in the decay may result in different slopes of the odd-pion spectra, but there can be no difference between the τ^+ and τ^- decay rates unless the final state of the three pions contains some $I=3$ state in addition to the $I=1$ states. However, at the level of precision achieved in this experiment, it is completely unknown whether these assumptions are justified.

[6]The separated beam (to the second bending magnet) was designed by B. Barrish and A. Maschke for another experiment. The help of both in the modification for the present experiment, as well as that of J. Fox and M. Webster, is gratefully acknowledged.

[7]This contribution amounted to less than 1% of the τ rate.

[8]At H the beams were 35 mm wide and 28 mm high (full width at half-maximum).

[9]At 3.0 GeV/c the detector efficiency was ~40%.

[10]For the $K_{\mu 2}$ only partial cancellation could be achieved; the fractional change of efficiency with momentum was $0.6\times|\Delta p/p|$.

[11]R. L. Cool, G. Giacomelli, T. F. Kycia, B. A. Leontić, K. K. Li, A. Lundby, and J. Teiger, Phys. Rev. Letters 17, 102 (1966); 16, 1228 (1966).

[12]This substitution reduced the detector efficiency for τ by ~20%.

[13]The details will be included in a report to be published elsewhere.

[14]This time corresponds to the interval between two rf pulses. However, no change was observed for an arbitrary time delay, thus indicating that the internal proton beam was thoroughly debunched.

[15]The goodness of the fit, as measured by χ^2 (Table I), shows that this first-order expansion is indeed adequate.

[16]A few runs were taken with greatly reduced beam intensity in order to improve the precision of this extrapolation.

[17]The slopes $(S/R)_+$ and $(S/R)_-$ agree within statistics, whereas the ratio $(T/R)_-/(T/R)_+=1.31\pm0.14$. This value is consistent with the ratio of the K^-- to K^+-nucleon total cross sections at 3 GeV/c ($\sigma_-/\sigma_+=1.43\pm0.07$).

[18]The assistance of Dr. A. J. S. Smith in writing the first version of the program is gratefully acknowledged.

[19]Taking the K lifetime to be 12.35 ± 0.06 nsec [A. H. Rosenfeld et al., Rev. Mod. Phys. 39, 1 (1967)], we obtain for the τ branching ratio (5.552 ± 0.045)%.

[20]Data computed by G. H. Trilling, Argonne National Laboratory Report No. ANL 7130, 1965 (unpublished).

[21]Unlike the τ data, the $K_{\mu 2}$ data did not quite reproduce within statistics, thus yielding for χ^2 too large a value. Therefore, the statistical error was increased to such a value as to make χ^2 equal to the number of degrees of freedom.

[22]The transmission of the system for protons (extrapolated to zero material) was 99.7%.

[23]V. L. Fitch, C. A. Quarles, and H. C. Wilkins, Phys. Rev. 140, B1088 (1965).

PRODUCTION OF PIONS, KAONS, AND ANTIPROTONS IN THE CENTER-OF-MASS SYSTEM IN HIGH-ENERGY PROTON-PROTON COLLISIONS*

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We have recently measured the differential production cross section $d^2\sigma/d\Omega dp$ for the production of various particles in 12.5-GeV/c proton-proton collisions. The experiment was effectively done in the center-of-mass system by making a series of measurements while holding fixed either P_l or $P_\perp$ of the produced particle in the center-of-mass system. We studied the production of π^+, π^-, K^+, K^-, and antiprotons while varying the center-of-mass longitudinal momentum between $P_l=0.0$ and 1.0 GeV/c and $P_\perp{}^2$ between 0.1 and 1.5 (GeV/c)^2. The quantity $d^2\sigma/d\Omega dp$ is the cross section for the production of a single particle in the phase-space region $\Delta\Omega\Delta p$, independent of what other particles are produced.

The experiment was performed on the slow extracted beam of the zero-gradient synchrotron (ZGS) at Argonne National Laboratory. About $(1\text{-}2)\times10^{11}$ protons of 12.50 GeV/c were extracted during the 300-msec spill every 2.5 sec. The angular divergence of the beam was about ±3 mrad and the momentum spread was less than ±10 MeV/c. The uncertainty in the absolute value of the momentum was less than $\frac{1}{2}$%. The beam spot at our target was a circle of about 1 cm.

The number of protons hitting our target was measured by two monitor telescopes, each made up of three small scintillation counters, $M = M_1M_2M_3$ and $N = N_1N_2N_3$. As shown in Fig. 1, these both looked at our target so that the number of counts in these monitors was proportional to the number of protons passing through the target.

