with a nuclear explosion used as a source of intense pulsed neutrons[11] confirm that fission in the slow-neutron resonances is strongly inhibited and is only about 1/100 of the capture process. Since the measurements of the capture and fission cross sections were made simultaneously with the same energy resolution, it is meaningful to compare the shape and peak heights of the cross-section curves. There is a striking similarity in the two partial cross sections, particularly below 200 eV, which implies a nearly constant value for the ratio Γ_f/Γ_γ. If Γ_γ is assumed to be constant, then the fission width varies little, indicating a large number of effective channels open for fission and thus also providing strong evidence for this reaction in Pu^{240}.

We believe that the data described here constitute substantial evidence that the $(n,\gamma f)$ reaction is present in Pu^{238} and that more accurate and complete studies on Pu^{238}, Pu^{240}, and other favorable nuclei should provide conclusive evidence of the existence of this reaction, and perhaps give quantitative information on the extent to which it is present.

---

†Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]V. Stavinsky and M. O. Shaker, Nucl. Phys. 62, 667 (1965).

[2]J. E. Lynn, Phys. Letters 18, 31 (1965).

[3]P. E. Vorotnikov, S. M. Dubrovina, G. A. Ostrochenka, and V. A. Shigin, Yadern. Fiz. 3, 479 (1966) [translation: Soviet J. Nucl. Phys. 3, 348 (1966)].

[4]L. Katz, A. P. Baerg, and F. Brown, in Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy (United Nations, Geneva, Switzerland, 1958), Vol. 15, p. 188.

[5]A. Bohr, in Proceedings of the United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955 (United Nations, New York, 1956), Vol. 2, p. 151.

[6]C. E. Porter and R. G. Thomas, Phys. Rev. 104, 483 (1956).

[7]W. F. Stubbins, C. D. Bowman, G. F. Auchampaugh, and M. S. Coops, Phys. Rev., to be published.

[8]T. E. Young, F. B. Simpson, J. R. Berreth, and M. S. Coops, Idaho Nuclear Corporation Report No. IN-1026 (unpublished).

[9]L. Wilets, Phys. Rev. Letters 9, 430 (1962); Theories of Nuclear Fission (Clarendon Press, Oxford, England, 1964), p. 117.

[10]M. De Vroey, A. T. G. Ferguson, and N. Starfelt, in Proceedings of the Symposium on the Physics and Chemistry of Fission, Salzburg, 1965 (International Atomic Energy Agency, Vienna, 1965).

[11]D. H. Byers, B. C. Diven, and M. G. Silbert, in Conference on Neutron Cross-Section Technology, Report No. CONF 660303, edited by P. B. Hemmig (Atomic Energy Commission, Washington, D. C., 1966), Paper F-5.

LOW-ENERGY THEOREMS AND INTERNAL SYMMETRY

A. Pais
The Rockefeller University, New York, New York

(Received 28 November 1966)

It is the purpose of this note to point out that the simultaneous use of (a) low-energy theorems and (b) internal symmetries leads to dynamical constraints without any approximation in regard to strong-interaction intermediate states. In other words, as we shall show first for isospin, then for SU(3), the simultaneous implementation of (a) and (b) leads to consistency conditions of a new kind which follow neither from (a) alone nor from (b) alone. When combined with nonsubtracted dispersion relations, the conditions take the form of integral relations between cross sections—see, e.g., Eqs. (5) and (8) below. They cannot, in general, be satisfied pointwise by cross sections at a given energy. We shall see in fact that they may interconnect with each other distinct multiplets of the internal symmetry.

As a first application, the truncation methods for sum rules will be discussed. Several attempts have been made recently to truncate cross-section integrals in sum rules by the approximation of the continuum by a finite set of more or less sharp resonant states. A main aim of this procedure is to find dynamical constraints which may serve to understand approximate dynamical symmetries such as SU(6). The consistency conditions are of interest for the understanding of the truncation method. It will be shown how they generate so-called "null solutions." An example of these is the following. It has been noted by many authors[1] that sum rules for the anomalous moments of the proton and the neutron give as a good leading approximation

$$\kappa(p)+\kappa(n)=0, \tag{1}$$

provided one assumes 33-resonance dominance. More recent estimates[2] for $\kappa(p)$ separately lend further credence to this approximation. However, the extension to SU(3) and decuplet dominance yields[3] the further restriction on (1) that $\kappa(p)=\kappa(n)=0$. The view will be adopted here that this is an undesirable result.[4] As we shall see, the consistency conditions are of use to understand this situation and to indicate approximations such that (1) is true but such that null solutions are avoided.

