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STUDY OF THE $\pi\pi$ S-WAVE PHASE SHIFT IN THE ρ REGION TAKING INTO ACCOUNT ABSORPTION EFFECTS*

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This note contains a determination of the $\pi\pi$ $I=0$, S-wave phase shift in the ρ region under certain assumptions about the effects of absorption on the differential cross section for the reactions $\pi^- + p \rightarrow \pi^+ + \pi^- + n$. The model which we employ is discussed, and the results of applying the model to data from π^-p interactions at 2.7 GeV/c are presented.

The data at 2.7 GeV/c show that the reactions

$$\pi^- + p \rightarrow \pi^+ + \pi^- + n, \qquad (1)$$

$$\pi^- + p \rightarrow \pi^0 + \pi^- + p, \qquad (2)$$

are dominated by ρ production.[1] When Reaction (2) was fitted to the absorption-modified one-pion-exchange model due to Gottfried and Jackson,[2] the theoretical cross section for ρ^- and ρ^0 and the density-matrix elements for ρ^- showed good agreement with our data at 2.7 GeV/c.[1] The Gottfried-Jackson model[2] does not include S-, P-wave interference effects, and thus cannot be applied to the study of the observed ρ^0-decay asymmetry.[1] Consequently, we have modified this absorption model to include an $I=0$, $J=0$ amplitude in the spirit of Gottfried and Jackson[2] and Durand and Chiu.[2,3]

Several authors have attributed this asymmetry to the interference of the $I=0, J=0$ and $I=1$, $J=1$ $\pi\pi$ elastic scattering amplitudes.[4] Durand and Chiu[3] have calculated the ρ^0 density-matrix elements including a $J=0, I=0$ resonance and find agreement with the experimental ρ^0-decay angular distribution averaged over the ρ^0 mass region. Hagopian and Selove[5] see evidence for an $I=0, J=0$ resonance in the $\pi^+\pi^-$ effective-mass spectrum. On the other hand, analyses by Jacobs[6] and Jabiol, James, and Nguyen[7] are in disagreement with Hagopian and Selove. A theoretical calculation by Finkelstein[8] shows a decreasing S-wave phase shift in accordance with a suggestion by Chew.[9] Dilley[10] has pointed out the possibility that a slowly increasing S-wave phase shift may go through 90° without being resonant. Estimates of the S-wave $\pi\pi$ phase shift have been calculated from experimental data by Jones et al.,[11] Wolf,[12] and Baton and Reignier.[13] None of these determinations has included absorption effects. But it is known that absorption effects introduce an isotropic term into the ρ-decay distribution[1,2]; thus in calculating the S-wave $\pi\pi$ phase shift as a function of the $\pi\pi$ effective mass, we have introduced absorption corrections.

Our model for calculating the S-wave phase shifts assumes the following:

(a) The absorption model, modified to include an S-wave amplitude, describes correctly the θ^* dependence of the $\pi\pi$-decay angular distribution in the region $0.8 < \cos\theta^* < 1$, where θ^* denotes the angle between the incident π^- and the outgoing di-pion in the over-all center-of-mass system.

(b) The ρ^0-decay angular distribution at the limit of the experimentally accessible region ($\cos\theta^* = 1.0$) is the same as the on-mass-shell distribution at the pole $\Delta^2 = -\mu^2$, where Δ^2 denotes the square of the four-momentum transfer to the recoil neutron and μ denotes the pion mass.

(c) Only $I=0, J=0$ and $I=1, J=1$ partial waves are considered.

(d) The P-wave phase shift for on-the-mass-shell $\pi\pi$ scattering is given by

$$\tan\delta_1^{\,1} = \frac{\omega_r}{\omega_r^{\,2} - \omega^2} \frac{2(q/q_r)^3}{1 + (q/q_r)^2} \Gamma_r, \qquad (3)$$

where $\omega_\gamma = 0.77$ GeV and $\Gamma_\gamma = 0.125$ GeV. The symbols q and q_γ are the momenta of the decay pions at $\pi^+\pi^-$ effective mass ω and the resonance peak, respectively. $\delta_J{}^I$ denotes the $\pi\pi$ phase shifts in the state with angular momentum J and isospin I.

