HADRONLIKE BEHAVIOR OF γ, ν-NUCLEAR CROSS SECTIONS*

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We explain how high-energy photon- or neutrino-nucleus reactions can exhibit an A dependence like that expected from a strongly interacting particle. The condition for this is a relation among the nucleon amplitudes implying that the total photon cross section on the nucleon is equal to a certain sum over forward vector-meson production amplitudes. This relation is compatable with presently available data.

In discussing Adler's partially conserved axial-vector current (PCAC) test[1] for collinear μ events in $\nu+\alpha\to\mu+\beta$, Bell[2] notes that since the matrix element is proportional to that for $\pi+\alpha\to\beta$, and since off-shell effects for the π are not great, the cross section when α is a nucleus (mass number A) should behave like that for π scattering ($\sim A^{2/3}$ on heavy nuclei), in striking contradiction to the fact that the nucleus is transparent to ν's ($\sim A$ behavior). We can produce a similar paradox for photoreactions by using the "ρ-photon analogy," an idea similar to PCAC[3] (Fig. 1) which seems to imply an A behavior like ρ-nuclear scattering. Since these results are theoretically puzzling and suggest experimental tests of the theories, it is interesting to see how they come about somewhat more explicitly.

The usual result that, regardless of strong final-state interactions, the total high-energy cross section for, say, a photon is $\sim A$ follows from calculating the imaginary part of the photon-nuclear forward scattering amplitude, F, in terms of the nucleon amplitude f. To order e^2, we need only take the first scattering of the photon on the nucleons; adding up the scattered waves[4] gives $\mathrm{Im}F=A\,\mathrm{Im}f$, or by the optical theorem $\sigma^{\mathrm{nucleus}}=A\sigma^{\mathrm{nucleon}}$. The suggestion as to how this simple and apparently inescapable conclusion may be altered can be found in the recent work by Wilkin[5] on isospin consistency for the screening effect in the deuteron. There we see how a quasielastic channel (albeit charge exchange) (e.g., $\pi^-+p\to\pi^0+n$ followed by $\pi^0+n\to\pi^-+p$ in π^-d scattering)

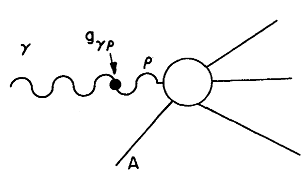

FIG. 1. Diagram for "ρ-photon analogy" in $\gamma+A\to$ hadrons.

must be considered on a par with the usual multiple scattering ($\pi^-+p\to\pi^-+p$ followed by $\pi^-+n\to\pi^-+n$) in calculating the screening. In gross nuclear matter, certain production channels can affect the shielding if they can be reached without break-up, i.e., with small momentum transfer and no quantum number exchange.[6] Now in high-energy (few BeV) photon reactions, the ρ^0 meson, having the same quantum numbers as the γ (of energy K) can be, and is, produced strongly at 0°, with small momentum transfer $\Delta=m_\rho^2/2K$. At high energy, ρ production is then a kind of quasielastic process,[7] and the γ will produce a coherent ρ wave in the nucleus, with amplitude $\sim e$. Reconversion of the ρ into a γ then gives a new e^2 contribution to the forward coherent nuclear amplitude, altering our earlier conclusion.

A natural way to treat this coupled γ-ρ problem is by the "regeneration" formalism recently used to discuss ω-φ production.[8] At energies where Δ is negligible, we simply consider the states γ', ρ' (like $K^0, \bar{K}^0$ in a regenerator) which are eigenstates of the forward scattering matrix in the material:

$$f=\begin{pmatrix} f_{\gamma\gamma} & f_{\gamma\rho} \\ f_{\gamma\rho} & f_{\rho\rho} \end{pmatrix}. \qquad (1)$$

Recombining into the physical particles behind the nucleus, we get for the forward coherent amplitude (and, therefore, for the total cross section) a linear combination of a volume effect (from the γ') and a surface effect (from the ρ'). If, however, the scattering for γ' is zero, the volume effect has zero amplitude and the photon shows absorption effects characteristic of a strongly interacting particle. The condition for this, for Eq. (1) to have a zero eigenvalue, is

$$f_{\gamma\gamma}=f_{\gamma\rho}^{\;2}/f_{\rho\rho}. \qquad (2)$$

But, Eq. (2) is just what results from naively applying the ρ-photon analogy (Fig. 2), which gives $f_{\gamma\gamma} = g_{\gamma\rho}^2 f_{\rho\rho}$, $f_{\gamma\rho} = g_{\gamma\rho} f_{\rho\rho}$. Thus all scattering goes through the $\rho' \cong \rho \pm |g_{\gamma\rho}|\gamma$, while the photonlike combination is completely decoupled. To see how this remarkable cancellation of the directly scattered photon wave comes about, it is instructive to construct the ρ wave function along a ray through the nucleus. The wave equation for the ρ to first order in e is given from the matrix index of refraction as $(\nabla^2 + n_\rho^2 K^2)\psi_\rho = -2n_{\gamma\rho}K^2\psi_\gamma$. Since $\psi_\gamma = e^{iKZ}$, the solution to this, vanishing at $-a$, is[9]

$$\psi_\rho(Z) = -(f_{\gamma\rho}/f_{\rho\rho})\{\exp(iKZ) - \exp[in_\rho K(Z+a) - iKa]\}. \quad (3)$$

