MEASUREMENT OF THE Σ^+ MAGNETIC MOMENT USING THE REACTION $\gamma+p\to\Sigma^+ + K^0$ NEAR THRESHOLD*†

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(Received 5 April 1967)

The sample of 51 high-information Σp events has a mean rotation of 32° per event and gives no evidence of bias; it yields $\mu_{\Sigma^+}=+3.0\pm1.2\,\mu_N$ and $\alpha\overline{P}=+0.85\pm0.25$ in the direction $\hat{P}_\gamma\times\hat{P}_\Sigma$. All magnetic field components along the path of each Σ^+ have been considered. The use of nuclear emulsion enables one to include the effects of the Σp c.m. angle on detection efficiency, and thereby on the calculated value of μ_{Σ^+}.

This communication includes the data presented in the first determination of μ_{Σ^+},[1] which will be referred to as I. The statistics have been improved, significant changes made in the data analysis, and approximately equal numbers of events obtained from exposures made with opposite signs of magnetic field. The value of field reversal in Σ^+ moment experiments has been shown in the spark chamber experiment of Cook et al.,[2] who obtained values of 0.0 and +4.0 μ_N for opposite sign of field before bias correction. They have pointed out that field reversal enables one to identify systematic biases and claim that the final result is relatively insensitive to such biases if field reversal is used.

Ilford K-5 emulsion stacks and a CH_2 target were placed inside a pulsed solenoid[3] with a measured axial field uniformity of better than 2% over the experimental region.[4] The magnetic axis of the solenoid was aligned with the photon beam to within a milliradian, thus we have $|\hat{B}\cdot\hat{P}_\gamma|>0.9998$ for all portions of the sigma path in the field. The 1.5-BeV bremsstrahlung beam from the Caltech electron synchrotron was collimated to a 3-mm diameter at the CH_2 target, and the emulsion exposed as described earlier.[1] Several exposures have been made at fields between 108 and 125 kG, with the magnetic axis aligned both parallel and antiparallel to the beam. The emulsions provide a detector which is insensitive to the magnetic field and which is the most precise of all available detectors for the measurement of angle and ionization changes.

The scanning criteria and methods of distinguishing sigma decays from background events (mostly proton scatters) are described in I and in the kaon paper.[5] A sample of 51 $\Sigma^+\to p+\pi^0$ decays has been selected for the magnetic moment analysis; their parameters are shown in Table I. The experiment is designed to detect high-information Σp events, whose magnetic moments have large precession angles in the field. The statistical error is inversely proportional to $\bar{\epsilon}N^{1/2}$, where $\bar{\epsilon}$ is the mean precession angle and N the number of events in the sample. Thus, this sample with $\bar{\epsilon}=32°$ has the same statistical significance as a 450-event sample with $\bar{\epsilon}=11°$ and the same polarization.

The assumption has been made in both of the earlier Σ^+ magnetic moment experiments[1,2] that the field components perpendicular to the hyperon momentum can be neglected. Under the assumption $\vec{P}_\Sigma\times\vec{B}=0$, the sigma spin $\vec{\sigma}_\Sigma$ remains perpendicular to its momentum $\vec{P}_\Sigma$, and there is no transverse field component to change the direction of the Σ^+ momentum. However, a Σ^+ moving at only 15° to a uniform 150-kG field sees a transverse field of 39 kG along with the 145-kG parallel component. In this experiment, although the incident beam

Table I. Parameters of the sigma sample.

Quantity	Average value/event for 51 Σp events
Laboratory	
B	117 kG
Bt_Σ'	21.3 kG nsec
KE_Σ	190 MeV
Σp decay angle	13.0°
Σ production angle	17.5°
$\lvert\hat{P}_\Sigma\times\hat{B}\rvert$ (at production)	0.30
$\lvert\hat{\sigma}_\Sigma\cdot\hat{P}_\Sigma\rvert$ (at decay)	0.07
Rest Frame	
ϵ (Precession angle)	32°
$\theta_{\min}-\theta_{\max}$	10°-90°
Missing mass	160 ± 70 MeV
τ_Σ (Lifetime)	$(0.96\pm0.35)\times10^{-10}$ sec

is well collimated and can be aligned with the magnetic field, the sigmas are produced over a 30° range of laboratory angles. We have found that neglect of the effects of transverse field components can cause differences of one or two nuclear magnetons in the calculated value for $\mu_{\Sigma}{}^{+}$ in this and the following experiment.[5]