In this note we exemplify the idea with the help of the low-energy theorem[5] for the amplitude $f_2(\omega^2)$ in Compton scattering and the related sum rule for anomalous magnetic moments given by Drell and Hearn.[2] Higher order electromagnetic effects are neglected throughout.

Consider a spin-$\frac{1}{2}$ particle with mass M and anomalous moment $e\kappa/2M$. The low-energy theorem says that

$$f_2(0)=-\alpha\kappa^2/2M^2. \tag{2}$$

Note that Eq. (2) is true provided the particle in question is not degenerate with others with the same quantum numbers. Likewise, it will be assumed in what follows that if a multiplet is discussed, it is not degenerate with other similar multiplets.

Isospin symmetry alone.—Consider a multiplet with isospin I. For $I>\frac{1}{2}$, $f_2(\omega^2)$ is a superposition of amplitudes with isospin $I+1$, I, $I-1$. On the other hand, the κ's within any multiplet satisfy

$$\kappa=a+bQ, \tag{3}$$

with constants a,b over the multiplet. Q is the charge variable within the multiplet. Thus the right-hand side of (2) contains two parameters a, b, while the left-hand side contains four amplitudes, for $I>\frac{1}{2}$. Hence the latter are constrained. (For the nucleon, no such conclusion can be drawn.)

This constraint leads to a sum rule for total cross sections if one assumes unsubtracted dispersion relations:

$$2\pi^2\alpha\kappa^2/M^2=X,$$
$$X=\int_0^\infty \omega^{-1}d\omega\,[\sigma_P(\omega)-\sigma_A(\omega)]. \tag{4}$$

For example, applied to the Σ triplet, one has

$$4X(\Sigma^0)=X(\Sigma^+)+X(\Sigma^-)\pm 2[X(\Sigma^+)X(\Sigma^-)]^{1/2}, \tag{5}$$

where the + (−) sign holds if $\kappa(\Sigma^+)$ and $\kappa(\Sigma^-)$ have the same (opposite) sign. Equation (5) does not follow from isospin alone. The following is an example of a null solution: If the continuum is approximated by a set of isoscalar states (Λ, Y_0^*, $\cdots$) then all Σ's have zero κ.

SU(3) neglecting SU(3) breaking except for lowest electromagnetic order.—For any multiplet, κ can be written as

$$\kappa=\kappa_F Q-\tfrac{1}{3}2k_D[U(U+1)-\tfrac{1}{4}Q^2-\tfrac{1}{3}F^2], \tag{6}$$

where κ_F, κ_D are the F- and D-type anomalous moments. U is the U-spin eigenvalue and $F^2=\frac{1}{3}(p^2+q^2-pq+3p)$ is the value of the quadratic Casimir operator for the representation (p,q). [For the octet, $(p,q)=(2,1)$ and $F^2=3$.] Thus the left-hand side of (4) now depends (at most) on two parameters, while the right-hand side depends (except for the case of a singlet) on a larger number of amplitudes, so the latter are again constrained.

Consider the baryon octet as an example, where one has four amplitudes after reduction with respect to U spin. Let $X(U,Q)$ denote the quantity defined in Eq. (4) for specified U, Q values. We have

$$X(1,0)=X(0,0)=8\pi^2\alpha\kappa_D^{\,2}/9M^2,$$
$$X(\tfrac{1}{2},1)=2\pi^2\alpha(\kappa_F+\tfrac{1}{3}\kappa_D)^2/M^2,$$
$$X(\tfrac{1}{2},-1)=2\pi^2\alpha(\kappa_F-\tfrac{1}{3}\kappa_D)^2/M^2. \tag{7}$$

The relation (5) is again implied by (7), while in addition

$$X(p)+X(n)-2[X(p)X(n)]^{1/2}=X(\Sigma^-). \tag{8}$$

[Here a sign has been fixed from the knowledge that $\kappa(p)$ and $\kappa(n)$ have opposite sign.] Equation (8) does not follow from U spin alone.

Equations (5) and (8) are not so much of interest from the point of view of direct verification. What does perhaps seem worthwhile to note is the mere fact that such cross-section relations are induced by an internal symmetry combined with the input that goes into the low-energy theorem together with the use of unsubtracted dispersion relations[6] and regardless of the detailed structure inside the "black boxes" for the reactions γ + particle → final states. Note that the present argument can also be applied to the isoscalar sum rule given by Bég.[7]

Next we give a few examples which indicate how the equations (7) work when one applies the truncation methods.