The second component, an attentuated wave, looks as if we had started with just an incoming ρ, while the first component is a plane wave because the γ penetrates to any depth to produce ρ's. Arriving at any nucleon there are then two plane waves, the photon with amplitude 1 and the first term of Eq. (3) with amplitude $-f_{\gamma\rho}/f_{\rho\rho}$. Then, in the production of any final state x, the photon contributes an amplitude $f_{\gamma x}$ and the plane-wave part of the ρ gives $-f_{\gamma\rho}f_{\rho x}/f_{\rho\rho}$. If we again have the factorization relation suggested by the ρ-photon analogy $f_{\gamma x} = f_{\gamma\rho}f_{\rho x}/f_{\rho\rho}$, then the two plane waves cancel and we have only the contribution from the second term of Eq. (3), as if we had simply started with a ρ in the first place.

Now for the collinear ν reaction, consider a fixed configuration of the ν with its collinear forward-going μ; according to PCAC, the missing four-momentum can be thought of as carried away by a collinear forward virtual π of small negative "mass"^{2} q^2. If q^2 is small, but the momentum $|\vec{q}|$ of the virtual π is high, then the materialization of a real forward pion on a nucleon ($\nu + p \to \mu + \pi + p$) will give only a small momentum to the nucleon, $\Delta = (m_\pi^2 + |q^2|)/2|\vec{q}|$; these conditions that the kinematics of the virtual π closely resemble that for a real π are just those of the PCAC test.[1] Consequently, for a given ν-μ configuration, the virtual π is accompanied by a coherent forward wave of real π's with the form of Eq. (3) and amplitude $-f_{\nu\pi}/f_{\pi\pi}$, where $f_{\nu\pi}$ is the amplitude for producing a forward π on a nucleon and $f_{\pi\pi}$ is the forward π-nucleon scattering amplitude.[10]

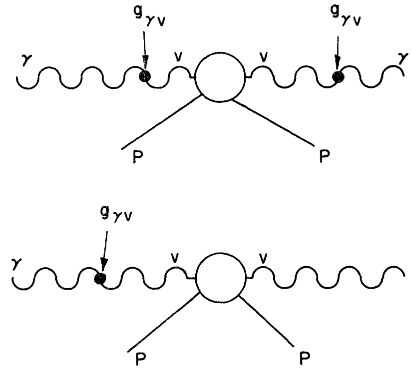

FIG. 2. Diagram for "ρ-photon analogy" in $\gamma + p \to \gamma + p$ and $\gamma + p \to$ (vector meson)$+p$.

A final state β can then be reached to order G in the weak interaction through the coherent π wave or by the direct action of the unattenuated incoming ν (the virtual π). If we again have the relation $f_{\nu\beta} = f_{\nu\pi}f_{\pi\beta}/f_{\pi\pi}$ as suggested by PCAC, then the plane waves cancel and the reaction proceeds through the second term of Eq. (3) as if we had started with an incoming π.

A case parallel to the collinear ν reaction exists in the electromagnetic problem if we take hadron production in nuclear electron scattering. In configurations where the momentum transfer to the nucleon necessary to produce the appropriate vector mesons is small, i.e., $\Delta = (m_v^2 + |\vec{q}|^2)/2|\vec{q}|$ is small (q is the four-momentum transfer to the electron), we have a situation like that for real high-energy photons, with similar results.

In discussing the photon case, we neglected for purposes of clarity the isoscalar photon channel, presumably related to the ω-φ production. If ω' and φ' are the eigenstates of the forward nuclear scattering, then clearly if

$$f_{\gamma\gamma} = \frac{f_{\gamma\rho}^2}{f_{\rho\rho}} + \frac{f_{\gamma\omega'}^2}{f_{\omega'\omega'}} + \frac{f_{\gamma\varphi'}^2}{f_{\varphi'\varphi'}}, \quad (4)$$

the "regenerated" photons from the vector mesons will again cancel the directly scattered photon amplitudes, giving hadronlike behavior for the nuclear cross section. We can even envision a generalized relation giving this behavior once the energy is high enough, in which we add more states to the right-hand side of Eq. (4) corresponding to a theory similar to the "ρ-photon analogy" but no longer dominated by the vector octet. Equation (4) can be compared with experiment if we use Fig. (2) and neglect the real parts of the amplitudes. $\mathrm{Im}F_{\gamma\gamma}$ is the total hadronic photo cross section, and expressing the right-hand side in terms of forward meson production, we obtain

