REGGE CUT IN PROTON-PROTON SCATTERING*

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A remarkable break in the differential cross section for p-p scattering as a function of energy at $\theta = 90°$ has been discovered recently.[1] In this note, we point out that this phenomenon may be explained in terms of a Regge cut, the existence of which has been postulated for independent theoretical reasons.[2] The experimental information may be summarized by the formula

$$(d\sigma/dt)_{90°} = \begin{cases} C_1 \exp(-3.29k^2), & k^2 < 3.40, \\ C_2 \exp(-1.51k^2), & k^2 > 3.40, \end{cases} \quad (1)$$

where C_1, C_2, are constants, and k is the c.m. momentum in BeV/c. The experimental points cover the region $2 < k^2 < 6$, corresponding to $4 < -t < 12$, where as usual, $t = -2k^2(1-\cos\theta)$.

For an exploratory investigation, we ignore complications due to spin and treat the protons as if they were scalar particles. Our purpose is not to produce a good numerical fit of the data, but to see whether a mechanism can be found that leads naturally to the observed phenomenon. We assume that the scattering amplitude is given by a Regge formula without the "background" term. Mandelstam[3] has shown in potential scattering that the background integral in the Sommerfeld-Watson transformation can be pushed arbitrarily far to the left in the J plane, provided one continues $P_J(\cos\theta)$ into the complex J plane in a specific manner, which we adopt. The contribution of a single Regge pole of positive signature is then given by

$$A_{\text{pole}}(s,t) = \bar{\beta}(t)(2\alpha+1)(1+e^{-i\pi\alpha}) \times Q_{-\alpha-1}(\cos\theta_t)/\cos\pi\alpha, \quad (2)$$

where $\cos\theta_t = -1 - s/2k_t^2$, with $k_t^2 = t/4 - m^2$. For $t < 4m^2$, $\bar{\beta}(t)$ is real and has no pole. It is related to the residue of the Regge pole $\beta(t)$ by $\bar{\beta}(t) = \beta(t)e^{-i\pi\alpha}$. If α is the leading trajectory, $\bar{\beta}(t)$ must have zeros at negative half-integer values of α to cancel the poles of $1/\cos\pi\alpha$. The function $Q_{-\alpha-1}$ is finite at negative integer α. Hence, $A_{\text{pole}} = 0$ at $\alpha = -1, -3, -5, \cdots$, owing to the signature factor. (In the usual Regge formula in which $Q_{-\alpha-1}/\cos\pi\alpha$ is replaced by $P_\alpha/\sin\pi\alpha$, only $\text{Re}A_{\text{pole}} = 0$ at these values of α.) At $\theta = 90°$, $-\cos\theta_t \to 3$ as $k \to \infty$. For $k^2 > 2$, we can replace $Q_{-\alpha-1}$ by its asymptotic form with an error of less than 10%. This leads to

$$A_{\text{pole}}(s,t) = \gamma(t) y^{\alpha} (1 + e^{-i\pi\alpha}), \quad (3)$$

where $y = 2\cos\theta_t$, and $\gamma(t) = \pi^{1/2}\bar{\beta}(t)(2\alpha+1)[\Gamma(\alpha+\frac{1}{2})/\Gamma(\alpha+1)]\csc\pi\alpha$. For our purpose, it is sufficient to know that for $t < 4m^2$, $\gamma(t)$ is real and has no poles. We assume it is a smooth function and slowly varying compared with the rest of the factors in (3).

Apart from the Pomeranchuk trajectory P, the trajectories relevant for p-p scattering are P' and ω. We assume that all these trajectories are well approximated by straight lines. The slope of P is taken to be between $\frac{1}{3}$ and $\frac{1}{5}$ (BeV/c)$^{-2}$, and those of P' and ω about 1 (BeV/c)$^{-2}$. The former is indicated by forward p-p scattering.[4] The latter is the known slope for A_2 and ρ,[5] which we assume to be in the same SU(3) octet as P' and ω, respectively. The trajectories are shown in Fig. 1, from which we can see that the effects of P' and ω can be neglected compared with that of P, for $t < -2$. The trajectory of P is real for $t < 4m^2$ and is taken to be