To obtain the ratio of protons to monitor counts we took calibration runs with a gold foil placed in the proton beam several feet upstream of the target. During each calibration run the number of monitor counts was recorded. We determined the number of protons passing through the Au foil (and thus the target) by doing a standard radiochemical analysis of the foil. The uncertainty in these calibrations was about 5%.

Our target was a vertical 2-in.-diam liquid-hydrogen flask. The flask window and vacuum window were, respectively, 0.003- and 0.005-in. Mylar. These windows resulted in a target-empty effect of about 30%. Target-empty runs were taken for every measurement. We also did a target-empty calibration of the monitors.

Our detection system for produced particles was a single-arm spectrometer. This was similar, in all ways except one, to the spectrometers that have been used in the many "beam survey" experiments[1,2] with θ_{lab} fixed. As shown in Fig. 1 it contained a telescope of three scintillation counters ($S_{123} = S_1S_2S_3$) which defined the solid angle $\Delta\Omega$; S_3 was $\frac{3}{4}$ in.$\times\frac{3}{4}$ in. and 240 in. from the target. It next contained a bending magnet which deflected the produced particles by about 12° for momentum analysis. The momentum bite was defined by the scintillation telescope $S_{45} = S_4S_5$; S_5 was 6 in.$\times$5 in. and 1100 in. from the target.

The Cherenkov telescope $C = C_1C_2C_3$ served to tag the particles as pions, kaons, or antiprotons. C_1 and C_2 were threshold Cherenkov counters filled with ethane and C_3 was a scintillation counter used only to reduce accidentals. C_2 was always run in coincidence and C_1 was run in anticoincidence to kill pions during the kaon runs and to kill pions and kaons during the antiproton runs. The appropriate ethane pressures for C_1 and C_2 were experimentally determined by running pressure curves. In order to subtract background pions from the kaon and antiproton measurements, it was necessary to take low-pressure runs in addition to the high-pressure runs. This was because our rejection efficiency against pions was only about 0.997. This pion background subtraction was always 20% or less.

The extra component in the spectrometer

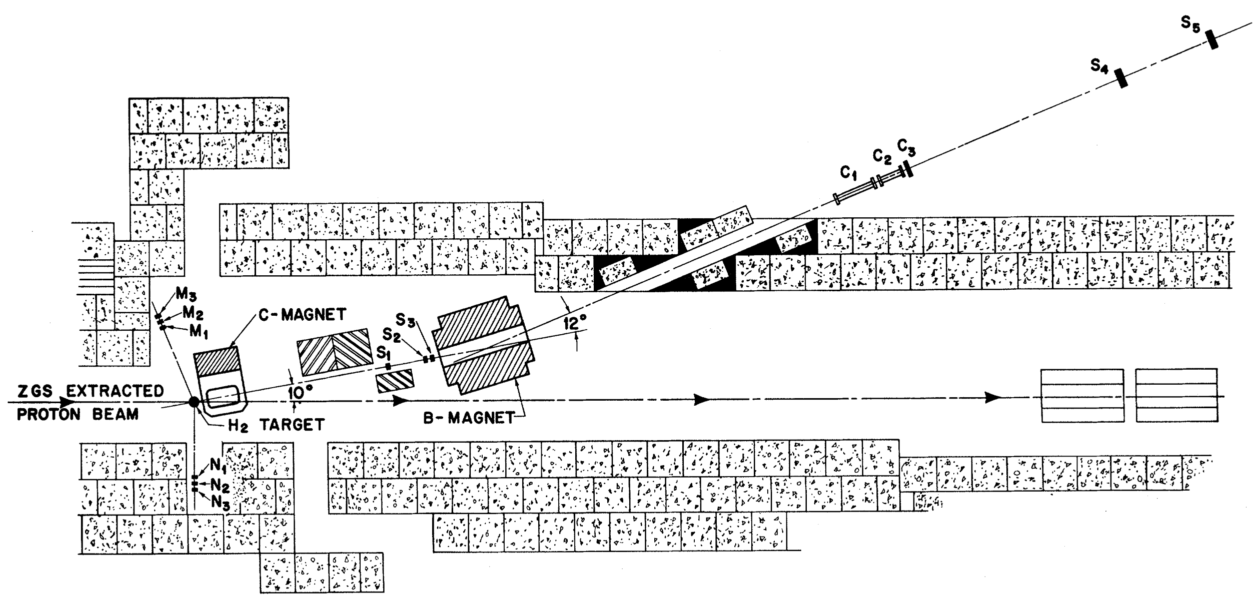

FIG. 1. Layout of experiment. The incident protons come down the extracted beam and strike the hydrogen target. The produced particles are detected by the spectrometer.

