ELASTIC SCATTERING OF 1-BeV PROTONS FROM HYDROGEN, HELIUM, CARBON, AND OXYGEN NUCLEI*

H. Palevsky, J. L. Friedes, R. J. Sutter, and G. W. Bennett
Brookhaven National Laboratory, Upton, New York

and

G. J. Igo
Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico

and

W. D. Simpson and G. C. Phillips
William Marsh Rice University, Houston, Texas

and

D. M. Corley and N. S. Wall
University of Maryland, College Park, Maryland

and

R. L. Stearns
Vassar College, Poughkeepsie, New York

and

B. Gottschalk
Northeastern University, Boston, Massachusetts
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The differential elastic-scattering cross sections of 1-BeV protons on hydrogen, helium, carbon, and oxygen have been measured. The nuclear data exhibit diffractionlike structure associated with the multiple scattering of protons by nucleons inside the nuclei. The experimental technique provides a new and powerful method for studying nucleon-nucleon correlations inside the nucleus.

The success of the Brueckner[1] theory in calculating properties of nuclear matter can be considered as strong evidence that nucleons inside the nucleus exist as entities not too dissimilar to free nucleons. The obvious question to the experimenter is, then, what kind of measurement will probe the motions of individual nucleons inside the nucleus. The answer in a general sense is very well known: (1) The wavelength of the probe must be comparable with or smaller than the characteristic spacings of nucleons inside the nucleus; (2) the energy resolution of the measuring system must be smaller than or comparable with the spacings of the energy levels of the nucleus under investigation. In particular, it is useful to be able to separate the elastic from the inelastic scattering.

Until just recently the only probe which satisfied the above criteria was the high-energy electron. Elastic electron scattering experiments have provided a large amount of data concerning the single particle or density distribution of nucleons.[2] In principle, inelastic electron scattering provides a measure of nucleon correlations inside the nucleus; in practice the radiation (bremsstrahlung) effects are so large that they obscure the desired nuclear effect.

This Letter presents the first results of a program undertaken at the Brookhaven Cosmotron using high-energy protons that satisfy the above two criteria. However, because both the mass and the interaction mechanism with the nucleon are different for the proton than for the electron, new and interesting experimental data were expected. In particular, it was hoped that from the inelastic data, effects of nucleon correlations could be extracted because the proton radiation cross section is small compared with the purely nuclear cross section.

A description of the apparatus, method of data reduction, together with the measured p-p elastic scattering at 1 BeV is in press.[3] It will suffice for this discussion to state that the measurements reported here were carried out at 1 BeV with an energy resolution of 3 MeV and an angular resolution of ~0.1°. The exact incident momentum as obtained from a calibration of the spectrometer bending magnets was 1.696 ± 0.015 BeV/c.

A plot of the helium data is shown in Fig. 1. The point at 0° marked "optical point" has a value equal to $(k\sigma_T/4\pi)^2$, where k is the wave number of the incident proton and σ_T is the measured total cross section. Total cross sections for helium, carbon, oxygen, and other nuclei have been measured using the same apparatus and are reported in a separate paper.[4] The solid line through the points is a best (least-squares) fit to the data points using a Saxon-Woods potential,

$$U = -\frac{V+iW}{1+\exp[(r-r_0)/a]},$$

with r_0 set equal to 1.60 F, the rms radius obtained from fitting a Gaussian distribution to the electron data.[5] The fitting of the proton

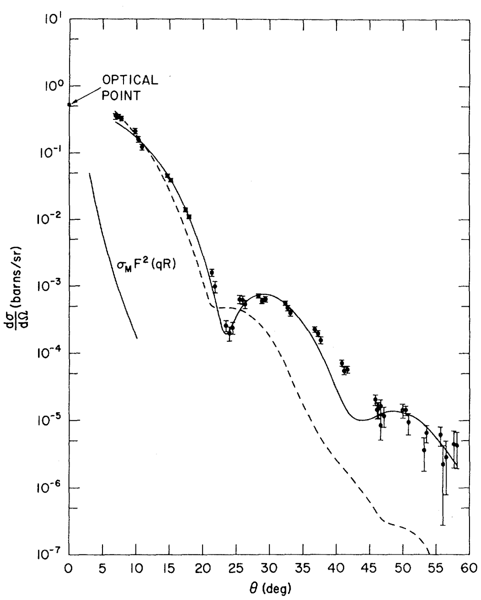

FIG. 1. 1-BeV proton elastic scattering from ^{4}He, in the center-of-mass system. The solid line is an optical-model fit to the data using a Saxon-Woods potential with the parameters given in the text. The calculated total cross section is 156 mb, in good agreement with the measured value of 152 ± 8 mb. The dashed line is an optical-model fit using a Gaussian potential with rms radius = 1.60 F, $V = -70$ MeV, and $W = 200$ MeV. The calculated total cross section with these Gaussian parameters is 206 mb.