$$\alpha(t) = 1 + \alpha' t. \quad (4)$$

Associated with P are branch cuts in the complex J plane, arising from the exchange of two or more P trajectories.[2] We only take into account the cut arising from two P trajectories. The moving branch point of this cut has a trajectory determined by the P trajectory, given by

$$\alpha_c(t) = 2\alpha(t/4) - 1 = 1 + \tfrac{1}{2}\alpha' t, \quad (5)$$

which is also shown in Fig. 1. The contribution of the cut to the scattering amplitude is taken to be

$$A_{\text{cut}}(s,t) = \gamma_c(t) y^{\alpha_c}[1 + \exp(-i\pi\alpha_c)], \quad (6)$$

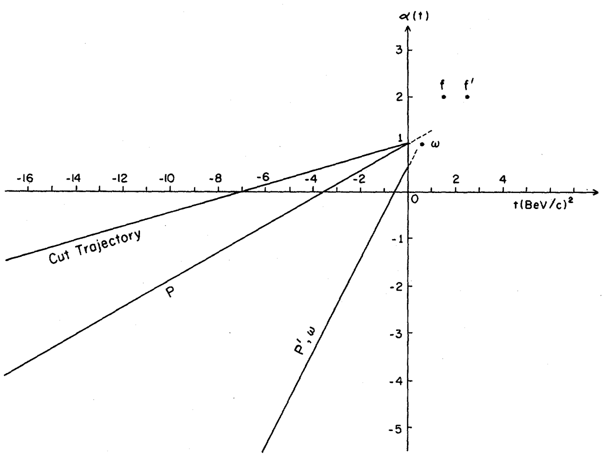

FIG. 1. Regge trajectories and cut trajectory relevant for p-p scattering. Our model takes into account only the cut and P.

where $\gamma_c(t)$ absorbs a factor $1/\ln(-t)$ and is a real function with no pole. Equation (6) represents only the leading contribution of the cut for large $-t$. The correction terms are of order $1/\ln(t/t_0)$, where t_0 is a unknown scale factor. These correction terms do not, in general, have a definite signature. We assume $\gamma_c(t)/\gamma(t)$ to be a constant:

$$\lambda = \gamma_c(t)/2\gamma(t), \tag{7}$$

which is an adjustable parameter. We note that in our approximation, $A_{\mathrm{cut}} = 0$ when α_c is an odd integer. This corresponds to $\alpha = -3$, -7, -9, $\cdots$. Our model consists of writing for the scattering amplitude

$$A(s,t) = A_{\mathrm{pole}}(s,t) + A_{\mathrm{cut}}(s,t), \tag{8}$$

which is related to the differential cross section by

$$d\sigma/dt = (\pi^2/k^4)\,|A(s,t)|^2. \tag{9}$$

Let us first see qualitatively how a sharp break can occur in $d\sigma/dt$. Since the effect of the cut is apparently absent in diffraction scattering, we expect $\lambda \ll 1$. As we go to large k^2 at negative t, however, the cut must eventually dominate over P because α_c has a smaller slope than α. The slope of $d\sigma/dt$, therefore, would be determined mainly by P at moderate values of k^2, and by the cut at large k^2. The changeover can occur sharply if it happens near $\alpha = -1$, where A_{pole} vanishes, thereby exposing the cut dramatically. Referring to Fig. 1, we see that $\alpha = -1$ at $t \approx -7$, corresponding to $k^2 \approx 3.5$, which is just where the break in $d\sigma/dt$ was observed. In our model, we can realize this by choosing λ appropriately.

At $\theta = 90°$, $t = -2k^2$, and (9) can be cast in the form

$$d\sigma/dt = \pi^4 \alpha'^2 \gamma_c^2 f_\lambda(\varphi), \tag{10}$$

$$f_\lambda(\varphi) = \left[\frac{\sin\frac{1}{2}\pi\varphi}{\frac{1}{2}\pi\varphi}\right]^2 y^{2-2\varphi} \times \left[\sin^2\tfrac{1}{2}\pi\varphi + \left(1 + \frac{1}{\lambda}y^{-\varphi}\right)^2 \cos^2\tfrac{1}{2}\pi\varphi\right], \tag{11}$$