(1) Consider the case where the cross sections in (4) are approximated by having as final states a set of $\underline{10}$ and $\underline{10}^*$ resonances which may have nonzero widths but such that there is no overlap between $\underline{10}$ and $\underline{10}^*$. Otherwise the set is arbitrary. From the (U, Q) content of $\underline{10}$ and $\underline{10}^*$, it follows that $X(0,0)=0$. Hence from (6), $X(1,0)=0$. On the other hand, one readily verifies that for the set of states at hand one has the stronger condition $X(1,0) = X(\frac{1}{2},1)+X(\frac{1}{2},-1)$. But all X's must be ≥ 0. Thus we find a "null solution:" All X's $=0$; $\kappa_F=\kappa_D=0$. Thus is true regardless of the spin values we assign to the resonances.

This example contains the special case where we saturate the cross sections with just the familiar $J=\frac{3}{2}$ decuplet which leads to $\kappa(p)=\kappa(n)=0$ in the SU(3) limit. With the help of (7), many other combinations of saturating states which yield null solutions are easily written down.

(2) Consider next the case of an arbitrary set of $\underline{10}$ and of $\underline{1}$ resonances. We allow any amount of overlap as well. Neither $\underline{10}$ nor $\underline{1}$ contains $U=\frac{1}{2}$, $Q=-1$. Therefore $X(\frac{1}{2},-1)=0$ hence, from (7),

$$\kappa_D=3\kappa_F-\kappa(p)+\kappa(n)=0. \tag{9}$$

Can this result be spoiled by the other relations in (7), i.e., do we have a null solution or not? A simple calculation shows that for this set,

$$X(1,0)=X(\tfrac{1}{2},1)=C(\underline{10}),$$
$$X(0,0)=C(\underline{1}),$$

where $C(\underline{10})$, $C(\underline{1})$ denotes the contribution to the respective integrals from the $\underline{10}$ and the $\underline{1}$ states, respectively. Note that $X(1,0)=X(\frac{1}{2},1)$ implies in particular that $\kappa^2(p)=\kappa^2(n)$.

We now see what the consistency conditions do: They imply that

$$C(\underline{10})=C(\underline{1}), \tag{10}$$

which shows again that if we drop the $\underline{1}$'s, we get a null solution. It is remarkable that however one approximates the continuum by selected unitary multiplets, the relation $\kappa(p)=-\kappa(n)\neq 0$ can be obtained only if the transition elements between the baryon and distinct SU(3) multiplets are correlated. Thus the consistency conditions contain information on SU(3) spectra. (Note that spins and parities are not specified at this stage.)

The saturation with at least one $\underline{10}$ and one $\underline{1}$ is the simplest way I know of to satisfy Eq. (1) with $\kappa\neq 0$. The neglect of contributions from $\underline{10}^*$'s, $\underline{27}$'s, and other $\underline{8}$'s always gives (1) without further ado. Their inclusion in a nonoverlapping way can be shown to yield $C(\underline{10})+C(\underline{10}^*)=C(\underline{1})+(5/9)C(\underline{27})$, where $C(\underline{27})$ and $C(\underline{10}^*)$ are the respective contributions to $X(0,0)$ and $X(\frac{1}{2},-1)$, but this is not enough to yield (1) without additional constraints. In this sense Eq. (10) appears to be the simplest possibility.

The present argument leads to three further questions:

(1) To examine how the implementation of Eq. (10), or an alternative thereof, can be justified further. Note that Eq. (4) for the octet together with Eq. (10) implies a connection between all possible three-triplet states, with a suppression of further states that moreover contain triplet-antitriplet pairs.

(2) To examine whether or not there exist other consistency conditions, compatible with but independent of those given above (due, for example, to stronger symmetry requirements), such that the class of null solutions will turn out to be even greater than is found here.

(3) To examine whether the nosubtraction Ansatz itself is justified for specific models.

It is a pleasure to thank M. A. B. Bég for illuminating discussions.

---

[1]I learned this first from a paper by L. D. Soloviev, to be published.

[2]S. D. Drell and A. C. Hearn, Phys. Rev. Letters 16, 908 (1966).

[3]This was first pointed out to me by M. A. B. Bég.

[4]In other words, it is assumed that anomalous moments do not exclusively arise from SU(3) breaking.

[5]F. E. Low, Phys. Rev. 96, 1428 (1954); M. Gell-Mann and M. L. Goldberger, Phys. Rev. 96, 1433 (1954).

[6]Consistency conditions also persist in the presence of subtractions, but then of course they can not be simply related to integrals over cross sections.

[7]M. A. B. Bég, Phys. Rev. Letters 17, 333 (1966), Eq. (12). In this case one uses again Eq. (6) but with $(U,Q)\to(I,-Y)$ and again obtains the corresponding Eqs. (7).