[12]L. Van Hove, in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, California, 1966 (University of California Press, Berkeley, 1967).

# SU(3) BREAKING AND CURRENT ALGEBRA IN NONLEPTONIC HYPERON DECAYS

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It is shown that the corrections due to octet mass splitting and finiteness of the parity-nonconserving spurion lead to significantly better agreement of the prediction with experiments for the eight independent nonleptonic hyperon-decay amplitudes. Comments are made on the reasons for the validity of Lee-Sugawara sum rules for the parity-nonconserving and parity-conserving decays.

Sugawara[1] and Suzuki[2] have shown that the $s$-wave hyperon decays are treated very well by the hypothesis of current algebra, partially conserved axial-vector current (PCAC), and soft-pion extrapolation. The extension of the same procedure to $p$-wave hyperon decays was considerably clarified by Brown and Sommerfield,[3] Hara, Nambu, and Schecter,[4] and Badier and Bouchiat.[4] The predicted values of the $p$-wave amplitudes, however, have not compared very well with experiments. In general, they were found to be two to three times smaller than the observed values.

In view of the above, it is pertinent to ask whether the success in case of the parity-nonconserving (pv) decays can still be maintained, while the apparent shortcomings of the method in case of the parity-conserving (pc) decays can be overcome by taking into account consistently the following SU(3)-breaking effects:

(a) Mass splitting within the baryon octet. This leads to certain corrections, which are of order $\Delta M/2M$ compared with Born terms and were dropped by Brown and Sommerfield[3] on the ground that these must be small.

(b) Finiteness of the parity-nonconserving spurion ($\langle B'|H_{\mathrm{pv}}|B\rangle \neq 0$). This enters into the equal-time commutator (ETC) term for pc decays and to the Born terms for pv decays. Hitherto,[3,4] it has been put to 0 at both places on the grounds that it is forbidden in the limit of SU(3) by $l$ invariance.[5] A priori, the violation of such an SU(3) selection rule could lead to large correction[6] inasmuch as the observed value of the $K_1 \to 2\pi$ decay amplitude, for example, is large, while it is forbidden[5,7] in the limit of SU(3) for the same reason as the pv spurion.

The purpose of this note is to point out that the correction mentioned in (a) turns out to be extremely important for pc decays in a rather unexpected manner and leads to significantly better agreement with experiments compared with that of any previous work,[3,4] while that due to (b) is found to be not as important even for large strength of the pv spurion. In the light of these corrections, we also comment on the reasons for the validity of the Lee-Sugawara[8] sum rules for the pv and pc decays, respectively.

Using reduction technique, PCAC, and partial integration, one obtains the by-now familiar relation[9]

$$\lim_{q^\mu \to 0} [q^\mu T_\mu + f_\pi M] = i\langle\beta|[F_5^{\,i}(0), H_W(0)]|\alpha\rangle, \tag{1}$$

where $M$ is the amplitude for hyperon decay $(\alpha \to \beta + \pi^i)$,

$$M \equiv -i\langle\beta+\pi^i|H_W(0)|\alpha\rangle; \tag{2}$$

$q^\mu$ is the momentum associated with the outgoing pion; $f_\pi$, $T_\mu$, and $F_5^{\,i}$ are defined by

$$\partial^\mu A_\mu^{\,i} = -if_\pi m_\pi^{\,2}\varphi^i(x), \tag{3}$$

$$T_\mu \equiv \int d^4x\, e^{iq\cdot x}\theta(x_0)\langle\beta|[A_\mu^{\,i}(x), H_W(0)]|\alpha\rangle, \tag{4}$$

and

$$F_5^{\,i}(t) = \int d^3x\, A_0^{\,i}(\vec{x}, t). \tag{5}$$

Following Brown and Sommerfield,[3] we split $M$ into a sum of a Born term $B$ and a remainder term $R$, defined by

$$M(q^\mu) \equiv [B^1(q^\mu) + B^2(q^\mu)] + R(q^\mu), \qquad (6)$$

where the Born term has been divided into two parts $B^1$ and $B^2$; in $B^1$, pion emission succeeds the weak transition whereas in $B^2$, it precedes the weak transition. We will assume that the remainder term $R(q^\mu)$ for the physical value of $q^\mu$ is well approximated by $R(0)$. We can then combine Eqs. (1) and (6) to write

$$M(q^\mu) \simeq \sum_{j=1}^{2} [B^j(q^\mu) - K^j] + (i/f_\pi)\langle\beta|[F_5{}^i(0), H_W(0)]|\alpha\rangle, \qquad (7)$$