data is performed using a modified form of Auerbach's ABACUS-2 code,[6] and a best fit is obtained with the following parameters:

$$r_0 = 1.60 \text{ F (fixed)}, \quad V = -30 \text{ MeV},$$
$$a = 0.31 \text{ F}, \quad W = 153 \text{ MeV}.$$

Note that with these parameters the shape of the potential describing the proton-He interaction exhibits a very sharp transition region at the nuclear surface and is in marked disagreement with the Gaussian charge distribution derived from the electron data.[7] If we require in the fitting of the proton data that the potential be Gaussian, a reasonable fit to the forward-scattering data can be obtained. This is shown as the dashed line in Fig. 1. Note, however, that (1) the sharp minimum at 24° is not reproduced by the fit, and (2) as the scattering angle increases, the calculated scattering from the Gaussian potential falls below experiment by two or three orders of magnitude. Physically this comes about because the Gaussian potential provides a smooth transition at the nuclear surface for the incident proton wave, which tends to wash out sharp diffraction effects and causes the reflection of protons for angles away from the forward direction to be minimized.

The solid line marked $\sigma_M F^2(qR)$ is the Coulomb scattering calculated from the Mott formula and multiplied by a Gaussian form factor obtained from electron-scattering data. It is seen that the Coulomb scattering is negligible compared with the nuclear scattering.

Figures 2 and 3 are plots of the carbon and oxygen elastic-scattering data. The solid lines through the points are obtained by fitting a Saxon-Woods potential with the parameters, r_0 and a, derived from a similar fit to the electron data,[8] and allowing only V and W as free parameters.

For the carbon data the parameters r_0 and a, and V and W derived from the fitting are as follows:

$$r_0 = 2.29 \text{ F (fixed)}, \quad V = -20 \text{ MeV},$$
$$a = 0.45 \text{ F (fixed)}, \quad W = 100 \text{ MeV}.$$

The calculated scattering exhibits for this case a well-defined minimum, but it is slightly displaced towards smaller momentum transfers than the experimentally observed minimum. In Born approximation, one would conclude

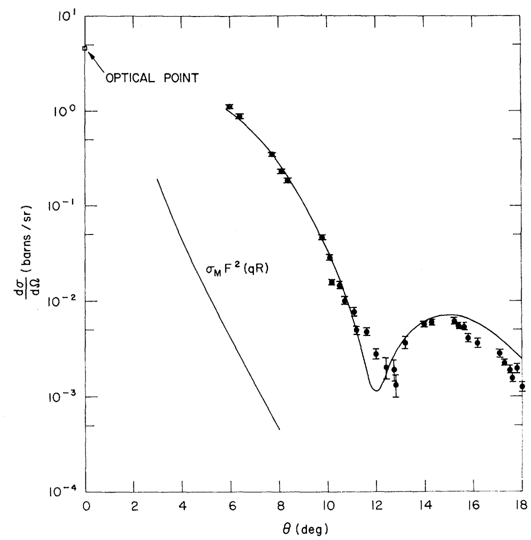

FIG. 2. 1-BeV proton elastic scattering from ^{12}C, in the center-of-mass system. The solid line is an optical-model fit to the data using a Saxon-Woods potential with the parameters given in the text. The calculated total cross section is 357 mb, in agreement with the measured value of 370 ± 9 mb.

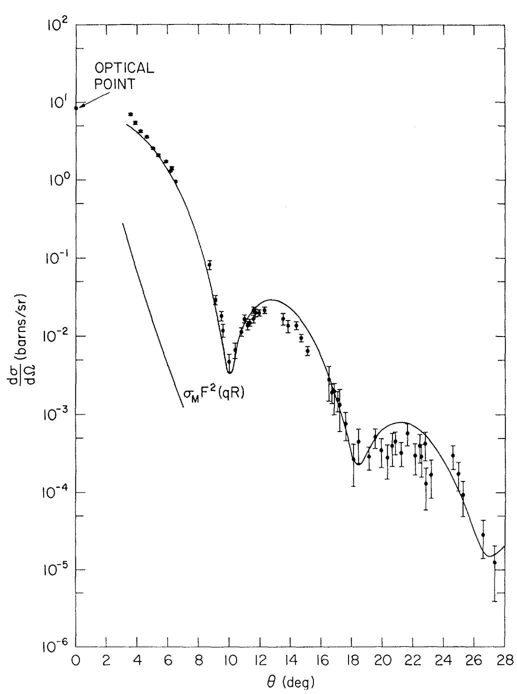

FIG. 3. 1-BeV proton elastic scattering from ^{16}O, in the center-of-mass system. The solid line is an optical-model fit to the data using a Saxon-Woods potential with the parameters given in the text. The calculated cross section is 509 mb as compared to the measured value of 475 ± 44 mb.

that the carbon nucleus appears to be smaller for protons than for electrons.