$$\sigma_{\mathrm{total}}(\gamma+p)=\left[g_{\gamma\rho}{}^{2}\frac{d\sigma^{0^\circ}(\gamma\to\rho)}{d\Omega}\left(\frac{4\pi}{K}\right)^{2}\right]^{1/2}+\left[g_{\gamma\omega'}{}^{2}\frac{d\sigma^{0^\circ}(\gamma\to\omega')}{d\Omega}\left(\frac{4\pi}{K}\right)^{2}\right]^{1/2}+\left[g_{\gamma\varphi'}{}^{2}\frac{d\sigma^{0^\circ}(\gamma\to\varphi')}{d\Omega}\left(\frac{4\pi}{K}\right)^{2}\right]^{1/2}. \quad (5)$$

At 4.4 BeV[11] we find 75 or 97 μb for the ρ term, depending on whether we use the $g_{\gamma\rho}$ found in Ref. 7 or that given by Sakurai.[12] For the isoscalar part, if the photon connects to the octet φ_0 by SU(3), $g_{\gamma\varphi_0}{}^2=\frac{1}{3}g_{\gamma\rho}{}^2$, and if the φ_0 scatters approximately like the ρ, we expect about $\frac{1}{3}$ the ρ contribution.[13] Thus it appears possible for the factorization relation dominated by ρ, ω, and φ to agree with a total photon cross section at these energies of 75-120 μb.[14] Obviously, more precise data on the various cross sections would help to clarify the situation regarding this interesting and somewhat mysterious relation. Similarly, although the necessary momentum transfer is not entirely negligible on the heaviest nuclei at perhaps 15 BeV, departure from simple $\sim A$ behavior for the total photohadronic cross section should become visible if such relations are at work.

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[1]S. L. Adler, Phys. Rev. 135, B963 (1964).

[2]J. S. Bell, Phys. Rev. Letters 13, 57 (1964).

[3]For a formulation of PCAC and the ρ-photon analogy along parallel lines, see M. Gell-Mann, Phys. Rev. 125, 1067 (1962). For a discussion, see J. J. Sakurai, Proceedings of the International School of Physics "Enrico Fermi," Course XXVI, Varenna Lectures, 1962, edited by M. Conversi (Academic Press, Inc., New York, 1963).

[4]We use the impulse approximation and the simplest model of the nucleus; we neglect correlations and assume spin, isospin saturation. Therefore, the f's used always refer to the spin- or isospin-independent parts of the forward amplitude if any such small dependent parts of the forward amplitude if any such small dependences can be present.

[5]C. Wilkin, Phys. Rev. Letters 17, 561 (1966).

[6]For a discussion of such nuclear coherence questions see L. Stodolsky, Phys. Rev. 144, 1145 (1966).

[7]M. Ross and L. Stodolsky, Phys. Rev. 149, 1172 (1966).

[8]M. Ross and L. Stodolsky, Phys. Rev. Letters 17, 563 (1966).

[9]The index of refraction is related to f by $n=1+(2\pi/K^2)\rho f$ (ρ = density of nucleons); $-a$ is the left-hand edge of the nucleus at a given impact parameter.

[10]Instead of using a matrix index of refraction, we can simply add up the waves of the produced π, propagating according to its index of refraction, n_π. If q is the three-momentum transfer to the leptons, then (neglecting Δ, the nuclear recoil)

$$\psi_\pi(Z)=\frac{2\pi i\rho f_{\nu\pi}}{n_\pi q}\int_{-a}^{Z}\exp[in_\pi q(Z-Z')]\exp(iqZ')dZ',$$

giving the same result as in Eq. (3).

[11]L. J. Lanzerotti et al., Phys. Rev. Letters 15, 210 (1965). We have used $d\sigma^0/d\Omega\simeq1$ mb/sr as a compromise with the smaller values indicated by the bubble-chamber experiments (F. M. Pipkin, private communication).

[12]J. J. Sakurai, Phys. Rev. Letters 17, 1021 (1966).

[13]Other methods of guessing this give about the same result or less. If we take the "ω-φ approximately diagonal" estimate of Ref. 8, Sec. III, we get 10 or 13 μb. Similarly if we try to use the bubble-chamber data (to be published) on ω^0 production near 0^0 we find $\sim10\mu b$ from the ω and should expect the same or less from the φ; but for these measurements the energy is very uncertain and there are special difficulties in observing forward production.

[14]Deutsches Elektronen Synchrotron bubble-chamber group in Electron and Photon Interactions at High Energies, Invited Papers Presented at the International Symposium, Hamburg, 1965 (Springer-Verlag, Berlin, Germany, 1965).