tribution of the data at the hypothetical K^*-p vertex fits the shape predicted for this angle assuming instead that we are observing OPE as in Fig. 1(a). The fit gives $\chi^2=14$ for 10 degrees of freedom. There is then no evidence for any mechanism other than OPE for events in this sample.

[7]Our results for the limits on Δ^2 for OPE dominance agree with those found by G. Goldhaber, in Proceedings of the Athens Topical Conference on Recently Discovered Resonant Particles, Ohio University, Athens Ohio, 1963, edited by B. A. Munir and L. J. Gallahan (Ohio University Physics Department, Miami, Ohio, 1963) p. 86, and Ref. 3, in K^+p interactions at 1.96 and 4.6 BeV/c to produce K^*+N^*.

[8]See compilation of elastic-scattering $\pi^\pm p$ data by E. Urvater and D. Alvarado, University of Colorado Report No. UA-3, 1967 (unpublished).

[9]Our $K^-\pi^-$ mass distribution is shown in Fig. 3(b). According to our hypothesis this represents a rough indication of the energy dependence of the $K^-\pi^-$ elastic-scattering cross section except for necessary phase-space modifications. The energy dependence corrected for phase space, according to the model discussed in the text, is illustrated in Fig. 3(a).

[10]E. Ferrari and F. Selleri, Nuovo Cimento 21, 1028 (1961); Nuovo Cimento Suppl. 24, 453 (1962); 27, 1450 (1963); we use

$$G(E,M_1,M_2)=\frac{M_1M_2}{(2\pi)^3E^2}\left[\frac{\lambda(E^2,M_1^2,M_2^2)}{\lambda(E^2,M^2,M_K^2)}\times\lambda(M_1^2,M_K^2,m^2)\lambda(M_2^2,M^2,m^2)\right]^{1/2},$$

where $\lambda(x,y,z,)=x^2+y^2+z^2-2xy-2xz-2yz$, M is the proton mass, M_K is the K-meson mass, and m is the pion mass. This is equivalent to the calculations of Ferrari and Selleri for the limiting case $\Delta^2\to-m^2$.

[11]See, for example, Stephen Gasiorowicz, Elementary Particles (John Wiley & Sons, Inc., New York, 1966), p. 482.

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POSSIBLE NONADDITIVITY OF QUARK AMPLITUDES IN HIGH-ENERGY CROSS SECTIONS*

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The possible nonadditive character of quark amplitudes and the resulting additional contributions to total cross sections are investigated. A sum rule is derived and it is suggested that the nonadditive contributions to some cross sections may be dominant.

High-energy hadron-nucleus cross sections exhibit the effects of multiple scattering to varying degrees, depending in part upon the reactions measured and the momentum transfers involved. For example, proton-deuteron elastic-scattering angular distributions exhibit a secondary maximum or shoulder and a backward peak which appear to be attributable to double scattering.[1-3] Proton-He^4 elastic scattering exhibits secondary and tertiary maxima which may be attributable to double and triple scattering, respectively.[4] Double-scattering effects in high-energy hadron-deuteron total cross sections are not quite so dramatic, but generally do amount to 5-20% of the corresponding hadron-nucleon cross sections.[5-7] The analyses of multiple scattering at high energies are usually made by means of the Glauber approximation.[8] In quark models, where hadrons may be thought of as bound states of three quarks (or antiquarks), or of a quark and an antiquark, one might expect multiple-scattering corrections to hadron-hadron total cross sections to be significant. We investigate the possible effects of "multiple scattering," i.e., nonadditivity of quark amplitudes, by applying the Glauber approximation to the quark model.

Most of the total cross-section relations obtained from the quark model with the assumption of additivity of the quark-quark and quark-antiquark amplitudes[9-15] appear to be in rather good or very good agreement with the measurements. This agreement need not, however, imply that the quark amplitudes are additive. The treatment of nonadditivity corrections in the quark model may be illustrated for $\pi^+\pi^+$ collisions for which the Glauber approximation leads naturally to a consideration of single, double, triple, and quadruple scattering. We use the notation $\mathcal{P}\mathcal{N}\lambda$ for the three quarks and we consider the π^+ meson to be a bound state of the antiquark-quark pair $\bar{\mathcal{N}}\mathcal{P}$. We assume that the internal velocities of the quarks may be neglected in comparison to the velocity of the incident π^+ meson. We also assume that the π^+ meson has an internal wave function and a corresponding form factor $S(\vec{q})$. The total cross section at a laboratory momentum k denoted by $\sigma(\pi^+\pi^+, k)$, may be written as