was a C magnet placed very close to the hydrogen target. This served as a steering magnet which compensated for the different laboratory angles of particles with different $P_\perp$ or P_l in the center of mass. For example, in the study of pions with $P_l = 0.60$ GeV/c, when $P_\perp^2$ was 0.5 (GeV/c)^2, then θ_{lab} was about 10° and the C magnet was essentially turned off. However, when $P_\perp^2$ was 1.5 (GeV/c)^2, then the lab angle was about 14° and the C magnet was bent in by about 4° so that the pion still passed through the S_3 counter. Similarly, when $P_\perp^2$ was 0.1 (GeV/c)^2, then θ_{lab} was about 5° and the C magnet was bent out by about 5° to put the pion through S_3. Thus the C magnet was always set to make the desired particle pass through the center of S_3. The B magnet was then set to make the particle pass through the center of S_5.

The use of the C magnet eliminated the need to move the counters and heavy magnet many times. It also resulted in the spectrometer being physically identical for all measurements; only the two magnet currents were changed. Thus many possible sources of systematic error were eliminated. The C magnet effectively allowed us to do the experiment in the center-of-mass system.

The phase space subtended by our spectrometer was the intersection of the two phase-space strips subtended by the S_3 and S_5 counters. The other counters in the spectrometer were all overmatched. The center-of-mass phase-space bite was typically $\Delta\Omega\Delta p = 5\times10^{-6}$ sr GeV/c.

The electronic logic began with the signals from S_1, S_2, and S_3 forming the S_{123} coincidence while the signals from S_4 and S_5 formed the S_{45} coincidence. Similarly, the signals from C_1, C_2, and C_3 formed the C coincidence. The number of particles passing through the spectrometer was then determined by the threefold coincidence SC. The accidental rate in SC was determined by a time-to-amplitude converter (TAC) which was triggered by the SC signal. This TAC was connected to a pulse-height analyzer so that the time-of-flight spectrum between S_3 and S_5 could be measured and displayed. In this spectrum the true events appeared as a large peak 1.5 nsec wide on top of a flat region 30 nsec wide caused by accidentals. The accidental rate was accurately determined from the flat region and subtracted from the peak. The subtraction varied between 2 and 20 % with an uncertainty between 1 and 5 %.

The differential production cross section was calculated from the formula

$$\frac{d^2\sigma}{d\Omega dp} = \frac{\text{events}}{I_0(N_0\rho t)\Delta\Omega\Delta p}. \tag{1}$$

The quantity I_0 is the number of incident protons as measured by our monitors. The uncertainty in I_0 was about 5 %. N_0 is Avogadro's number; ρ is the density of liquid hydrogen, taken as 0.07; t is the target length, taken as 5.08 cm; $\Delta\Omega\Delta p$ is the c.m. phase-space volume.

There were several corrections and uncertainties involved in determining the number of events. The statistical error varied from 1 to 2 % for the pions and 4 to 10 % for the kaons, and was as much as 50 % for the antiprotons. In addition to the accidental correction of 2-20 %, we made a target-empty subtraction which was about 30 % as mentioned above. There was essentially no systematic error in this determination of the target-empty effect. We also made a subtraction for unrejected pions in the kaon and antiproton runs. This was always less than 20 % and was experimentally determined with a systematic error of no more than 2 %. A correction was made for the nuclear interactions of the particles in the early components of the spectrometer. This correction was calculated to be 1.14 for pions, 1.15 for kaons, and 1.30 for antiprotons. There was a 2 % error in this correction. No correction was made for multiple Coulomb scattering because inscattering is equal to outscattering in a single-arm spectrometer with a small $\Delta\Omega\Delta p$.

We also made a correction for the decay of the pions and kaons before reaching S_5. This correction varied between 1.09 and 1.28 for pions and between 1.93 and 5.80 for kaons. The uncertainty in this correction varied between 0 and 5 %.