One important simplifying condition which holds at all points along the Σ^+ path is that

$$\vec{\sigma}_{\Sigma}\cdot\vec{B}=0. \tag{1}$$

This results from precise beam alignment along the axis of the magnet and from the uniformity of the magnetic field throughout the sensitive volume.[6]

Since $\vec{P}_{\Sigma}\times\vec{B}\neq 0$, the Σ^+ particle travels through the field in a spiral path. The proton distribution function and the kinematic detection limits, however, depend on the sigma direction $\vec{P}_{\Sigma}$ at decay. Thus, $\vec{P}_{\Sigma}$ at decay is used to define the coordinate axes as follows:

$$\hat{x}=\hat{P}_{\Sigma},$$

$$\hat{z}=\hat{B}\times\hat{P}_{\Sigma},$$

$$\hat{y}=\hat{z}\times\hat{x}, \tag{2}$$

Coordinates and typical orientations of $\vec{P}_{\Sigma}$, $\vec{P}_p$, $\vec{B}$, and $\vec{\sigma}_{\Sigma}$ relative to these axes are given in Fig. 1.

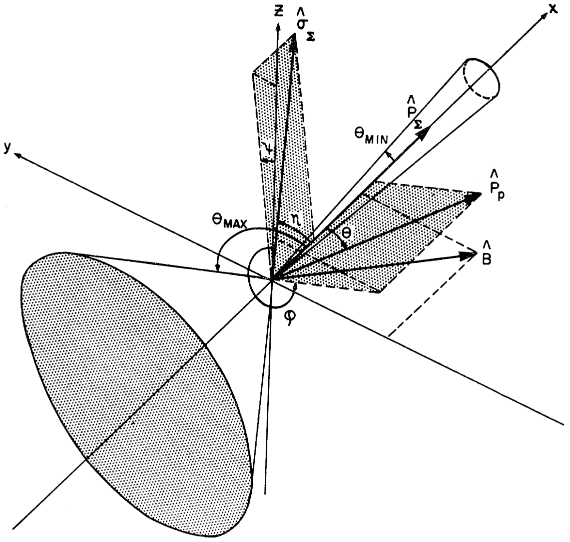

FIG. 1. Typical orientations of $\hat{P}_{\Sigma}$, $\hat{P}_p$, $\hat{B}$, and $\hat{\sigma}_{\Sigma}$ relative to the coordinate axes. The vectors representing the directions of the sigma spin $\bar{\sigma}_{\Sigma}$ and the proton's momentum $\bar{P}_p$ lie in the shaded planes and satisfy the following relations: (1) $\hat{\sigma}_{\Sigma}\cdot\hat{P}_{\Sigma}=\cos\eta$, $\hat{\sigma}_{\Sigma}\cdot\hat{Z}=\cos\psi\sin\eta$, (2) $\hat{P}_p\cdot\hat{P}_{\Sigma}=\cos\theta$, $\hat{P}_p\cdot\hat{Z}=\cos\varphi\sin\theta$, and (3) $\hat{\sigma}_{\Sigma}\cdot\hat{P}_p=\cos\theta\cos\eta+\sin\theta\sin\varphi\sin\eta\sin\psi+\sin\theta\times\sin\varphi\sin\eta\cos\psi$.