$$\sigma(\pi^+\pi^+, k) = 2\sigma(\mathcal{P}\bar{\mathfrak{N}}, \mu k) + 2\sigma(\mathcal{P}\mathcal{P}, \mu k) + \delta\sigma_2 + \delta\sigma_3 + \delta\sigma_4, \quad (1)$$

where $\sigma(\mathcal{P}\bar{\mathfrak{N}}, \mu k)$ is the $\mathcal{P}\bar{\mathfrak{N}}$ "total cross section" at an incident momentum μk for the $\mathcal{P}$ quark in the laboratory system, with a corresponding meaning given to $\sigma(\mathcal{P}\mathcal{P}, \mu k)$.[16] The factor μ is a constant equal to the ratio of the effective mass of the quasifree quark $\mathcal{P}$ to the mass of the π^+ meson. The quantities $\delta\sigma_2$, $\delta\sigma_3$, and $\delta\sigma_4$ represent, respectively, the double-, triple-, and quadruple-scattering corrections which we shall investigate.

We let $f(\mathcal{P}\bar{\mathfrak{N}}, \vec{q}, \mu k)$ denote the $\mathcal{P}\bar{\mathfrak{N}}$ elastic-scattering amplitude for an incident momentum μk and a momentum transfer $\vec{q}$ for the quark $\mathcal{P}$, with a corresponding notation for the $\mathcal{P}\mathcal{P}$ amplitude. An extension of the Glauber approximation for hadron-deuteron collisions[6] yields[17]

$$\delta\sigma_2 = (2/\mu^2k^2)\,\mathrm{Re}\int\{4S(\vec{q})f(\mathcal{P}\bar{\mathfrak{N}}, \vec{q})f(\mathcal{P}\mathcal{P}, -\vec{q}) + S^2(\vec{q})[f(\mathcal{P}\mathcal{P}, \vec{q})f(\mathcal{P}\mathcal{P}, -\vec{q}) + f(\mathcal{P}\bar{\mathfrak{N}}, \vec{q})f(\mathcal{P}\bar{\mathfrak{N}}, -\vec{q})]\}d^2q, \quad (2)$$

$$\delta\sigma_3 = -(1/\pi\mu^3k^3)\,\mathrm{Im}\int 2S(\vec{q})S(q')[f(\mathcal{P}\bar{\mathfrak{N}}, \vec{q}+\vec{q}\,')f(\mathcal{P}\mathcal{P}, -\vec{q}\,') + f(\mathcal{P}\mathcal{P}, \vec{q}+\vec{q}\,')f(\mathcal{P}\bar{\mathfrak{N}}, -\vec{q}\,')f(\mathcal{P}\bar{\mathfrak{N}}, -\vec{q})d^2q\,d^2q', \quad (3)$$

and

$$\delta\sigma_4 = -(1/2\pi^2\mu^4k^4)\,\mathrm{Re}\int S(\vec{q})S(\vec{q}\,')f(\mathcal{P}\mathcal{P}, \vec{q}\,''-\tfrac{1}{2}\vec{q}-\tfrac{1}{2}\vec{q}\,')f(\mathcal{P}\mathcal{P}, \vec{q}\,''+\tfrac{1}{2}\vec{q}+\tfrac{1}{2}\vec{q}\,')$$
$$\times f(\mathcal{P}\bar{\mathfrak{N}}, -\vec{q}\,''+\tfrac{1}{2}\vec{q}-\tfrac{1}{2}\vec{q}\,')f(\mathcal{P}\bar{\mathfrak{N}}, -\vec{q}\,''-\tfrac{1}{2}\vec{q}+\tfrac{1}{2}\vec{q}\,')d^2q\,d^2q'\,d^2q'', \quad (4)$$

for $\pi^+\pi^+$ collisions, where the quark amplitudes are to be evaluated at the momentum μk (which we have suppressed). We have chosen units in which $\hbar = 1$.