where

$$K^j \equiv \lim_{q^\mu \to 0} \frac{1}{f_\pi}[q^\mu T_\mu{}^j + f_\pi B^j(q^\mu)], \qquad (8)$$

$$T_\mu{}^1 \equiv \int d^4x\, e^{iq\cdot x}\theta(x_0)\langle\beta|A_\mu{}^i(x)H_W(0)|\alpha\rangle, \qquad (9)$$

and

$$T_\mu{}^2 \equiv -\int d^4x\, e^{iq\cdot x}\theta(x_0)\langle\beta|H_W(0)A_\mu{}^i(x)|\alpha\rangle. \qquad (10)$$

Let us assume a vector-axial-vector current-current theory for the nonleptonic weak interaction in the form

$$H_W(\Delta S=1) = Gd\sigma_{ij}J^iJ^j = H_{\rm pc} + H_{\rm pv}. \qquad (11)$$

We define the pc and pv spurions by

$$\langle j|H_W(0)|i\rangle = \bar{u}_j H_W{}^{ij} u_i, \qquad (12)$$

where

$$H_W{}^{ij} = \begin{pmatrix} C^{ij} \\ v^{ij}\gamma_5 \end{pmatrix} \text{ for } H_W(0) = \begin{pmatrix} H_{\rm pc} \\ H_{\rm pv} \end{pmatrix}. \qquad (13)$$

By the assumed octet property of the Hamiltonian we may represent the pc spurion in the SU(3) limit[10] by (we choose $d'+f'=1$)

$$C^{ij} = (g'/\sqrt{2})[d'D^6 + f'F^6]_{ij}. \qquad (14)$$

About the pv spurion $v^{ij}$ we comment later.

Following the procedure of Refs. 3 and 9 it is straightforward to evaluate the $K^j$ terms of Eq. (7) including the corrections (a) and (b). This leads to the following general form for the pc and pv decay amplitudes:

$$M_{\rm pc}(q^\mu) \simeq \sum_{j=1}^{2} B_{\rm pc}{}^j(q^\mu)\left[1 - \frac{\Delta M_j}{2M}\right] + \frac{i}{f_\pi}\langle\beta|[F_5{}^i(0), H_{\rm pc}]|\alpha\rangle, \qquad (15)$$

$$M_{\rm pv}(q^\mu) \simeq \sum_{j=1}^{2} B_{\rm pv}{}^j(q^\mu)\left[1 - 1 - \frac{\Delta M_j'}{2M}\right] + \frac{i}{f_\pi}\langle\beta|[F_5{}^i(0), H_{\rm pv}]|\alpha\rangle, \qquad (16)$$

where $\Delta M_j$ and $\Delta M_j'$ stand symbolically for differences of two masses within the baryon octet, while $2M$ stands for the sums of two masses within the same octet. (These differ from one process to another.) The Born terms $B_{\rm pc}{}^j(q^\mu)$ and $B_{\rm pv}{}^j(q^\mu)$ are given by

$$B^1(q^\mu) = -i\bar{u}_\beta(p')(M_\alpha \mp M_\gamma)^{-1}\begin{pmatrix} C^{\alpha\gamma}\gamma_5 \\ v^{\alpha\gamma} \end{pmatrix} u_\alpha(p) g[fF^i + dD^i]_{\gamma\beta}, \qquad (17)$$

$$B^2(q^\mu) = -i\bar{u}_\beta(p')(M_\beta \mp M_\delta)^{-1}\begin{pmatrix} C^{\delta\beta}\gamma_5 \\ v^{\delta\beta} \end{pmatrix} u_\alpha(p) g[fF^i + dD^i]_{\alpha\delta}, \qquad (18)$$

where the upper sign holds for $B_{\rm pc}{}^j$ and the lower sign for $B_{\rm pv}{}^j$. The indices $\gamma$ and $\delta$ denote the intermediate baryon states for $B^1$ and $B^2$, respectively. In writing the above expression we have used SU(3)-symmetric pseudoscalar coupling for the $BBP$ vertices. We normalize to $d+f=1$, so that $g^2/4\pi \simeq 14.6$.

