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FIXED POLES IN THE COMPLEX ANGULAR-MOMENTUM PLANE*

Reinhard Oehme
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics,
The University of Chicago, Chicago, Illinois
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It is shown that fixed poles in the l plane are forbidden for strong interactions unless there exist moving branch points with very specific properties. A fixed pole with protective cuts is considered as a possible model for diffraction scattering.

Several years ago, we formulated a theorem concerning fixed poles in the complex angular-momentum plane.[1,2] Assuming the validity of some form of elastic unitarity in the neighborhood of a fixed pole at $\lambda=\lambda_0$, we used the continuity theorem for functions of two or more complex variables[3] in order to show that such poles are generally not allowed.

It is the purpose of this paper to show that there exists a very special, but physically most interesting, case where a fixed pole at a point $\lambda=\lambda_0$ is made possible by a closely associated set of moving branch points[4] $\lambda=\alpha_C(S)$ [$S=S_C(\lambda)$], which have the following properties: For every relevant two-particle threshold $S=S_0$, there is a moving branch point $S=S_C(\lambda)$. At $\lambda=\lambda_0$, the branch point $S_C(\lambda_0)$ coincides with S_0 and has the same character as the threshold. A threshold is relevant if the amplitude loses the pole by continuation through the corresponding cut into a secondary sheet. On the basis of these results, we propose that a fixed pole with protective cuts may be used for the construction of a model for nonshrinking diffraction peaks with constant asymptotic cross sections,[5] which otherwise would be difficult to obtain within the framework of causal dispersion theory.[6]

In order to exhibit the special relevance of the branch points $S_C(\lambda)$, we prove in the following that in most cases fixed poles are forbidden even in the presence of moving branch cuts which overlap the right-hand cuts in the S plane. For reasons of simplicity, we consider the scattering of spinless particles with mass μ (e.g., $\pi\pi$ scattering), and we concentrate on the elastic threshold at $S=4\mu^2$. Let $F(S,\lambda)=F_+(S,\lambda)$ be the analytic function which is uniquely determined by $F_+(S,l)=F_l(S)$ for $l=$ even, $l>N$. It follows from general postulates that there is a number N such that for $\mathrm{Re}\lambda>N$ the function F is analytic in S and λ with the usual fixed cuts along the real axis in the S plane.[2] It satisfies the continued unitarity condition due to the elastic threshold at $S=4\mu^2$. We write this condition in the analytic form

$$F_{\mathrm{II}}^{-1}(S,\lambda)=F^{-1}(S,\lambda)+2i\rho(S), \tag{1}$$

where $\rho(S)=[(S-4\mu^2)|S]^{1/2}$, and the subscript II indicates the amplitude in the second sheet.[7] If we continue F into the region $\mathrm{Re}\lambda<N$, λ-dependent singularities can appear in the physical sheet of the S plane. Moving branch points can only come up through inelastic thresholds like $S_{\mathrm{I}}=16\mu^2$.[8] Let $S_C(\lambda)$ be such a branch point, and suppose that $F(S,\lambda)$ has a fixed pole at $\lambda=\lambda_0$ for S in the physical sheet (I). It follows from Eq. (1) that F_{II} has no such pole at $\lambda=\lambda_0$. If we assume that $|S_C(\lambda_0)-4\mu^2|>\epsilon$, $\epsilon>0$, we can continue F_{II} for all $|\lambda-\lambda_0|<\delta(\epsilon)$ from sheet II into sheet I, and we find that F is regular at $\lambda=\lambda_0$. Hence we see that the pole is not allowed on the basis of the continuity theorem. We note that our proof is applicable even if $S_C(\lambda_0)$ is real and to the left of the threshold $S=4\mu^2$ so that the elastic cut is completely blanketed. Our argument is also applicable to other nonmoving singularities with S-independent character, provided there exists a sequence

$\{\lambda_n\} \to \lambda_0$ such that $F^{-1}(S, \lambda_n) \to 0$ for $n \to \infty$. This includes essential singularities of fixed character.

The arguments given above can be generalized to all cases where the unitarity condition makes it possible to find a continuation in s along which the pole would disappear. Simple exceptions are amplitudes involving weak or electromagnetic interactions up to a finite order in the coupling constants[9] and certain reactions with coupled spin channels.[2]

We now consider the special case where $S_C(\lambda_0) = 4\mu^2$. It is instructive to introduce the function

$$\varphi^{-1}(S, \lambda) = F^{-1}(S, \lambda) + i\rho(S), \tag{2}$$

which does not have the fixed square-root cut for $4\mu^2 < S < S_{\rm I}$.[10] If F has a fixed pole at $\lambda = \lambda_0$, we find the relation

$$\varphi^{-1}(S, \lambda_0) = i\rho(S), \tag{3}$$

which requires that φ^{-1} has a branch point of the type $(S - 4\mu^2)^{1/2}$ at $\lambda = \lambda_0$. For $S_C(\lambda_0) \neq 4\mu^2$ this is not possible, as we have seen before, but if $S_C(\lambda_0) = 4\mu^2$, the function $\varphi^{-1}(S, \lambda_0)$ can have a branch point corresponding to $\rho(S)$, provided the moving cut has the same character for $\lambda = \lambda_0$ as the unitarity cut.[11]

The partial wave function $F(S, \lambda)$ may have further two-particle thresholds for $S > 4\mu^2$, like, for example, the $K\bar{K}$ threshold at $S = 4m_K^2$. In this case, we must generalize our considerations to the many-channel problem.[11,2] If the fixed pole is also present in elements of the transition matrix other than F, like for instance in $G(\pi + \pi \to K + K)$ and $K(K + \bar{K} \to K + \bar{K})$, then we find that additional moving branch cuts are needed in order to protect the fixed pole. At $\lambda = \lambda_0$, these cuts must coincide in position and character with the corresponding higher two-particle cuts.

