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¹A. W. Overhauser, Phys. Rev. **101**, 1702 (1956).

²R. S. Knox and N. Inchauspé, Phys. Rev. **116**, 1093 (1959).

³J. C. Phillips, Phys. Rev. Letters **12**, 142 (1964).

⁴J. C. Phillips, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press, Inc., New York, 1966), Vol. 18, p. 55.

⁵C. B. Duke and B. Segall, Phys. Rev. Letters **17**, 19 (1966).

⁶B. Velický and J. Sak, Phys. Status Solidi **16**, 147 (1966).

⁷U. Fano, Phys. Rev. **124**, 1866 (1961).

⁸M. Inoue, M. Okazaki, Y. Toyozawa, T. Inui, and E. Hanamura, Institute for Solid State Physics, University of Tokyo, Technical Report No. 203A 1966 (unpublished), p. 1.

⁹G. F. Koster and J. C. Slater, Phys. Rev. **95**, 1436 (1954).

¹⁰J. C. Phillips, Phys. Rev. **136**, A1714 (1964).

¹¹G. Baldini, Phys. Rev. **128**, 1562 (1962).

¹²R. J. Elliott, Phys. Rev. **108**, 1384 (1957).

¹³J. Callaway, J. Math. Phys. **5**, 783 (1964).

¹⁴J. Hermanson, Phys. Rev. **150**, 660 (1966).

¹⁵J. C. Phillips, Phys. Rev. **136**, A1705 (1964).

¹⁶Note that the normalization convention used in Ref. 5 assumes incoming and outgoing particle fluxes far from the origin equal to $(2\pi\hbar)^{-1}$ for all energies. This leads to an asymptotic $(E_0 - E)^{-1/2}$ behavior of the strength function for $\epsilon \rightarrow -\infty$ and a weak, asymmetric

peak for adiabatic potentials that are flat at the origin. However, for finite-range potentials the probability density and not the current is constant at large distances [the envelope function for noninteracting Bloch pairs $F^0(\vec{\beta}) = \Omega^{1/2}/(2\pi)^{3/2} e^{i\vec{k}\cdot\vec{\beta}}$, where Ω is the volume of the unit cell] and a flat potential produces at most a shoulder in $\alpha(E)$.

¹⁷This is the ground-state binding energy E_{2D} for the two-dimensional hydrogenic problem in Xe. The tunneling mechanism of Ref. 5 can lead to "peak" structure only at energy $E_r \approx E_0 - V_0$, where V_0 , the well depth of the adiabatic potential, is approximately given by E_{2D} (owing to cancellation effects discussed in Ref. 14, it is doubtful that central cell corrections would alter this estimate significantly).

¹⁸The exact formal relation between our finite-range approach and a treatment including the full Coulomb interaction may be nontrivial. See R. G. Newton, J. Math. Phys. **1**, 319 (1960), where the analytic properties of the scattering matrix near a parabolic edge are reviewed for infinite- and finite-range potentials. It should be noted, however, that the asymptotic behavior of the final-state wave function as $r \rightarrow \infty$, through important for scattering experiments, plays a minor role in optical response, which depends only on the probability amplitude at the origin. Moreover, the well-known accumulation of poles in the S matrix of the hydrogenic problem probably does not occur at an M_1 edge, because within the adiabatic approximation the effective one-dimensional potential $V_n(x_3)$ is repulsive for $r > a$ and so does not contain resonant states for large radial quantum numbers n of the two-dimensional (light mass) motion.

TWO-BODY SPECTRUM IN THE $1f_{7/2}$ SHELL

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Considering only two-body forces, the wave functions of all nuclear states of the $(f_{7/2})^n$ configuration may, in principle, be determined from a knowledge of the eight energy levels and spins of the $(f_{7/2})^2$ states of Sc^{42} or of the $(f_{7/2})(f_{7/2})^{-1}$ states of Sc^{48} . Thus, a great deal of effort has very recently been expended in studies of $\text{Sc}^{42, 1-4}$ and, to a somewhat lesser extent, in studying $\text{Sc}^{48, 5, 6}$. A major difficulty has been the initial problem of identifying those states which have these configurations. The measurements reported here identify six of these states in Sc^{48} and, through a particle-hole transformation to Sc^{42} , suggest plausible excitation energies of the remaining unknown $(f_{7/2})^2$ and $(f_{7/2})(f_{7/2})^{-1}$ levels of Sc^{42} and Sc^{48} .

A target enriched to 68% in Ti^{49} and a target of natural titanium were bombarded with a 20-MeV triton beam from the Los Alamos three-stage Van de Graaff accelerator. The spectrum of emergent alpha particles was recorded at six angles from 21° to 46° using the enriched target and at 36° and 46° using the natural target. Alpha particles were detected with a surface-barrier counter and the spectra stored in an SDS computer. The full width at half-maximum of a typical alpha-particle peak was 60 keV.

Shown in Fig. 1 are the angular distributions of the more prominent transitions to levels of Sc^{48} and of transitions to the ground state and the 0.765- and 1.4-MeV levels of Sc^{47} .