In order to obtain a rough estimate of the pv spurion we will adopt a model suggested before in Ref. 6 (although the results are found to be largely independent of the model). In this model the $K_1 \to 2\pi$ decay amplitude is essentially given by the appropriate extrapolation of the strong $KK\,\pi\pi$ vertex times the matrix element for $K_1 \to$ vacuum transition (which we will denote by $f_{K_1} m_K{}^3$), while the pv spurion $v^{ij}$ will similarly be given by

$$v^{ij} \simeq (g/\sqrt{2})[d(D^6 - iD^7) + f(F^6 - iF^7)]_{ij}(f_{K_1} m_K{}^3/\sqrt{2}\, m_K{}^2). \tag{19}$$

In the present analysis we treat $f_{K_1}$ as a parameter.[11]

Writing the hyperon-decay matrix element in the general form

$$M(\alpha \to \beta + \pi^i) \equiv -i\bar{u}_\beta (V + C\gamma_5) u_\alpha, \tag{20}$$

it is straightforward to write down the various $V$ and $C$ amplitudes using Eqs. (14)-(20). Two such typical amplitudes are

$$C(\Lambda_-) = gg'\left(\frac{2}{3}\right)^{1/2}\left[\frac{(-1-2f')}{\Lambda - N}\left(1 + \frac{\Delta - N}{2N}\right) + \frac{2(1-f)(1-2f')}{N-\Sigma}\left(1 + \frac{N-\Sigma}{\Lambda+\Sigma}\right)\right] + gf_{K_1} m_K (12^{1/2} f_\pi)^{-1}(1+2f),$$

$$V(\Lambda_-) = g^2 f_{K_1} m_K \left(\frac{1}{3}\right)^{1/2}\left[\frac{(-1-2f)}{\Lambda+N}\left(\frac{N-\Lambda}{2N}\right) + \frac{2(1-f)(1-2f)}{N+\Sigma}\left(\frac{\Lambda - N}{\Lambda+\Sigma}\right)\right] + g'(6^{1/2} f_\pi)^{-1}(1+2f'). \tag{21}$$

On the right-hand sides of these equations, the particle symbols denote the corresponding physical masses.

Thus all the pv and pc amplitudes are given in terms of three parameters ($g'$, $f'$, and $f_{K_1}$). We vary $f$ between[12] 0.3 and 0.4; on the whole the results are found to be insensitive[13] to this variation. The theoretical prediction together with the experimental values are shown in Table I. The fit given is one of the best that we are able to find and corresponds to a choice[14] of $g' = -6 \times 10^{-6}$ MeV, $f' = 6$, and $f_{K_1} m_K \simeq 1.4 \times 10^{-6}$ MeV with $f = 0.34$. As remarked earlier, the fit in Table I is markedly better compared with that obtained before[15] by Brown and Sommerfield.[3]

The following comments are of interest with regard to the results shown in Table I:

(1) The correction term $(K^1 + K^2)$ that was dropped in Ref. 3 makes significant contribution to the pc amplitudes. The reason for this is that the main terms $B^1$ and $B^2$ are always opposite in sign (see Table I), while the terms

Table I. Decay amplitudes.[a]

| $p$-wave amplitude | $B^1 + B^2$ | $-(K^1 + K^2)$ | ETC | Total Theory | Total Expt.[b] |
|---|---|---|---|---|---|
| $C(\Lambda_-) \times 10^6$ | 4.9 − 3.8 | 0.46 + 0.42 | 0.08 | 2.1 | 2.3 |
| $C(\Xi_-{}^-) \times 10^6$ | −0.7 + 2.4 | −0.04 − 0.18 | −0.02 | 1.5 | 1.4 |
| $C(\Sigma_-{}^-) \times 10^6$ | −2.6 + 2.4 | 0.20 − 0.25 | −0.04 | −0.3 | −0.03 ± 0.08 |
| $C(\Sigma_+{}^+) \times 10^6$ | 7.0 − 5.0 | 0.95 + 0.45 | 0 | 3.4 | 4.1 |
| $[C(\Lambda_-) + 2C(\Xi_-{}^-)] \times [\sqrt{3}C(\Sigma_0{}^+)]^{-1}$ | | | | 1.10 | 1.02 |

| $s$-wave amplitude | $B^1 - K^1$ | $B^2 - K^2$ | ETC | Total Theory | Total Expt. |
|---|---|---|---|---|---|
| $V(\Lambda_-) \times 10^6$ | −0.12 + 0.13 | 0.03 − 0.03 | −0.27 | −0.26 | −0.33 |
| $V(\Xi_-{}^-) \times 10^6$ | 0.08 − 0.09 | 0.01 − 0.01 | 0.49 | 0.48 | 0.43 |
| $V(\Sigma_-{}^-) \times 10^6$ | −0.07 + 0.06 | −0.02 + 0.02 | −0.57 | −0.58 | −0.40 |
| $V(\Sigma_+{}^+) \times 10^6$ | 0.05 − 0.06 | −0.05 + 0.04 | 0 | −0.02 | ~0 |
| $[V(\Lambda_-) + 2V(\Xi_-{}^-)] \times [\sqrt{3}V(\Sigma_0{}^+)]^{-1}$ | | | | 1.02 | 1.08 |

[a]Columns 3 and 4 give the correction terms for the pc amplitudes, while columns 2 and 3 denote the correction terms for the pv amplitudes.