For the oxygen data the parameters are as follows:

$$r_0 = 2.60 \text{ F (fixed)}, \quad V = -21 \text{ MeV},$$

$$a = 0.45 \text{ F (fixed)}, \quad W = 120 \text{ MeV}.$$

In this case the shape parameters for the proton-oxygen potential seem to be in reasonable agreement (Fig. 3) with those obtained from the analysis of electron data.

It is useful in comparing the electron data with the proton data to discuss the two types of measurements in Born or impulse approximation. In this approximation the elastic scattering amplitude $f(\theta)$ is given by a product of two terms:

$$f(\theta) = f_c(\theta) f_1(\theta).$$

Here $f_c(\theta)$ represents the coupling mechanism of the particle used to probe the nucleus and $f_1(\theta)$ is the Fourier transform of the charge distribution for electrons and the matter distribution for protons:

$$f_1(\theta) \sim \int \rho(r)\, e^{i\vec{K}\cdot\vec{r}}\, d\vec{r},$$

where K is the momentum transferred in the scattering process and is related to the scattering angle θ by the simple relation

$$\Delta p = \hbar K = 2p_0 \sin\tfrac{1}{2}\theta,$$

where p_0 is the incident momentum.

If isospin is a good quantum number for the light nuclei, as we expect it is,[9] then the charge and matter distributions should be the same, and in this framework $f_1(\theta)$ derived from both electron and proton data should be similar. The attempt to fit the proton-nuclei data with optical potentials having the same shape parameters as the nuclear charge distributions derived from electron data was an attempt to see if the matter distribution probed by the proton has the same shape as the charge distribution seen by the electron, the implicit assumption being that the optical potential has the same shape as the matter distribution. The fact that a Gaussian optical potential cannot in any way be made to fit the p-helium scattering data just reflects that the observed $f_1(\theta)$ for protons is entirely different from $f_1(\theta)$ derived from electron data. What is puzzling is that if one argues the impulse approximation is at fault, one would expect the disagreement to get worse as the target nucleus increased in size, whereas the experimental results seem to converge with increasing atomic weight.

It should be mentioned that in all the attempted optical-model fits to the data, best fits were obtained with the sign for the real part of the potential negative. This means a repulsive potential and probably reflects that the real part of the nucleon-nucleon potential is negative at $T = 1$ BeV.

Recently Bassel and Wilkin[10] at Brookhaven and Czyż and Leśniak[11] at Krakow have independently proposed that the electron and proton results can be reconciled. These authors claim that because of the strong proton-nucleon interaction, the proton incident upon the nucleus scatters from more than one nucleon before escaping from the nucleus; that is, the impulse approximation breaks down. A more nearly correct description of the process, therefore, is that given by the multiple scattering formalism of Glauber.[12] Figure 4 shows the results of such an analysis carried out by Bassel and Wilkin at Brookhaven. (The same result was obtained by Czyż and Leśniak.) These authors started with a Gaussian density distribution derived from the electron-scattering data together with an over-all δ function to describe the c.m. motions. The effect of double and triple scattering in the calculations qualitatively reproduces the two experimentally observed minima. The agreement with data is extremely good considering the very simple form of the correlation used. Attempts are now being made by Bassel and Wilkin to take account of the particle correlations in a more realistic form.[13]

In conclusion, this work has demonstrated the feasibility of using high-energy protons as probes for studying nucleon correlations inside the nucleus. Because the proton-nucleon interaction is strong, the scattering amplitude is not correctly given by the impulse approximation but can be expressed in the form of a series expansion, the first (impulse) term describing the single scattering, the second term the double scattering, etc.,

$$f(\theta) = f_c(\theta) f_1(\theta) + f_c(\theta_1) f_c(\theta_2) f_2(\theta) + \cdots,$$

where $f_c(\theta)$ and $f_1(\theta)$ are as defined above, and $\theta_1 + \theta_2 = \theta$. Because the single and multiple scattering is coherent, there is interference in the scattering between the various processes,