The latter two states are known^{7,8} to have $J^\pi = \frac{3}{2}^+$ and $\frac{1}{2}^+$ and are thus reached by d and s waves, respectively. The angular distributions of transitions to these levels differ markedly from those of the Sc^{48} ground-state ($J^\pi = 6^+$) transitions, which are necessarily f -wave transitions. Also, p -wave transitions are expected to be comparatively weak, as has been found in (t, α) reactions on the nickel isotopes.⁹ Thus, the more prominent transitions to predominant-

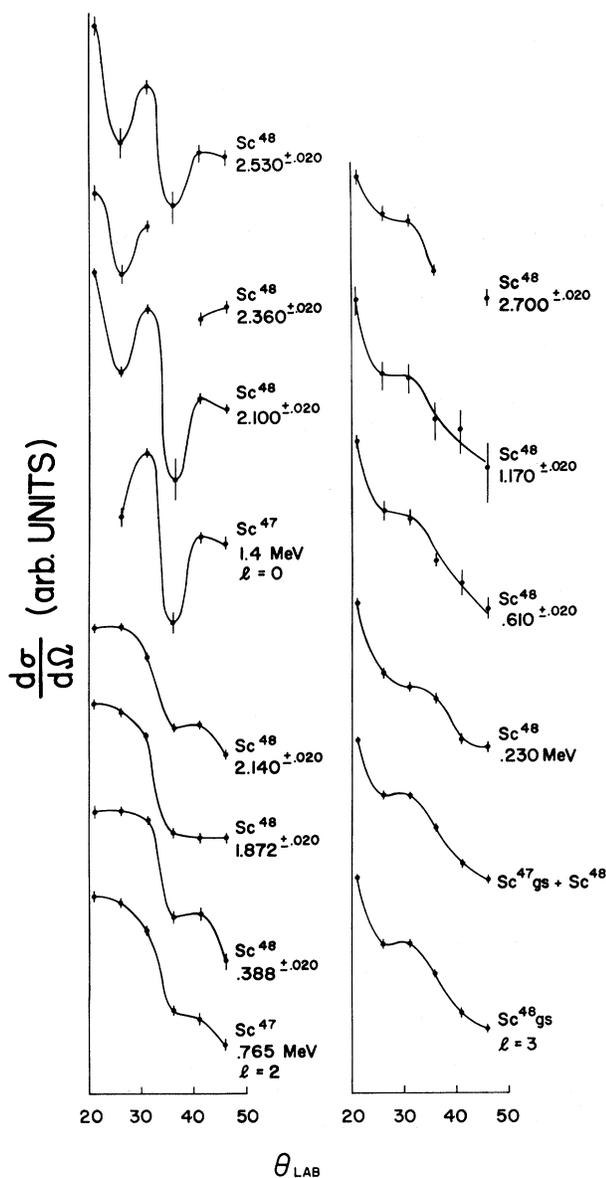


FIG. 1. Angular distributions of the more prominent transitions in the reactions $\text{Ti}^{49,48}(t, \alpha)\text{Sc}^{48,47}$ at 20.0 MeV. Cross sections relative to the Sc^{48} ground-state transition are indicated for f -wave transitions.

ly $(f_{7/2})(f_{7/2})^{-1}$ configurations are readily distinguishable.

From Fig. 1 it appears that levels at 0.230, 0.610, 1.170, and 2.700 MeV in Sc^{48} are reached with f waves. An additional Sc^{48} f -wave transition appears to be contributing strongly to the group of f -wave transitions corresponding to the Sc^{47} ground-state transition. The ratio of the intensity of the Sc^{47} ground-state transition using the enriched target to that using the natural target is twice the ratio of the intensities of all other Sc^{47} groups. This is further evidence for the existence of a Sc^{48} level at 131 keV suggested by Chasman, Jones, and Ristinen.⁵ A seventh $(f_{7/2})(f_{7/2})^{-1}$ state of Sc^{48} is estimated to lie at 7.150 ± 0.05 MeV. This is the $(J, T) = (0, 4)$ state, and its position is estimated from Coulomb and binding energy differences between Sc^{48} and Ca^{48} .

A recent $(p, n\gamma)$ study⁵ suggests spins 5, 4, and 3 for the 0.131-, 0.250-, and 0.610-MeV levels of Sc^{48} . To investigate the spins of the 2.700- and 1.150-MeV levels, a least-squares fit has been made to the particle-hole transformation equations¹⁰ relating the $(f_{7/2})(f_{7/2})^{-1}$ Sc^{48} spectrum to the $(f_{7/2})^2$ Sc^{42} spectrum. In making this fit, seven of the eight $(f_{7/2})(f_{7/2})^{-1}$ Sc^{48} levels were assumed to lie at 0.0, 0.131, 0.230, 0.610, 7.150, 1.170, and 2.700 MeV with spins of 6, 5, 4, 3, and 0 for the first five states, respectively. Also taken as fixed parameters were five $(f_{7/2})^2$ states of Sc^{42} at 0.0, 0.600, 0.620, 2.750, and 3.191 MeV with spins 0, 7, 1, 4, and 6, respectively. The variable parameters of interest were the spins of the 1.170- and 2.700-MeV levels and the unknown eighth $(f_{7/2})(f_{7/2})^{-1}$ state in Sc^{48} . The least-squares fit assigned $J = 1, 7$ to the 2.700- and 1.170-MeV states of Sc^{48} and placed the unknown eighth state of Sc^{48} at about 0.580 MeV with $J = 2$. Thus, the $J = 2$ level may possibly be either degenerate with the 0.610-MeV state or very weakly excited in this reaction or both, since the only level significantly populated between 0.3 and 1.0 MeV in excitation is the 0.610-MeV state.