First we consider the validity of assumption (a). The S- and P-wave helicity amplitudes[14] are calculated in the Born approximation[2,3] and the absorption effects are included as described by Högaason and Högaason[15] and coded by Keyser and Donahue.[16] The ρ^0-decay angular distribution in terms of the density-matrix elements is[17]

$$W(\theta,\varphi) = 1/4\pi + (3/4\pi)\{(\rho_{00}-\rho_{11})(\cos^2\theta - \tfrac{1}{3}) - \sqrt{2}\,\mathrm{Re}\rho_{10}\sin 2\theta\cos\varphi - \rho_{1,-1}\sin^2\theta\cos 2\varphi\}$$
$$+ (\sqrt{3}/4\pi)\{-2\sqrt{2}\,\mathrm{Re}\rho_{10}{}^{\mathrm{int}}\sin\theta\cos\varphi + 2\mathrm{Re}\rho_{00}{}^{\mathrm{int}}\cos\theta\}, \quad (4)$$

where ρ_{ik} denotes the ρ density-matrix element, the subscripts denote the helicity state of the ρ^0 amplitude, and $\rho_{ik}{}^{\mathrm{int}}$ denotes the density-matrix element arising from the S-, P-wave interference. The polar and aximuthal angles are θ and φ and the choice of the coordinate system is identical to that of Jackson.[18] The values of the density-matrix elements are determined experimentally by fitting our data to Eq. (4). The results are shown in Fig. 1(a), where the solid curves are the predictions of the S- and P-wave absorption model. The theory is in reasonable agreement with our data except for the ρ_{00}-ρ_{11} curve, where the theory underestimates the peak value of ρ_{00}-ρ_{11}.

Assumption (b) is the most crucial. Experimental data are available only in the region where one of the pions is virtual; thus one has to find a relation between angular distributions where one of the pions is off the mass shell and where both pions are on the mass shell. The classical method is the Chew-Low extrapolation method, first carried out by Carmony and Van de Walle.[19] To use it in its original form to perform an extrapolation for S- and P-wave amplitudes as a function of ω would require prohibitively large statistics. A way out of this problem was indicated by Selleri.[20] He proposed that the off-mass-shell $\pi^+\pi^-$-scattering angular distribution and the phase shifts are related by

$$\frac{d^3\sigma_{\pi+N\to\pi+\pi+N'}(\omega,\Delta^2,\cos\theta)}{d\Delta^2 d\cos\theta\, d\omega} = R(\Delta^2,\omega)2\pi\lambda\!\!\!^{-2}\{F_0(\Delta^2,\omega)(4/9)\sin^2\delta_0{}^0 + 4F_1(\Delta^2,\omega)$$
$$\times\cos(\delta_0{}^0-\delta_1{}^1)\sin\delta_0{}^0\sin\delta_1{}^1\cos\theta + 9F_2(\Delta^2,\omega)\sin^2\delta_1{}^1\cos^2\theta\}, \quad (5)$$

where $\lambda\!\!\!^-$ is the pion wavelength and $R(\Delta^2,\omega)$ is some function of Δ^2 and ω. In Selleri's method the functions F_i are

$$F_0(\Delta^2,\omega) = 1,$$
$$F_1(\Delta^2,\omega) = q_{\mathrm{off}}/q,$$
$$F_2(\Delta^2,\omega) = (q_{\mathrm{off}}/q)^2, \quad (6)$$

where q (q_{off}) is the outgoing (incoming) momentum in the $\pi\pi$ rest frame. The condition at the pole is

$$F_i(-\mu^2,\omega) = R(-\mu^2,\omega) = 1. \quad (7)$$

Selleri suggested that formula (5) would hold for $\Delta^2 \lesssim 10\mu^2$. Using maximum likelihood techniques in conjunction with Eq. (5), we can estimate the S-wave phase shift if the P-wave phase shift is known. In Eq. (5) let A, B, and C denote the coefficients of the isotropic, interference, and P-wave terms, respectively. We note that the ratios of A/C and B/C are given by

$$\frac{A}{B} = \frac{F_0(\Delta^2,\omega)}{F_2(\Delta^2,\omega)}\frac{4\sin^2\delta_0{}^0}{81\sin^2\delta_1{}^1},$$
$$\frac{A}{B} = \frac{F_1(\Delta^2,\omega)}{F_2(\Delta^2,\omega)}\frac{4\cos(\delta_0{}^0-\delta_1{}^1)\sin\delta_0{}^0}{9\sin\delta_1{}^1}. \quad (8)$$