[b]N. Cabibbo, Rapporteur's talk at the Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, California, 1966 (University of California Press, Berkeley, 1967).

$K^1$ and $K^2$ always [except in case[16] of $C(\Sigma_-^-)$] have the same sign and the sum $(K^1+K^2)$ has the right sign relative to $(B^1+B^2)$ so as to improve[17] the agreement with experiments.

(2) The corrections arising due to the finiteness of the pv spurion are not as important as that due to the mass-splitting effect. One can place a rough upper limit on the value of $f_{K_1}$ (i.e., $f_{K_1}m_K \simeq 6\times10^{-6}$ MeV) by demanding that the predicted amplitudes fit their observed values at least to within, say, 30%. This upper limit is consistent with previous estimates[11] of $f_{K_1}$. Suffice it to say that, even for this upper limit, the corrections due to the pv spurion are not more than 15%[18] of the corresponding observed values of the amplitudes.

(3) The fit shown in Table I satisfies the Lee-Sugawara[8] sum rule quite well for the pv as well as pc amplitudes. One can foresee the approximate validity of the sum rule for pv[19] amplitudes on general grounds, since for these amplitudes the ETC terms satisfy the sum rule exactly, while the $B^j$ and $K^j$ terms satisfy the sum rule separately in the approximation of the sums of masses being equal (i.e., $\Lambda+N=\Sigma+N=\Xi+\Lambda$). For the pc amplitudes, on the other hand, while the ETC terms satisfy the sum rule exactly, neither the $B^j$ nor the $K^j$ terms satisfy the sum rule separately. (The $B^j$ terms satisfy the sum rule only in the limit of the bad approximation[20] $\Lambda-N=\Sigma-N=\Xi-\Lambda$.) In spite of this the term $(B^j-K^j)$ leads to the following ratio for the two sides of the sum rule[21]:

$$\frac{C(\Lambda_-)+2C(\Xi_-^-)}{\sqrt{3}C(\Sigma_0^+)}\simeq\frac{11.2-12.9f'}{5.7-11.4f'}. \quad (22)$$

The above ratio is unity for $f'\approx4$ and continues to be nearly unity (within 12%) for any higher value of $f'$. This shows that the reason for the validity of the sum rule for pc amplitudes is related to typical special values of baryon masses, and to the $d/f$ ratios for the $BBP$ coupling as well as for the pc spurion.

(4) We have introduced SU(3) breaking only into those entities which vanish in the limit of SU(3), namely mass splitting and the pv spurion, while we have used SU(3) symmetric[22] values for the pseudoscalar[23] baryon-meson coupling and the pc spurion. These latter entities are finite in the limit of SU(3) and the corrections arising due to SU(3) breaking in them may hopefully be small. The discrepancies in Table I may partly be attributed to such corrections and partly to the use of PCAC and the soft-pion extrapolation.

To conclude, the results suggest that the current-algebra approach can account for the main features of both the pv and the pc nonleptonic hyperon-decay amplitudes reasonably well, provided the ratios of the pseudoscalar baryon-pion couplings are nearly SU(3) symmetric.

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Note added in proof.—It should be noted that if one chooses to use pseudovector coupling for the baryon-meson vertices instead of pseudoscalar coupling, as used here, the $K^j$ terms vanish for both the pv and the pc amplitudes; however, in this case the Born terms $B^j$ incorporate the $(\Delta M/2M)$ corrections of the pseudoscalar theory exactly. The use of SU(3) for the pseudovector[24] couplings leads to results which differ from those of the pseudoscalar theory inasmuch as the ratios of the sums of baryon masses are not SU(3) symmetric. Although this does not alter the qualitative features (i.e., the relative signs of various terms) as mentioned in the text, the over-all fit is found to be worse than that given in Table I.

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[1]H. Sugawara, Phys. Rev. Letters 15, 870, 997 (1965).

[2]M. Suzuki, Phys. Rev. Letters 15, 986 (1965).