This coordinate system has the advantage that the kinematic detection limits are expressed as a function of the single variable θ, the polar angle between $\vec{P}_p$ and $\vec{P}_{\Sigma}$. In earlier Σ^+ measurements,[1,2] effects due to the θ detection limits were neglected since it was assumed that $\vec{\sigma}_{\Sigma}\cdot\vec{P}_{\Sigma}=0$.[7] In the general case, $\vec{\sigma}_{\Sigma}\cdot\vec{P}_{\Sigma}\neq 0$; hence the θ detection limits result in a proton bite which is not symmetrical with respect to a plane perpendicular to $\vec{\sigma}_{\Sigma}$; the θ detection limits therefore become important in evaluating μ_{Σ^+}. While the averaging of values of μ_{Σ^+} with reversed field orientations is helpful in reducing these effects, such procedures are not exact for finite angular displacements due to transverse field components.

The 7072 computer program takes the measured directions and ionizations for each sigma and its proton at the decay vertex and transforms them to the rest system of the Σ^+, defined by (2). The expected distribution of the proton in this system has the form

$$f(\mu_{\Sigma},\alpha\bar{P},\theta\varphi)=[1+\alpha\bar{P}(\hat{P}_p\cdot\hat{\sigma}_{\Sigma})]/\int_0^{2\pi}\int_{\theta\min}^{\theta\max}[1+\alpha\bar{P}(\hat{P}_p\cdot\hat{\sigma}_{\Sigma})]\sin\theta d\theta d\varphi, \tag{3}$$

where α is the Σp decay asymmetry parameter $\alpha_0=(0.960\pm 0.067)$[8] and $\bar{P}$ is the average polarization. The computer program simultaneously determines $\alpha\bar{P}$ and μ_{Σ^+} by a maximum-likelihood fit of the distribution.[9] The fact that B is constant and can be used in (1) and (2) results in a considerable simplification, namely, that irrespective of the sigma's path the sigma moment μ_{Σ^+} undergoes a simple rotation ϵ about a fixed B given by

$$\epsilon=9.57\times 10^6\mu_{\Sigma}Bt_{\Sigma},$$

where μ_{Σ} is in nuclear Bohr magnetons, B is in kG, and t_{Σ}, the time in magnetic field in the Σ rest frame, is in seconds. The total rotation of $\vec{\sigma}_{\Sigma}$ relative to the coordinate axes defined in (2) also includes a term resulting from the rotation of $\vec{P}_\Sigma$ about $\vec{B}$ (i.e., from a precession of the coordinate axis as the particle moves through the field).

The kinematic detection limits, $\theta_{\rm min}$ and $\theta_{\rm max}$, are determined from the observed θ distribution of the Σ events. Reasons for the existence of these limits are discussed in the kaon paper.[5] If the simple assumption is made that Σp decays are uniformly detected over all c.m. θ, then the same data lead to a value for μ_{Σ^+} approximately 1 μ_N less than that obtained using the measured detection limits.

Three separate tests were made to detect possible systematic bias in scanning, but none gave any evidence of such bias. The area scanning used in I and the line scanning used for the rest of the data are described elsewhere.[5] The φ distribution at production (before spin precession) provides a strong test for bias according to Cook et al.[2] The observed distribution of c.m. φ at production is compared in Fig. 2 with the distribution predicted if there were no bias. The agreement is excellent with $P(\geq\chi^2)=90\,\%$ for no bias. The second indication that the sample is unbiased results because events in the parallel and antiparallel field orientations give values of μ_{Σ^+} which are in surprisingly close agreement, differing by only 5 % for the same $\alpha\bar{P}$. In contrast, Cook et al.[2] had a 2:1 asymmetry in the equivalent φ distribution and got very different μ_{Σ^+} from their two field orientations, before bias corrections. The third test comes from a comparison of μ_{Σ^+} values measured using the photon and kaon reactions, since these reactions produce Σ^+ particles which have polarizations of nearly equal magnitude but opposite sign. This final check is discussed elsewhere,[5] along with the results of an independent measurement of μ_{Σ^+} using the reaction $K^- + p \rightarrow \Sigma^+ + \pi^-$.