From Eq. (6) the Δ^2 or $\cos\theta^*$ dependence of A/C and B/C is

$$\left(\frac{A}{C}\right)_{\mathrm{Selleri}} = \left(\frac{q}{q_{\mathrm{off}}}\right)^2\frac{4\sin^2\delta_0{}^0}{81\sin^2\delta_1{}^1},$$
$$\left(\frac{B}{C}\right)_{\mathrm{Selleri}} = \left(\frac{q}{q_{\mathrm{off}}}\right)\frac{4\cos(\delta_0{}^0-\delta_1{}^1)\sin\delta_0{}^0}{9\sin\delta_1{}^1}. \quad (9)$$

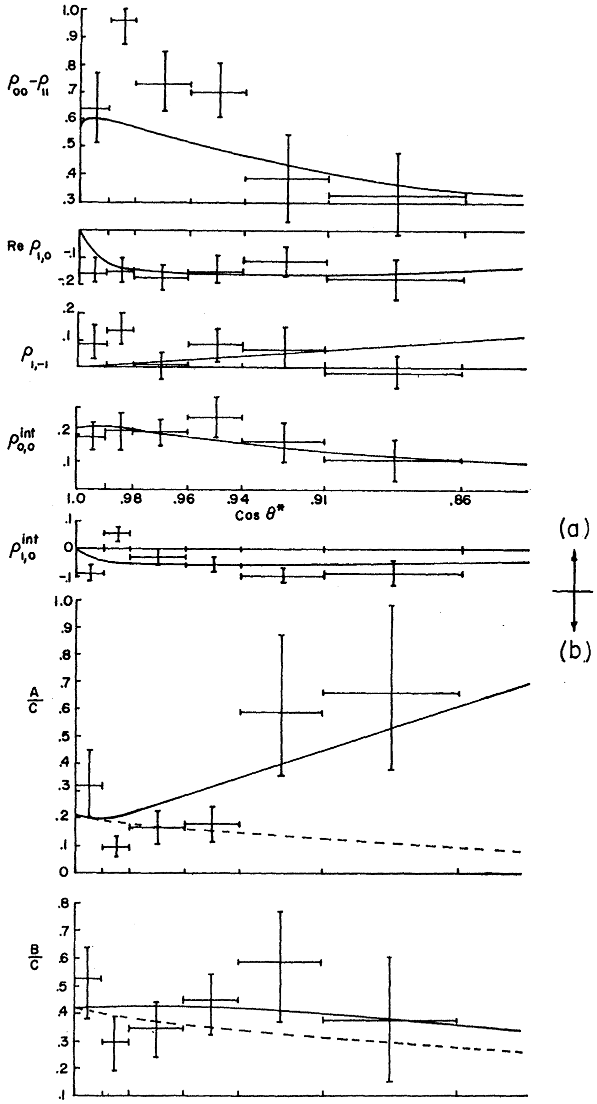

FIG. 1. (a) Density-matrix elements as a function of $\cos\theta^*$. The solid curves are the predictions of the absorption model. (b) A/C and B/C, the ratios of the isotropic and interference terms to the P-wave term in Eq. (5), plotted as functions of $\cos\theta^*$. The solid curves are the predictions of the absorption model [Eq. (10)]. The broken curves are predicted by the Selleri model [Eq. (9)].

The ratios A/C and B/C in the absorption model are given in terms of density-matrix elements by

$$\left(\frac{A}{C}\right)_{\text{absorption}} = \frac{1-(\rho_{00}-\rho_{11})}{3(\rho_{00}-\rho_{11})},$$

$$\left(\frac{B}{C}\right)_{\text{absorption}} = \frac{2}{\sqrt{3}}\frac{\mathrm{Re}\rho_{00}^{\text{int}}}{\rho_{00}-\rho_{11}}. \qquad (10)$$

In Fig. 1(b) the Selleri and absorption models are compared with our data. The θ^* dependence of A/C is seen to be quite different for the two models. In the absorption model the justification for assumption (b) is based upon the observations that A/C and B/C are slowly varying functions near $\theta^*=0$ and that $\theta^*=0$ corresponds to $\Delta^2 \approx \mu^2$ for our data.