[3]L. S. Brown and C. M. Sommerfield, Phys. Rev. Letters 16, 751 (1966). Since we mostly follow the procedure of this paper, our comments will be primarily with regard to the results in this paper. However the comments with regard to other contemporary work [Y. Hara, Y. Nambu, and J. Schecter, Phys. Rev. Letters 16, 380 (1966); S. Badier and C. Bouchiat, Phys. Letters 20, 529 (1966)] will be quite similar.

[4]Hara, Nambu, and Schecter, Ref. 3; Badier and Bouchiat, Ref. 3.

[5]M. Gell-Mann, Phys. Rev. Letters 12, 155 (1964).

[6]J. C. Pati and S. Oneda, Phys. Rev. 140, B1351 (1965); D. Loebbaka, S. Oneda, and J. C. Pati, Phys. Rev. 144, 1280 (1966); D. Loebbaka and J. C. Pati, Phys. Rev. 147, 1046 (1966).

[7]N. Cabibbo, Phys. Rev. Letters 12, 62 (1964).

[8]B. W. Lee, Phys. Rev. Letters 12, 83 (1964); H. Sugawara, Progr. Theoret. Phys. (Kyoto) 31, 213 (1964).

[9]V. A. Alessandrini, M. A. B. Bég, and L. S. Brown,

Phys. Rev. 144, 1137 (1966).

[10]See comment (4) at the end on the use of SU(3) for such vertices.

[11]Previous estimates of $f_{K_1}$ are based on the observed amplitude of the decay $K_1 \to 2\pi$ (see Loebbaka, Oneda, and Pati, Ref. 6) and independently on a dynamical model of hyperon decays (see Loebbaka and Pati, Ref. 6). Both these estimates yield $f_{K_1} m_K \simeq$ (3-4) $\times 10^{-6}$ MeV.

[12]W. Willis et al., Phys. Rev. Letters 13, 291 (1964); A. W. Martin and K. C. Wali, Phys. Rev. 130, 2455 (1963); N. Brene, L. Vege, M. Roos, and C. Cronstrom, Phys. Rev. 149, 1288 (1966).

[13]The only amplitude which is sensitive to the variation of $f$ is $C(\Xi_-^-)$.

[14]The fit is not a very sensitive function of the various parameters $g'$, $f'$, and $f_{K_1}$. The stated interrelationships between the $B^1$, $B^2$, $K^1$, and $K^2$ terms hold as long as $f' > \frac{1}{2}$.

[15]We compare our results with that of Brown and Sommerfield (Ref. 3), since that is closest to our work. Similar remarks apply to other previous work (Ref. 4).

[16]In this case, the terms $K^1$ and $K^2$ are opposite in sign which is precisely what is desired, since $C(\Sigma_-^-)$ is experimentally very small.

[17]See Ref. 14.

[18]That the pv spurion may not lead to important corrections can be inferred qualitively from the structure of Eqs. (15) and (16), in which the ETC terms for pc decays and the Born terms for pv amplitudes involve the pv spurion.

[19]It is well known that the sum rule for pv amplitudes is expected to be exact in the limit of SU(3) (see Ref. 5).

[20]B. W. Lee, Phys. Rev. 140, B152 (1965); D. Loebbaka and J. C. Pati, Phys. Rev. 147, 1047 (1966).

[21]The given ratio corresponds to the use of physical masses for the baryon octet together with $f = 0.33$.

[22]For the present analysis, it is only important that the ratios of baryon-pion couplings be nearly SU(3) symmetric. Certain models of SU(3) nonconservation do lead to such a behavior [see, for example, S. K. Bose and Y. Hara, Phys. Rev. Letters 17, 409 (1966); R. H. Graham and Walter A. Simmons, Phys. Rev. 153, 1458 (1967)].

[23]In as much PCAC is exact, it should be noted that there exists an ambiguity in the use of SU(3)-symmetric couplings for the $BBP$ vertices. This is because the baryon-meson pseudoscalar couplings are proportional to the weak axial-vector couplings by relations of the form $f_\pi g_B{}^i{}_B{}^j{}_P{}^K = -g_A{}^{i\to j}(m_i + m_j)$, so that the use of SU(3) for the strong pseudoscalar couplings is not equivalent to the use of SU(3)-symmetric weak axial-vector couplings due to mass splittings. For further comments in this respect, see the note added in proof.

[24]Note that the use of SU(3) for pseudovector baryon-meson couplings is equivalent to the use of SU(3) for the weak axial-vector couplings, since they are directly proportional to each other (without involving any mass factors) through the Goldberger-Treiman relation.