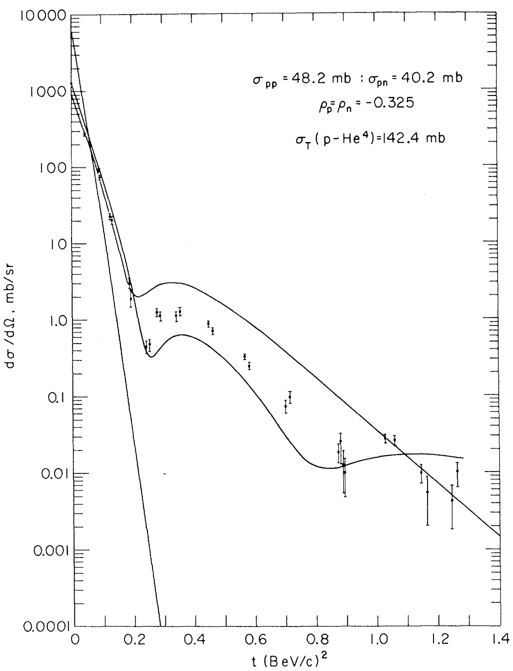

FIG. 4. A multiple scattering calculation of the proton-helium cross section by Bassel and Wilkin, based upon the high-energy approximation of Glauber. The ordinate is the laboratory differential cross section and t is the invariant four-momentum transfer. The nucleon-nucleon parameters used in the calculation are shown in the drawing. The steeply falling curve is the single-scattering contribution, and the upper curve, which contains only one diffraction dip, is the result of including double scattering. The lower curve, which has two diffraction dips similar to the experimental data, is from a calculation which includes double and triple scatterings.

and the interference terms give rise to the structure observed in the proton scattering measurements. The expression for $f_2(\theta)$ is of the form

$$f_2(\theta) \sim \int G(\vec{r}_1 \vec{r}_2)\, e^{ik(\vec{r}_2 - \vec{r}_1)} d(\vec{r}_2 - \vec{r}_1),$$

where $G(\vec{r}_1 \vec{r}_2)$ is the nucleon-pair correlation. It is in this way that elastic scattering of protons provides a means for probing nucleon correlations in the nucleus.

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SIMULTANEOUS OBSERVATIONS OF SOLAR PROTONS INSIDE AND OUTSIDE THE MAGNETOSPHERE

S. M. Krimigis, J. A. Van Allen, and T. P. Armstrong
Department of Physics and Astronomy, University of Iowa, Iowa City, Iowa
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Simultaneous observations of low-energy (~0.5 MeV) protons emitted in a solar flare of 7 July 1966 were made with detectors on board the earth satellites Explorer 33 and Injun IV, located outside and inside the earth's magnetosphere, respectively. We find that such protons have full and essentially immediate access from interplanetary space to the polar caps of the earth.

Although it has been established that the earth's magnetospheric boundary is greatly distorted by the flow of the solar wind, there is essential disagreement regarding the topology of the magnetic field at the boundary between the magnetosphere and the interplanetary medium. Figure 1 illustrates two contrasting models. The model shown in Fig. 1(a) envisions considerable merging[1-3] between the geomagnetic and interplanetary magnetic fields, such that charged particles approaching the earth on an interplanetary magnetic field line have immediate access to points over the earth's polar caps.

The model shown in Fig. 1(b) envisions no merging between the geomagnetic and interplanetary-magnetic fields[4,5] near the earth. Proponents of this model suggest that solar-emitted protons having $E_p \lesssim 5$ MeV must diffuse into the very long tail of the magnetosphere and spread slowly from the auroral zone over the polar caps after a delay or "diffusion time"

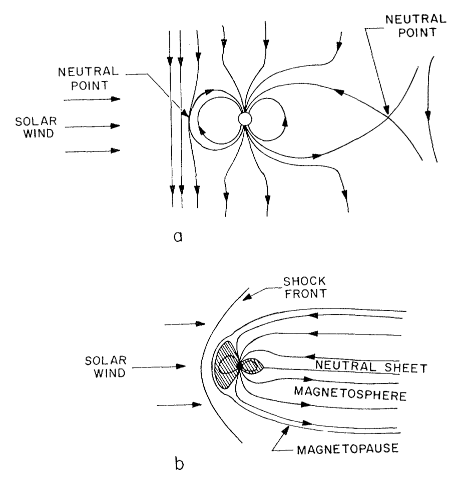

FIG. 1. (a) Magnetospheric model (Refs. 1-3) that envisions merging between the geomagnetic and interplanetary fields ("open model"). (b) Magnetospheric model (Refs. 4 and 5) in which merging of lines of force does not occur until at least several A.U. on the antisolar side of the earth ("long-tail model").