Assumption (c) is found to be in agreement with our data. To estimate the possible $I=2$, S-wave contribution we have studied Reaction (2). From the $\pi^-\pi^0$ data in the ρ^- region, the decay angular distribution is symmetrical and above $M_{\pi\pi}=820$ there is a sudden rise in the asymmetry. From this we conclude that the $I=2, J=0$ contribution is negligible in our effective-mass region. The validity of Eq. (3) under (d) has been discussed by Jackson.[18]

Now we use our assumptions to study $\delta_0^{\ 0}$. From a maximum likelihood fit to (5) we obtain A/C and B/C, evaluated at $\theta^*=0$, as functions of ω. For on-the-mass-shell $\pi\pi$ scattering, Eq. (8) can be written as

$$\tan\delta_1^{\ 1} = \frac{\frac{1}{2}\sin 2\delta_0^{\ 0}}{(9B/4C)-\sin^2\delta_0^{\ 0}}. \qquad (11)$$

To determine $\delta_0^{\ 0}$ we invoke assumption (b) by using B/C at $\theta^*=0$ in Eq. (11). For a given energy, there are two solutions to Eq. (11), $\delta_0^{\ 0}$ and $\delta_0^{\ 0\prime} = \frac{1}{2}\pi-(\delta_0^{\ 0}-\delta_1^{\ 1})$. Both solutions are shown in Fig. 2. In principle the isotropic term should distinguish between the two solutions. However, the isotropic term is subject to larger errors than the interference term because the contribution of the S wave to the total $\pi\pi$ cross section is small and because noncoherent background effects, if present, contaminate the isotropic term more than the $\cos\theta$, $\cos^2\theta$ terms. Thus, we do not attempt to say which set of phase shifts is preferred. There is also a trivial ambiguity of $\delta_0^{\ 0}\pm n\pi$, where n is an integer. However, the analysis of Jones et al.[11] indicates that a positive sign for $\delta_0^{\ 0}$ is correct. Figure 2 also contains for comparison the results obtained by Wolf,[12] Jones et al.,[11] and Baton and Reignier.[13] The calculations of Wolf are based upon the Selleri model. It is interesting to note that our results are in good agreement with those of Baton and Reignier who, while not taking absorption explicitly into account, found it necessary to introduce two form factors in order to fit the data. Jones et al.[11] suggest that the phase shifts quoted in their

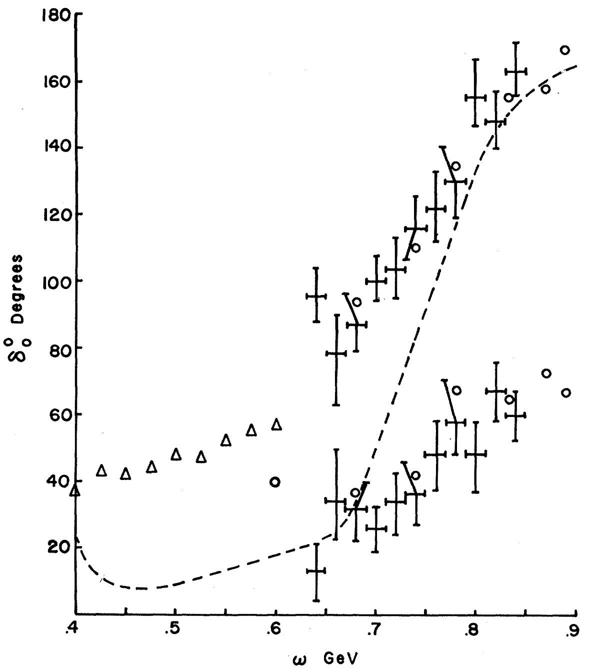

FIG. 2. The $I=0$, S-wave phase shift, δ_0^0, as a function of ω, the effective mass of the $\pi^+\pi^-$ system. The errors are statistical. The dashed curve is due to Wolf. ○, from Baton and Reignier. △, from Jones et al.

paper should be regarded as upper limits for δ_0^0. Our data are in agreement with this interpretation if it is the lower set which is correct.

In conclusion, it appears that (i) the absorption model is in good agreement with the data; (ii) the S-wave phase shift in the ρ region appears to be a slowly increasing function of energy; and (iii) the determination of the preferred set of phase shifts δ_0^0 or $\delta_0^{0\prime}$ requires larger statistics than are available in our data at 2.7 GeV/c.

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