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DETERMINATION OF THE Σ^+ MAGNETIC MOMENT USING THE REACTION $K^- + p \to \Sigma^+ + \pi^-$ AT 1.15 BeV/c*

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A sample of 52 Σp decays satisfying hydrogen-like production kinematics yields $\mu_{\Sigma^+} = +3.5 \pm 1.5\ \mu_N$ and $\alpha\overline{P} = -0.69 \pm 0.15$ relative to $\hat{P}_K \times \hat{P}_\Sigma$. The effect of all magnetic field components along the Σ^+ path has been included, as well as that of the c.m. decay angle on detection efficiency. The measurement of μ_{Σ^+} can be averaged with an earlier result to give $\mu_{\Sigma^+} = +3.2 \pm 0.9\ \mu_N$, consistent with the prediction by SU(3).

The reaction $K^- + p \to \Sigma^+ + \pi^-$ has certain unique features which make it preferable to photon[1] and pion[2] reactions for measurement of the Σ^+ magnetic moment. The exothermic kaon reaction produces highly polarized Σ^+ particles at lower energies and larger laboratory angles than those produced in the photon and pion reactions at equivalent laboratory momenta. Furthermore, the Σ^+ cross sections are relatively large, so that the Σ^+/background ratio for the kaon reaction is well over an order of magnitude greater than that for the other two reactions. The main disadvantage has been the low intensity of separated, high-purity kaon beams, but this situation has been improving in recent years.

Recently, the F-20 separated beam of the Brookhaven alternating-gradient synchrotron (AGS) was modified to deliver about 10^6 K^- ($P_K = 1150$ MeV/c) onto a polyethylene target. This was the only suitable momentum where polarization measurements were available at the time of the exposure.[3] Two stacks of Ilford K-5 nuclear emulsion were placed next to the target, at 42° to the beam (Fig. 1). Compensated coil windings in the magnet held the 150-kG axial magnetic field uniform to 2 % over

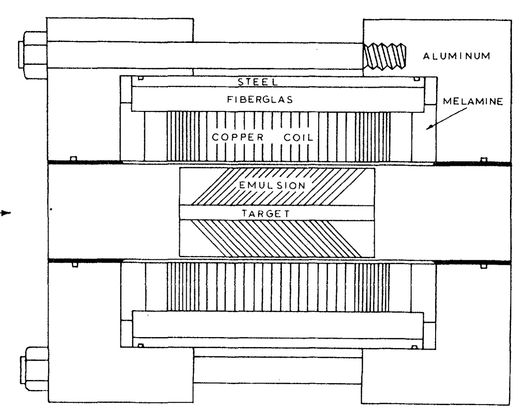

FIG. 1. Pulsed magnet and orientation of experimental apparatus.

the 100 cm³ of sensitive volume. The axis of the magnet was carefully aligned along the beam direction, so that $(\hat{P}_K \cdot \hat{B}) \gtrsim 0.9998$. Rapid beam deflection and special machine tuning were used to obtain a spill time of less than 150 μsec for the entire circulating beam of the AGS. Since the magnet pulse had a 1.5-msec half-period, proper timing insured that the field was essentially constant during the spill.

Scanners followed all tracks which entered the emulsion with ionization greater than 1.5 × minimum, and space angle between 30° and 50° with respect to the beam (nearly half of the Σ^+ are produced within these kinematic limits). They line-scanned for an abrupt change in track direction with projected angle ⩾3° and/or with space angle ⩾5°.[4] From this vertex, the projected angles, dip angles (relative to the emulsion plane), and grain densities of primary and secondary tracks were measured. This served as input to a kinematics program (PRETILT) which calculated the energies and space angles of the tracks and the expected secondary grain density, assuming the event to be a Σp decay (Fig. 2).

The event was then adjudged a Σp decay if the following two criteria were satisfied:

(1) Given the grain density of the primary track, its angle relative to the beam had to satisfy the kinematics of production off hydrogen to within ±7°. This criterion helped eliminate those sigmas which were most likely to have undergone depolarization after production – namely, those which interacted strongly with the nuclei in which they were produced or which scattered off other nuclei in the target.

(2) The primary-secondary space angle and grain densities had to agree sufficiently well with Σp decay kinematics that the event have at least a 5 times greater probability of being a Σp decay than being an elastic proton scatter. Most events had a probability ratio ⩾100, so that on the basis of the probabilities alone, the Σ sample is expected to contain only a 2% contamination due to proton scatters.