Shown in Table I is the Sc^{42} spectrum calculated with the particle-hole transformation of the Sc^{48} spectrum suggested by the least-squares fit. The calculated and known levels of Sc^{42} agree remarkably well, having a rms deviation of only 70 keV. The unknown 3^+ and 5^+ states of Sc^{42} are calculated to lie at 1.556 and 1.540 MeV, respectively. Such a degen-

Table I. The known spectrum of Sc^{42} as compared with the Sc^{42} spectrum calculated from a particle-hole transformation of assumed Sc^{48} spectrum.

Assumed Sc^{48} spectrum		Calculated Sc^{42} spectrum		Known Sc^{42} spectrum	
E (MeV)	J	E (MeV)	J	E (MeV)	J
g.s.	6	g.s.	0	g.s.	0
0.131	5	0.420	7	0.6 ± 0.1	7
0.230	4	0.704	1	0.620	1
0.580	2	1.540	5		
0.610	3	1.556	3		
1.170	7	1.607	2	1.590 ^a	2
2.700	1	2.835	4	2.750	4
7.150	0	3.127	6	3.191	6

^aIt is possible, of course, that the $\text{Sc}^{42} (f_{7/2})^2 2^+$ state is split, as in Ca^{42} , into two levels with a centroid at about 2.0 MeV. However, no such splitting has as yet been identified in Sc^{42} .

eracy would resolve the conflicting results of recent Sc^{42} studies, one of which indicates that a state of spin 3 lies at about 1.5 MeV,⁴ and one of which indicates that a state of spin 5 lies at about 1.5 MeV.³ In any event, if the 0.131-, 0.230-, and 0.610-MeV Sc^{48} states have spins of 5, 4, and 3, respectively, as suggest-

ed by Chasman, Jones, and Ristinen, then the measurements reported here do not support suggestions^{1,3} that either the 3^+ or the 5^+ ($f_{7/2}$)² state of Sc^{42} lies above 2.0 MeV.

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¹J. W. Nelson, J. D. Oberholtzer, and H. S. Plendl, Nucl. Phys. **62**, 434 (1965).

²D. Cline, H. E. Gove, and B. Cujec, Bull. Am. Phys. Soc. **10**, 25 (1965).

³E. Rivet, R. H. Pehl, J. Cerny, and B. G. Harvey, Phys. Rev. **141**, 1021 (1966).

⁴R. S. Zurmuhle, C. M. Fou, and L. W. Swenson, Nucl. Phys. **80**, 259 (1966).

⁵C. Chasman, K. W. Jones, and R. A. Ristinen, Phys. Rev. **140**, 212 (1965).

⁶J. B. Ball, Bull. Am. Phys. Soc. **11**, 349 (1966).

⁷J. L. Yntema and G. R. Satchler, Phys. Rev. **134**, B976 (1964).

⁸R. E. Holland, F. J. Lynch, and K. E. Nysten, Phys. Rev. Letters **13**, 241 (1964).

⁹A. G. Blair and D. D. Armstrong, Bull. Am. Phys. Soc. **11**, 365 (1966).

¹⁰A. DeShalit and I. Talmi, *Nuclear Shell Theory* (Academic Press, Inc., New York, 1963).

ENERGY DEPENDENCE OF THE FORM FACTOR IN K_{e3}^0 DECAY*

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A sample of 762 decays $K_L \rightarrow \pi + e + \nu$ in the 80-inch hydrogen bubble chamber have been identified using electron detection by shower production in a tantalum plate. The results are consistent with vector interaction and constant form factor.

The $|\Delta I| = \frac{1}{2}$ rule for the semileptonic weak decay predicts that the dependence of the vector form factor on pion energy be the same in the K_{e3}^0 and K_{e3}^+ decays. Experiments on the K_{e3}^+ decay show no evidence for an energy dependence of the form factor.¹ Of the two previous experiments on the K_{e3}^0 decay, one allows for an energy dependence within large errors,² and the other indicates a large

energy dependence in the form factor.³ The experimental data appear to be in good agreement with the $|\Delta I| = \frac{1}{2}$ rule for the decay rates, i.e., $\Gamma(K_L \rightarrow \pi + e + \nu) = 2\Gamma(K^+ \rightarrow \pi + e + \nu)$.

We report here results of an experiment on K_L decays, in which the K mesons are produced from a 7-BeV/c π^- beam incident on an aluminum target and are detected with the Brookhaven 80-inch liquid-hydrogen bubble chamber.⁴