The production cross sections that we measured are plotted in Figs. 2 and 3. In the measurements shown in Fig. 2 we held the c.m. longitudinal momentum fixed and studied the dependence of $d^2\sigma/d\Omega dp$ on $P_\perp^2$. In Fig. 3 we held $P_\perp^2$ fixed and studied the dependence of the cross section on P_l. The errors shown are a combination of statistical and systematic errors. There is an additional normalization uncertainty in all cross sections of 5 % because of the uncertainty in the calibration of the number of incident protons. The cross sections and errors shown are preliminary. A table of final values will appear in a paper to follow. These final values will all lie with-

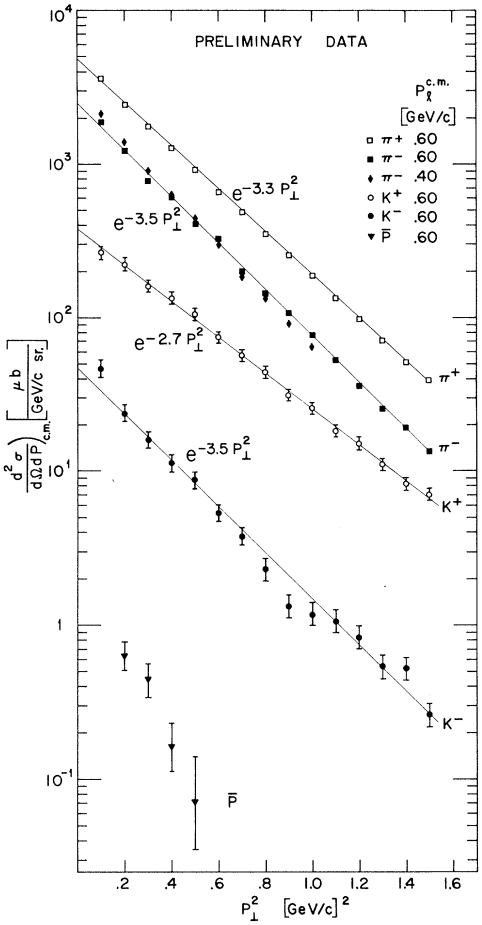

FIG. 2. Plot of $d^2\sigma/d\Omega dp$ against $P_\perp{}^2$ for P_l held fixed. The production cross sections for π^+, π^-, K^+, K^-, and antiprotons are shown. The lines are straight-line fits to the data.

in the error bars.

We see from Fig. 2 that when $d^2\sigma/d\Omega dp$ is plotted against $P_\perp{}^2$ on semilog paper, we get a straight line. Thus the cross section is a Gaussian in $P_\perp$:

$$\frac{d^2\sigma}{d\Omega dp} = B \exp(-AP_\perp{}^2). \tag{2}$$

This Gaussian dependence on $P_\perp$ is in disagreement with the exponential dependence on $P_\perp$ first suggested by Cocconi, Koester, and Perkins[3] on a phenomenological basis. A Gaussian dependence has been suggested in a model[4] to explain elastic scattering.

The slopes of the straight-line fits are indicated on Fig. 2. The uncertainty in these slopes is ± 0.2 GeV/c and the slopes all appear to be consistent with $A \approx 3.5$ (Gev/c)$^{-2}$ except for the

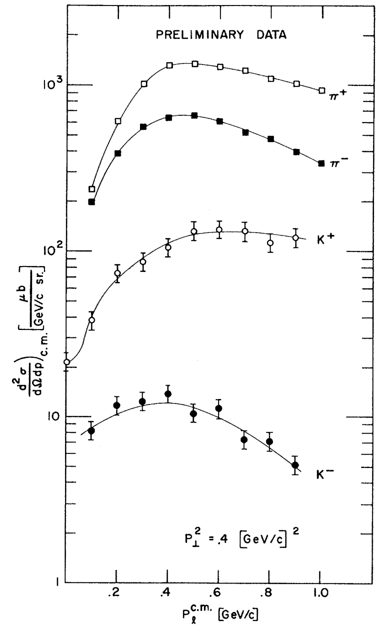

FIG. 3. Plot of $d^2\sigma/d\Omega dp$ against P_l for $P_\perp{}^2$ held fixed. The cross sections for π^+, π^-, K^+, and K^- production are shown. The lines are freehand fits to the data.

K^+ slope, which is about 2.7 (GeV/c)$^{-2}$. It will be interesting to speculate about why the K^+ slope is different. It is also interesting to note that $d^2\sigma/d\Omega dp$ continues to drop as a Gaussian all the way out to $P_\perp{}^2 = 1.5$ (GeV/c)^2, in contrast to elastic cross sections which all seem to break sharply around $P_\perp{}^2 \approx 0.9$ (GeV/c^2).