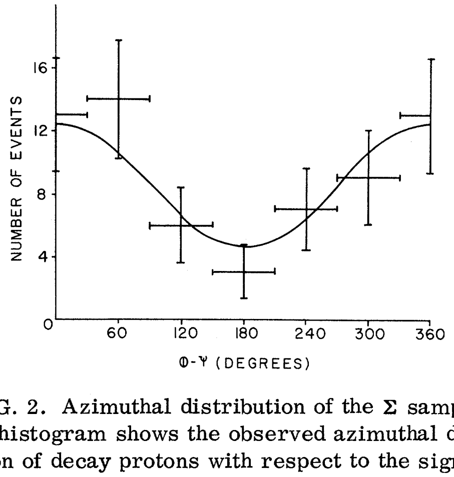

FIG. 2. Azimuthal distribution of the Σ sample. The histogram shows the observed azimuthal distribution of decay protons with respect to the sigma production normal. The effect of magnetic moment precession has been removed. The solid line shows the distribution expected if there is no bias.

In summary, the sample of 51 Σp events from the reaction $\gamma + p \rightarrow \Sigma^+ + K^0$ gives no evidence of bias, and yields

$$\mu_{\Sigma^+} = +3.0 \pm 1.2\ \mu_N,$$

$$\alpha\bar{P} = +0.85 \pm 0.25,$$

where the positive direction of Σ spin is defined as $\hat{P}_\gamma \times \hat{P}_\Sigma$. The errors are purely statistical. The symmetry of the likehood distribution indicates that μ_{Σ^+} and $\alpha\bar{P}$ are essentially uncorrelated at these values (Fig. 3).

It should be emphasize that this new determination includes the data in I, and should not be averaged with it. One can, however, quote the conclusions of I with increased confidence: "The anomalous moment of the Σ^+ has the same sign as the charge moment and is comparable in magnitude to the nucleon moments." After presentation of an independent measurement of the Σ^+ moment, using the kaon reaction, the measured values will be compared with various theoretical predictions.

The personnel of the Caltech electron synchrotron were extremely helpful when the exposures were made; it would be difficult to thank all who have made substantial contributions to this experiment. Professor R. F. Bacher and Professor R. L. Walker have provided encouragement and support when needed. Dr. O. Maloy and Dr. J. Mullins have been especially helpful and the overtime work of F. Scarlino is appreciated. At Vanderbilt, Dr. E. R. Goza has made major contributions to this work

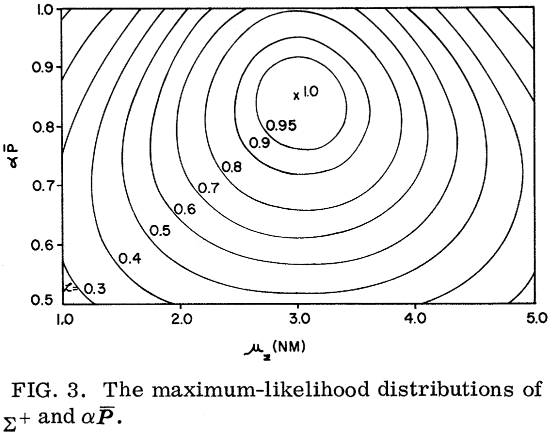

FIG. 3. The maximum-likelihood distributions of μ_{Σ^+} and $\alpha\bar{P}$.

and the scanning efforts of Mrs Anuradha Amtey, Mrs. Takako Davis, Mr. Chinkook Lee, Mr. William May, and Mr. Jospeh Mulcahey are especially commended.

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*Work supported by grant from the National Science Foundation.

†Taken in part from a Ph.D. dissertation submitted by C.R.S. to Vanderbilt University.

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[1] A. D. McInturff and C. E. Roos, Phys. Rev. Letters 13, 246 (1964).

[2] V. Cook, T. Ewart, G. Masek, R. Orr, and E. Platner, Phys. Rev. Letters 17, 223 (1966).

[3] C. E. Roos and V. Z. Peterson, Phys. Rev. 135, B1012 (1964).

[4] The accuracy of field measurement techniques, pulse reproducibility, and field homogeneity are demonstrated by an experiment on high-field Zeeman effects. [S. H. Lott, C. E. Roos, and M. Ginter, J. Opt. Soc. Am. 56, 775 (1966).]

[5] D. Kotelchuck, E. R. Goza, C. R. Sullivan, and C. E. Roos, following paper [Phys. Rev. Letters 18, 1166 (1967)].