$$\varphi = \alpha' k^2, \tag{12}$$

$$y = 6(\varphi + \tfrac{2}{3}\alpha' m^2)/(\varphi + 2\alpha' m^2). \tag{13}$$

The factor $y^{2-2\varphi}$ in front of the bracket in (11) is due to the cut alone, while the bracket represents the effects of the pole and the interference between cut and pole. If $\lambda \ll 1$, the slope of $f_\lambda(\varphi)$ changes rapidly in the neighborhood of $\varphi = 1$. In fact, the curve exhibits a nearly discontinuous change of slope. We choose α' to make the point $\varphi = 1$ correspond to $k^2 = 3.4$. The value of λ is then chosen to make the difference in slope before and after the rapid change agree with experiments.

To obtain correct absolute slopes, it is necessary to assume that $\gamma_c = \gamma_0 \exp(-ck^2)$, where γ_0 and c are constants. We find in this manner

$$c = 0.35\ (\mathrm{BeV}/c)^{-2},$$
$$\alpha' = 0.29\ (\mathrm{BeV}/c)^{-2},$$
$$\lambda = 0.11, \tag{14}$$

which are all consistent with our original assumptions. We note that if the sign of λ is changed, then $d\sigma/dt$ does not possess the desired break. A comparison between theory and experiment is shown in Fig. 2. The dashed portions of the curve correspond to regions in which the model is inapplicable.

For $k^2 < 2$, the model should not be used because the effects of P' and ω have been neglected. For $k^2 > 4.5$, the theoretical curve falls rapidly because (10) vanishes at $\varphi = 2$ ($k^2 = 6.8$), where $\alpha = -3$, $\alpha_c = -1$. Clearly, near this point, we must take into account the higher order corrections to the cut as well as the cuts arising from the exchange of three or more P trajectories. Since these cuts lie higher than the first cut, they may dominate the cross section at large k^2.

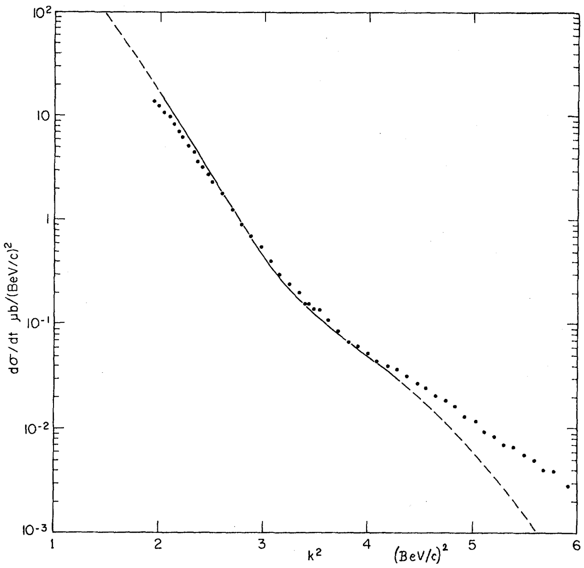

FIG. 2. Comparison between theory (the curve) and experiment (the dots). Experimental errors vary from 3 to 6%.

There exists an independent suggestive theoretical argument that the Regge cuts should be important at the value of t where $\alpha = -1$. It has been shown by Gribov and Pomeranchuk[6] that in the absence of cuts, there will exist an essential singularity at $J=-1$ where an infinite number of Regge poles accumulate. The Regge cut proposed by Mandelstam shields the essential singularity and allows poles to pass through the point $J=-1$. It is, therefore, not surprising that the cut becomes important at a point where the pole trajectory passes through $J=-1$.

The model used here can be extended to angles other than 90°. For an order-of-magnitude estimate, we may extend (11) in an obvious way to $\varphi \neq 90°$ and symmetrize with respect to $\cos\theta$. There are unfortunately only a few experimental points available for comparison, for the bulk of existing large angle data[7] pertains to $k^2 > 7$, where our model must be supplemented by other contributions, such as a higher Regge cut. For the few points that comparison can be made, there is consistency within a standard deviation. However, we do not regard such a comparison significant, because at $\theta \neq 90°$, the scattering amplitude can acquire a term antisymmetric in $\cos\theta$, which will be independent of the extension of (11). A careful treatment of the complete angular range, with spin taken into account, is in progress.

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