An analysis of the background indicated that ⩾90% of the background tracks in the angular region studied were due to protons, primarily from πp interactions in the target ($\pi/K \approx 5/1$ in the beam). Events which might be confused with Σp decays were mostly elastic proton scatters. Pion scatters and $\Sigma\pi$ decays were easy to identify on the basis of ionization criteria and/or the magnetic curvature of the track in the emulsion. The probability that a given event is a Σp decay is given by the product of two factors. The first is a ratio of the number of Σp decays with laboratory space angle between θ' and $\theta'+d\theta'$ to the number of proton Coulomb scatters in the same angular interval. The second factor is a ratio of the probability that the primary and secondary grain densities are a statistical fluctuation from the predicted values for Σp decay to the probability that these grain densities are a fluctuation from the predicted values for an elastic proton scatter.

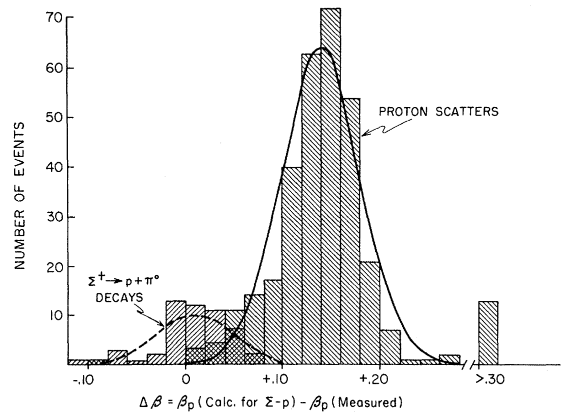

FIG. 2. Δβ distribution for all events. The 52 Σp decays and all topologically similar events in the kinematically allowed regions are shown as a function of Δβ. The actual selection of Σp decays was based on the probability ratio (see text) which also includes the correlation of β_Σ and the decay space angle, as well as the effects of grain-density fluctuations. As expected, the Σp decays give values of Δβ near 0. The number of proton scatters which can be confused with Σp events is small for the kaon reaction [it is 30% lower for the photon reaction (Ref. 5)]. The seven probable Σp events, which did not satisfy the criterion for unambiguous identification, have been treated as proton scatters.

Using the two above-mentioned criteria, 52 Σp decays were found–the protons from these hydrogenlike Σp events were emitted between 10° and 120° in the sigma rest system. This lower limit was chosen to eliminate the small angular region in which the scanning efficiency was known to be low due to small-angle scanning criteria (see above discussion). The upper limit resulted from our inability to count accurately grains (hence to determine energy) of the slow protons emitted backward in the Σ rest system. Furthermore, the topology of these backward events appeared quite different from those in the forward hemisphere;

Table I. Parameters of the sigma sample.

Quantity	Average value/event for 52 Σp events
Laboratory	
B	150 kG
Bt_{Σ}'	21.8 kG nsec
KE_{Σ}	96 MeV
Σp decay angle	22.9°
Σp production angle	42.8°
$\lvert\hat{P}_{\Sigma}\times\hat{B}\rvert$ (at production)	0.68
$\lvert\hat{\sigma}_{\Sigma}\cdot\hat{P}_{\Sigma}\rvert$ (at decay)	0.33
Rest Frame	
ϵ (Precession angle)	36°
$\theta_{\min}-\theta_{\max}$	10°-115°
Missing mass	150 ±50 MeV
τ_{Σ} (Lifetime)	$(0.84\pm0.31)\times10^{-10}$ sec

hence they were less readily recognized by the scanners. Other relevant data on the Σ sample is given in Table I.

Once the Σ sample was chosen, μ_{Σ^+} and $\alpha\overline{P}$ were simultaneously determined by a maximum-likelihood fit to the c.m. distribution

$$f(\mu_{\Sigma}, \alpha\overline{P}, \theta, \varphi) = N^{-1}[1+\alpha\overline{P}(\hat{P}_p\cdot\hat{\sigma}_{\Sigma})],$$

where

$$N=\int_0^{2\pi}\int_{\theta_{\min}}^{\theta_{\max}}[1+\alpha\overline{P}(\hat{P}_p\cdot\hat{\sigma}_{\Sigma})]d\Omega,$$

which is described in the preceding paper.[5]

The accuracy of the distribution function and the analysis program was checked with a Monte Carlo sample of 1726 Σ_p events having assumed values of μ_{Σ^+} and $\alpha\overline{P}$, and distribution of beam momentum, production cross section, scanning criteria, etc., similar to those for the observed events. The analysis programs correctly chose values of μ_{Σ^+} and $\alpha\overline{P}$ in agreement with the input values $\mu_{\Sigma^+}=+3.0\ \mu_N$ and $\alpha\overline{P}=-0.70$ in all cases. A systematic study is presently under way to investigate the effects of transverse field components and detection limits on the value of μ_{Σ^+}.