We can see from Fig. 3 that the cross section does not have a maximum at zero longitudinal momentum, but instead peaks at about 0.45 GeV/c and drops rapidly as P_l goes to 0. This behavior indicates that there is no tendency for particles to be produced at rest in the center-of-mass system. Instead they tend to appear in two clouds surrounding one or another of the two protons. Thus the protons themselves appear to emit essentially all of the produced particles.

We would like to thank the entire ZGS staff for their help and encouragement during the experiment. We also thank Dr. E. Steinberg and the radiochemistry group for their assistance. We are grateful to Professor D. I. Meyer and Professor M. H. Ross for their helpful comments.

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FIXED POLES IN THE COMPLEX ANGULAR-MOMENTUM PLANE*

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It is shown that fixed poles in the l plane are forbidden for strong interactions unless there exist moving branch points with very specific properties. A fixed pole with protective cuts is considered as a possible model for diffraction scattering.

Several years ago, we formulated a theorem concerning fixed poles in the complex angular-momentum plane.[1,2] Assuming the validity of some form of elastic unitarity in the neighborhood of a fixed pole at $\lambda=\lambda_0$, we used the continuity theorem for functions of two or more complex variables[3] in order to show that such poles are generally not allowed.

It is the purpose of this paper to show that there exists a very special, but physically most interesting, case where a fixed pole at a point $\lambda=\lambda_0$ is made possible by a closely associated set of moving branch points[4] $\lambda=\alpha_C(S)$ $[S=S_C(\lambda)]$, which have the following properties: For every relevant two-particle threshold $S=S_0$, there is a moving branch point $S=S_C(\lambda)$. At $\lambda=\lambda_0$, the branch point $S_C(\lambda_0)$ coincides with S_0 and has the same character as the threshold. A threshold is relevant if the amplitude loses the pole by continuation through the corresponding cut into a secondary sheet. On the basis of these results, we propose that a fixed pole with protective cuts may be used for the construction of a model for nonshrinking diffraction peaks with constant asymptotic cross sections,[5] which otherwise would be difficult to obtain within the framework of causal dispersion theory.[6]

In order to exhibit the special relevance of the branch points $S_C(\lambda)$, we prove in the following that in most cases fixed poles are forbidden even in the presence of moving branch cuts which overlap the right-hand cuts in the S plane. For reasons of simplicity, we consider the scattering of spinless particles with mass μ (e.g., $\pi\pi$ scattering), and we concentrate on the elastic threshold at $S=4\mu^2$. Let $F(S,\lambda)=F_+(S,\lambda)$ be the analytic function which is uniquely determined by $F_+(S,l)=F_l(S)$ for $l=$ even, $l>N$. It follows from general postulates that there is a number N such that for $\mathrm{Re}\lambda>N$ the function F is analytic in S and λ with the usual fixed cuts along the real axis in the S plane.[2] It satisfies the continued unitarity condition due to the elastic threshold at $S=4\mu^2$. We write this condition in the analytic form

$$F_{\mathrm{II}}^{-1}(S,\lambda)=F^{-1}(S,\lambda)+2i\rho(S), \qquad (1)$$

where $\rho(S)=[(S-4\mu^2)|S]^{1/2}$, and the subscript II indicates the amplitude in the second sheet.[7] If we continue F into the region $\mathrm{Re}\lambda<N$, λ-dependent singularities can appear in the physical sheet of the S plane. Moving branch points can only come up through inelastic thresholds like $S_{\mathrm{I}}=16\mu^2$.[8] Let $S_C(\lambda)$ be such a branch point, and suppose that $F(S,\lambda)$ has a fixed pole at $\lambda=\lambda_0$ for S in the physical sheet (I). It follows from Eq. (1) that F_{II} has no such pole at $\lambda=\lambda_0$. If we assume that $|S_C(\lambda_0)-4\mu^2|>\epsilon$, $\epsilon>0$, we can continue F_{II} for all $|\lambda-\lambda_0|<\delta(\epsilon)$ from sheet II into sheet I, and we find that F is regular at $\lambda=\lambda_0$. Hence we see that the pole is not allowed on the basis of the continuity theorem. We note that our proof is applicable even if $S_C(\lambda_0)$ is real and to the left of the threshold $S=4\mu^2$ so that the elastic cut is completely blanketed. Our argument is also applicable to other nonmoving singularities with S-independent character, provided there exists a sequence