlo event is then analyzed in a manner identical to that used for the real events.

The matrix M_{ij} is primarily diagonal and shows that the experimental resolution is less than 20 MeV. The calculation also shows that, in about 10% of the events, an incorrect resolution of the zero-constraint ambiguity is made, in agreement with the previous estimate based on the actual events. In order to check the validity of the calculation of M, we have studied the distributions of detected events with respect to position in the chamber, visible momentum, and visible mass. The agreement is excellent in all distributions.

The predicted Dalitz plot distribution is given by $P_j = \sum_i M_{ij} T_i$, where T_i is the number of events in the ith box predicted by theory, and P_j is the actual expected number of events in the jth box, including all the effects of measurement errors, scattering in the lead, shower probability, wrong zero-constraint solution, etc.

For the following analysis we divide the Dalitz plot as shown in Fig. 2. Background events could arise from the high-mass tails of the Dalitz pairs, from π^0 mesons produced by $K_L \rightarrow 3\pi^0$, or by inelastic $K_L p$ interactions with invisibly short protons. Detailed calculations have shown that these events as well as background events due to the high-mass tail of externally converted γ rays appear in box 1 of the Dalitz plot. We therefore have excluded box 1 (37 events) from the subsequent analysis. The expected distributions are normalized to the observed number of 762 events in boxes 2 through 12.

Figure 3 shows the result of a chi-squared fit to the predictions of various theories. We

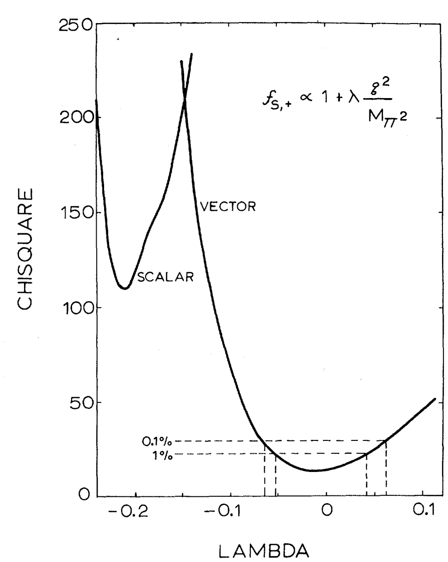

FIG. 3. Chi-squared fit to the data (with 10 degrees of freedom) for scalar and vector interactions in which the form factor is written as $f(q^2) = f(0)[1+\lambda(q^2/M_\pi^2)]$. The chi-squared for the tensor interaction is always greater than 250.

have assumed pure scalar, pure vector, and pure tensor interactions,[8] and have written the form factor as $f(q^2) = f(0)\,[1+\lambda q^2/M_\pi^2]$, in which q is the invariant mass of the leptons, or $q^2 = M_K^2 + M_\pi^2 - 2M_K E_\pi$. Scalar and tensor interactions do not fit at all. The results are in good agreement with vector interaction and show $\lambda = -0.010 \pm 0.02$, where the error represents one standard deviation and includes an estimate of systematic errors.

If the energy dependence of f is due to a single intermediate state of mass M_{K^*}, the form factor will have the form $f(q^2) = f(0)\,[M_{K^*}^2/(M_{K^*}^2 - q^2)]$. In this case, $M_{K^*} \geqslant 740$ MeV at the 99% confidence level and $M_{K^*} \geqslant 650$ MeV at the 99.9% confidence level.

The results of this experiment are in good agreement with the $|\Delta I| = \frac{1}{2}$ rule and the K_{e3}^+ data.[1] A more complete account of the K_L decay experiment will be published elsewhere.

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QUARK DECOMPOSITION OF HADRON SCATTERING AMPLITUDES*

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The quark model of hadrons,[1] combined with the additivity postulate for the quark-quark scattering amplitudes,[2] has had remarkable success in the analysis of high-energy scattering data. In particular, many sum rules have been derived between meson-baryon total cross sections which are satisfied to within or near experimental limits.[3] However, in sum rules relating meson-baryon to baryon-baryon total cross sections, such as[4]

$$\sigma_T(\bar{p}p) + \sigma_T(pn) = 2\sigma_T(\pi^- p) + \sigma_T(\pi^+ p) \qquad (1a)$$

or

$$\sigma_T(\bar{p}n) + \sigma_T(pp) = \sigma_T(\pi^- p) + 2\sigma_T(\pi^+ p), \qquad (1b)$$

there exists a large discrepancy (~15-20%) with experiment. This is particularly unfortunate since the sum rules concerned depend only on additivity, involve only the sums of cross sections (and, therefore, have relatively small experimental errors), and altogether provide a particularly "clean" test of the model. We show here how this discrepancy is re-