A check on the purity of our sample was made by calculating μ_{Σ^+} and $\alpha\overline{P}$ for all the 20° angle bins between $\theta_{\min}$ and $\theta_{\max}$. Within statistics the results of all the bins were consistent; the various values of μ_{Σ^+} were consistent with the mean value $\mu_{\Sigma^+}=+3.5\pm1.5\ \mu_N$, with $P(\geq\chi^2)=40\,\%$.

Determination of possible sources of bias is of major importance in any magnetic-moment experiment. Evidence for scanning bias in choice of the Σ sample was checked, as in the preceding paper, by comparing the observed sigma distribution versus φ with that predicted if there were no bias (Fig. 3). The curves shown are in good agreement, with $P(\geq\chi^2)=65\,\%$; hence this test gives no indication of scanning bias. Since a reversed field run was not feasible here due to kaon intensity, possible scanning bias was also studied by looking for asymmetries in the proton scatters. The 341 proton scatters which "satisfy" hydrogen-like production kinematics give a right-left ratio of 1.1 ± 0.2. The absence of left-right bias is particularly encouraging because a true sigma polarization results in a right-left decay asymmetry.[6]

While the absence of right-left asymmetry was not unexpected, given the scanning procedure, a depleted number of scatters both up and down relative to the emulsion plane might be expected, since emulsion shrinkage causes a 50% "demagnification" of the dip angle. The effect would show up as an asymmetry in the azimuthal distribution of the secondary tracks, of the form $f(\varphi)=1+b\cos2\varphi$.[7] The parameter b was expected to be a function of θ, the polar angle between the primary and secondary tracks, since events with larger θ would be more likely to be detected than those with smaller θ. Analysis of 341 scatters gives a maximal val-

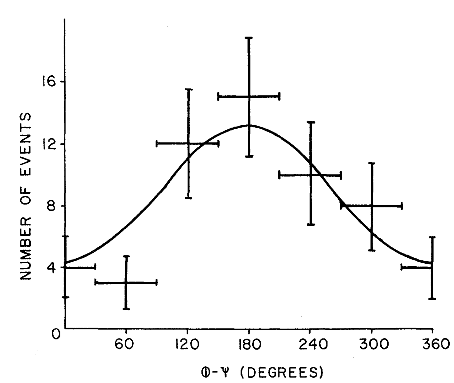

FIG. 3. Azimuthal distribution of the Σ sample. The histogram shows the observed azimuthal distribution of decay protons with respect to the sigma production normal. The effect of magnetic-moment precession has been removed. The solid line shows the distribution expected if there is no bias.

ue $b=1.0\pm0.2$ for $\theta<10°$, $b=0.35\pm0.2$ for θ from 10° to 36°, and $b=0.0\pm0.2$ for $\theta>36°$, where θ is measured in the rest system of the primary particle, as was done for the Σ^+ sample. Since events in the biased regions ($\theta<36°$) represent only 15% of our total sigma sample, the average value of b for the scatter sample, weighted over the sigma distribution, is 0.06 $\pm$0.16. Hence no correction for b was necessary.

Finally, it should be noted that the values of $\alpha\overline{P}$ in this and the preceding experiment are both large in magnitude, but have opposite sign. Since the same scanning procedures and analysis programs were used in both experiments, and since sources of bias which increase the value of $\alpha\overline{P}$ in one experiment would be expected to decrease it in the other, the large magnitude of $\alpha\overline{P}$ in both experiments further rules against the possibility that there exists a major source of bias not previously considered. Thus, each experiment has been triple checked for possible bias.