Without a fixed pole at $\lambda = +1$, it is rather difficult to construct a simple model with a nonshrinking diffraction peak and a constant asymptotic total cross section. But we can allow such a pole provided we also introduce the required moving branch points like $\alpha_C(S)$ with $\alpha_C(4m_\pi^2) = 1$ as well as others for higher two-particle thresholds. With the fixed pole and the cut $\alpha_C(S)$, we obtain for the asymptotic form of the invariant amplitude $F(S, t)$ in the t channel

$$F(S, t) \sim ib(S)t + C(S)\frac{1 + \exp[-i\pi\alpha_C(S)]}{\sin\pi\alpha_C(S)} \times t^{\alpha_C(S)}(\ln t)^{-\beta} + \cdots, \tag{4}$$

where $b(S)$ and $C(S)$ are real, and β depends upon the character of the branch point for $S \leq 0$; we have $\beta = \frac{3}{2}$ for a square-root cut. Since $\alpha_C(4m_\pi^2) = 1$, we may expect that $\alpha_C(0) < 1$. It is a nice feature of the fixed-pole diffraction model that the protective cuts, and hence also the pole, are intimately connected with inelastic processes. The asymptotic form (4) gives rise to constant elastic cross sections. Apart from the question of shrinkage, the model can therefore be distinguished from a Regge-pole model with a Pomeranchuk trajectory, which would give rise to a logarithmically decreasing elastic cross section. At nonasymptotic energies, the cut contribution may cause characteristic effects for specific processes.

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RADIATIVE CORRECTIONS TO β DECAY AND THE QUANTUM NUMBERS OF FIELDS UNDERLYING CURRENT ALGEBRA*

K. Johnson and F. E. Low
Laboratory for Nuclear Science and Physics Department,
Massachusetts Institute of Technology, Cambridge, Massachusetts

and

H. Suura
School of Physics, University of Minnesota, Minneapolis, Minnesota
(Received 3 May 1967)

It is shown that the second-order radiative corrections to $\pi^\pm\beta$ decay are finite, provided one assumes a $V-A$ interaction and a current algebra constructed from spin-$\frac{1}{2}$ SU(3) triplet fields with the quantum numbers of Λ, Ξ^0, and Ξ^-, or Ω^-, Ξ^0, and Ξ^-, and $\frac{1}{3}$-integral hypercharge; or, equivalently, $V+A$ together with p, n, Λ or p, n, Ω'^+ triplets.

Bjorken[1] has shown that the chiral U(6)⊗U(6) algebra of current densities implies a logarithmically divergent second-order electromagnetic correction to the process $\pi^\pm \to \pi^0 + e^\pm + \nu$, independent of the nature of the strong interactions.

One may take three points of view towards the occurence of such divergences. The first is that of conventional renormalization theory: That is, the renormalized coupling constants and masses are finite, but not calculable. The approximate equality of masses in isotopic multiplets would then appear as a lucky accident, unrelated to the smallness of the fine-structure constant. The second is that quantum electrodynamics and/or weak couplings must be modified at high energies, so that the divergences are cut off, and the corrections, in practice, are small. This possibility may well occur, but in the present Letter we reject it in favor of a third, that is, that the logarithmic divergences in fact do not occur with conventional quantum electrodynamics. In this spirit, we restrict ourselves to consider only point Fermi couplings.

We then ask, how must the U(6)⊗U(6) algebra be modified to give finite answers? A strong hint, as observed by Berman and Sirlin,[2] is contained in the finiteness of the α corrections to μ decay. They observe that a $V-A$ interaction with an initial negative fermion decaying to an electron and neutral fermions is finite to second order in α and lowest order in the strong interactions. Equivalently, a $V+A$ interaction with an initial neutral fermion decaying to an electron, a positive fermion, and a neutral fermion is also finite, since it is related to the first alternative by crossing. The algebraic modification of U(6)⊗U(6) required to generalize this observation consists in adding an SU(3) singlet to the electromagnetic current density. The new equal-time commutation rules are

$$[V_k^{\mathrm{em}}(\vec{x}), A_j^\alpha(\vec{y})] = \mp i\epsilon_{kjl} V_l^\alpha(\vec{x})\delta(\vec{x}-\vec{y}), \qquad (1)$$

$$[V_k^{\mathrm{em}}(\vec{x}), V_j^\alpha(\vec{y})] = \mp i\epsilon_{ujl} V_l^\alpha(\vec{x})\delta(\vec{x}-\vec{y}), \qquad (2)$$

for k and j different space components, and α a charge-raising or -lowering SU(3) index which occurs in the semileptonic weak processes. The ∓ sign refers to a $V-A$ or $V+A$ weak hadronic current. No commutators of charge densities with each other or with current densities need be modified.

It is obvious that the relations (1) and (2) can be achieved in a model where the currents are made up (in the $V-A$ case) of a (Λ, Ξ^0, Ξ^-)-like SU(3) triplet[3] with quantum numbers $I = (0, \frac{1}{2},$