[6] Since $\hat{\sigma}_\Sigma = \hat{P}_\gamma \times \hat{P}_\Sigma$ at production, and $\vec{P}_\gamma \times \vec{B} = 0$, then $\vec{\sigma}_\Sigma \cdot \vec{B} = 0$ at production. During the time between Σ production and decay, $\hat{\sigma}_\Sigma$ precesses about the constant field $\vec{B}$, independent of $\vec{P}_\Sigma$, hence $\vec{\sigma}_\Sigma \cdot \vec{B} = 0$, holds at all points along the Σ^+ path.

[7] This follows from $\hat{\sigma}_\Sigma = \hat{P}_\gamma \times \hat{P}_\Sigma$ at production, and the assumption that $\hat{P}_\Sigma \times \hat{B} = 0$ over the entire Σ path.

[8] R. O. Bangerter, A. Barbaro-Galtieri, J. P. Berge, J. J. Murray, F. T. Solmitz, M. L. Stevenson, and R. D. Tripp, Phys. Rev. Letters 17, 495 (1966).

[9] J. Orear, University of California Radiation Laboratory Report No. UCRL-8417 (unpublished).

DETERMINATION OF THE Σ^+ MAGNETIC MOMENT USING THE REACTION $K^- + p \to \Sigma^+ + \pi^-$ AT 1.15 BeV/c*

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(Received 5 April 1967)

A sample of 52 Σp decays satisfying hydrogen-like production kinematics yields $\mu_{\Sigma^+} = +3.5 \pm 1.5\ \mu_N$ and $\alpha\overline{P} = -0.69 \pm 0.15$ relative to $\hat{P}_K \times \hat{P}_\Sigma$. The effect of all magnetic field components along the Σ^+ path has been included, as well as that of the c.m. decay angle on detection efficiency. The measurement of μ_{Σ^+} can be averaged with an earlier result to give $\mu_{\Sigma^+} = +3.2 \pm 0.9\ \mu_N$, consistent with the prediction by SU(3).

The reaction $K^- + p \to \Sigma^+ + \pi^-$ has certain unique features which make it preferable to photon[1] and pion[2] reactions for measurement of the Σ^+ magnetic moment. The exothermic kaon reaction produces highly polarized Σ^+ particles at lower energies and larger laboratory angles than those produced in the photon and pion reactions at equivalent laboratory momenta. Furthermore, the Σ^+ cross sections are relatively large, so that the Σ^+/background ratio for the kaon reaction is well over an order of magnitude greater than that for the other two reactions. The main disadvantage has been the low intensity of separated, high-purity kaon beams, but this situation has been improving in recent years.

Recently, the F-20 separated beam of the Brookhaven alternating-gradient synchrotron (AGS) was modified to deliver about 10^6 K^- ($P_K = 1150$ MeV/c) onto a polyethylene target. This was the only suitable momentum where polarization measurements were available at the time of the exposure.[3] Two stacks of Ilford K-5 nuclear emulsion were placed next to the target, at 42° to the beam (Fig. 1). Compensated coil windings in the magnet held the 150-kG axial magnetic field uniform to 2% over

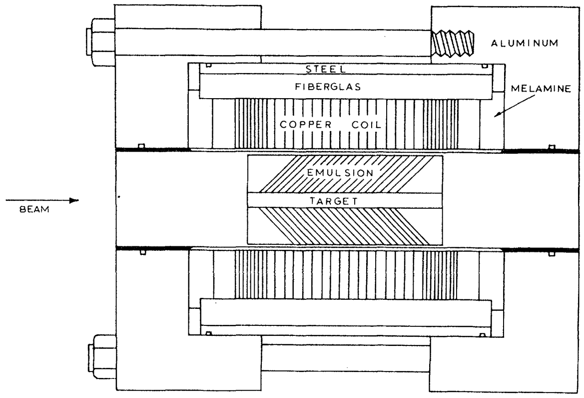

FIG. 1. Pulsed magnet and orientation of experimental apparatus.