In summary of the experimental results, the sample of 52 Σp events satisfying hydrogen-like production kinematics yields

$$\mu_\Sigma=+3.5\pm1.5\ \mu_N,$$

$$\alpha\overline{P}=-0.69\pm0.15,$$

where the positive spin direction is $\hat{P}_K\times\hat{P}_\Sigma$. The value of $\alpha\overline{P}$ found in this experiment is in close agreement with the recent bubble-chamber result $\alpha\overline{P}=-0.76\pm0.23$ found at 1113 Mev/c over the same range of c.m. production angles ($-0.8\leq\cos\theta\leq-0.4$).[8]

The value of μ_{Σ^+} reported in this experiment is in excellent agreement with the result $\mu_{\Sigma^+}=+3.0\pm1.2\ \mu_N$, reported in the preceding paper.[5] The weighted average of these two results is $\mu_{\Sigma^+}=+3.2\pm0.9\ \mu_N$. This value is consistent with a previous emulsion measurement,[1] as well as with the spark-chamber measurement $\mu_{\Sigma^+}=+1.4\pm1.1\ \mu_N$.[2] Both of these experiments[1,2] used the approximation $\vec{P}_\Sigma\times\vec{B}=0$. this ignores transverse field components, variation of magnetic field, and the effects of the kinematic detection limits. In the case of the first μ_{Σ^+} measurement[1] this approximation led to a 1.2-μ_N systematic error. No other experimental values of μ_{Σ^+} have been published.[9]

The present result, with its improved accuracy, is in good agreement with the value +2.79 μ_N, found for the proton magnetic moment. The equality of the Σ^+ and proton magnetic moments is just the result predicted by SU(3).[10] Recently, other theoretical calculations have given rise to predictions for the Σ^+ moment that range from +2.2 μ_N[11] to +3.5 μ_N,[12] while also predicting Λ^0 values well within present experimental errors.[13-17] Clearly the present experiments cannot distinguish between these theories; experiments of greatly improved accuracy will be necessary to do that.

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[7]This has been observed in a similar CERN magnetic-moment experiment using sigmas produced in the reaction $\pi^-+p\to\Sigma^++K^-$ (W. T. Toner, private communication). Neither in the CERN experiment nor the present one was an appreciable contribution of $\cos\varphi$-type terms found.

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η DECAY AND CURRENT ALGEBRA

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It has been argued[1] that in the usual picture of current-current electromagnetic interaction, current algebra and the assumption of a linear dependence of the matrix element on the energy variables imply that the decays $\eta\to 3\pi$ are forbidden. In this note we show that, when the partially conserved axial-vector current (PCAC) hypothesis and current algebra are systematically and carefully applied, one finds, within the framework of conventional electrodynamics, that (1) the decays $\eta\to 3\pi$ are allowed; (2) the slope[2] a of the Dalitz plot for $\eta\to\pi^+ +\pi^- +\pi^0$ is -0.49, in excellent agreement with experiment; and (3) the rate for $\eta\to 3\pi^0$ is at most of order of 10^2 eV. In addition to the usual assumptions about PCAC and the current algebra, one essential assumption in our calculation is that[3]

$$[Q_5{}^\alpha(x_0), D^\beta(x)] = i\delta^{\alpha\beta}\sigma(x), \tag{1a}$$

$$[Q_5{}^\alpha(x_0), \sigma(x)] = -iD^\alpha(x), \tag{1b}$$

where $Q_5{}^\alpha(x_0)=\int d^4x\, A_0{}^\alpha(x,x_0)$ is the axial charge, $D^\alpha(x)\equiv\partial^\mu A_\mu{}^\alpha(x)$, and $\sigma(x)$ is an isoscalar, scalar field. Our final results make no reference to the detailed properties of $\sigma(x)$, however.

In this picture the branching ratio $\Gamma(\eta\to 3\pi^0)/\Gamma(\eta\to\pi^+ +\pi^- +\pi^0)$ is of course predicted to be approximately $\frac{3}{2}$. We shall further comment on this point later.

To exploit PCAC and current algebra systematically, we define a symmetric off-shell matrix element for $\eta(p)\to\pi^\alpha(q_1)+\pi^\beta(q_2)+\pi^\gamma(q_3)$:

$$T^{\alpha\beta\gamma}(q_1,q_2,q_3)=(2\pi)^{\frac{3}{2}}(2M\eta)^{\frac{1}{2}}\left[\prod_{i=1}^{3}(q_i{}^2-\mu^2)\int d^4x_i\exp(iq_ix_i)\right]\langle 0|T\{D^\alpha(x_1)D^\beta(x_2)D^\gamma(x_3)H_{\mathrm{em}}(0)\}|\eta\rangle, \tag{2}$$

where[4]

$$H_{\mathrm{em}}(0)=e^2\int dx\, D_{\mu\nu}(x)T\{V_\mu{}^3(x)V_\nu{}^0(0)\},$$