Let us consider two extreme cases regarding the binding of the quark-antiquark pair. If we assume that the $\mathcal{P}\bar{\mathfrak{N}}$ system is very strongly bound, we may approximate $S(\vec{q})$ by unity in the integrals (2)-(4). Thus $\delta\sigma_2$, for example, would become

$$\delta\sigma_2^{(s)} = (2/\mu^2k^2)\,\mathrm{Re}\int[4f(\mathcal{P}\bar{\mathfrak{N}}, \vec{q})f(\mathcal{P}\mathcal{P}, -\vec{q}) + f(\mathcal{P}\mathcal{P}, \vec{q})f(\mathcal{P}\mathcal{P}, -\vec{q}) + f(\mathcal{P}\bar{\mathfrak{N}}, \vec{q})f(\mathcal{P}\bar{\mathfrak{N}}, -\vec{q})]d^2q, \quad (5)$$

where the superscript (s) denotes the strong-binding limit.

The other extreme, in which the $\mathcal{P}\bar{\mathfrak{N}}$ system is very weakly bound, leads to formulas containing expectation values of expressions involving the quark-antiquark separation r (or ρ) in the π^+ meson. For example, $\delta\sigma_2$ is given simply by

$$\delta\sigma_2^{(w)} = (4\pi/\mu^2k^2)\,\mathrm{Re}\{4f(\mathcal{P}\bar{\mathfrak{N}}, 0)f(\mathcal{P}\mathcal{P}, 0)\langle r^{-2}\rangle_{\pi^+} + [f^2(\mathcal{P}\mathcal{P}, 0) + f^2(\mathcal{P}\bar{\mathfrak{N}}, 0)]$$
$$\times\langle\langle(1/2r\rho)\ln[(r+\rho)/|r-\rho|]\rangle_{\pi^+}\rangle_{\pi^+}\}, \quad (6)$$

where the symbol $\langle\;\rangle_{\pi^+}$ denotes the expectation value in the $\mathcal{P}\bar{\mathfrak{N}}$ bound state, i.e., in the π^+ state. The superscript (w) indicates the weak-binding limit.

By including multiple-scattering corrections, cross-section sum rules may be derived under certain simplifying assumptions. It should be clear that for a given set of assumptions regarding the various quark amplitudes, any sum rule we obtain by including multiple-scattering effects would have to be a linear combination of the sum rules obtained from the usual additivity assumption. The reason for this may be seen by noting that the sum rules are independent of the values of the various quark amplitudes. Therefore if the multiple-scattering amplitudes happened to all vanish, we would be in a situation where the additivity assumption was correct, and consequently the "new" sum rules would not be independent of those already derived. However, it is important to note that the inverse statement is not correct. A sum rule derived from additivity would not in general be any linear combination of sum rules derived by including multiple-scattering effects. Consequently, a sum rule derived by including multiple-scattering corrections would perhaps indicate which combinations of the familiar sum rules are to be preferred.

For the remainder of our discussion we consider the strong-binding limit, treat only double-scattering "corrections" to the additivity assumption, and assume $f(\mathcal{P}\mathcal{P}) = f(\mathcal{P}\mathcal{N}) = f(\bar{\mathcal{P}}\mathcal{N}) = f(\lambda\mathcal{P}) = f(\bar{\lambda}\mathcal{P}) \neq f(\bar{\mathcal{P}}\mathcal{P})$. We exclude $f(\bar{\mathcal{P}}\mathcal{P})$ from this assumption since an isosinglet annihilation channel may be expected to give significant contribution and break the Pomeranchuk theorem for amplitudes which can have an isosinglet component.[12] The sum rule we derive under these assumptions is

$$\sigma(K^-n)-\tfrac{1}{2}\sigma(K^-p)-\tfrac{1}{2}\sigma(K^+p) = \tfrac{1}{4}\sigma(\bar{p}n)-\tfrac{1}{5}\sigma(\bar{p}p)-\tfrac{1}{20}\sigma(pp). \quad (7)$$

The comparison with the measurements[7] is shown in Table I. It is seen that five of the six predictions agree well with experiment. The remaining one lies within two standard deviations of the measurements.[18]

We have calculated separately the single- and double-scattering contributions to the total cross sections. The single-scattering contributions may consist of two types of terms. One corresponds to $\bar{\mathcal{P}}\mathcal{P}$ collisions and is proportional to the antiquark-quark "total cross section" $\sigma(\bar{\mathcal{P}}\mathcal{P})$. The other corresponds to $\mathcal{P}\mathcal{P}$, $\mathcal{P}\mathcal{N}$, $\bar{\mathcal{P}}\mathcal{N}$, $\lambda\mathcal{P}$, and $\bar{\lambda}\mathcal{P}$ collisions and is proportional to $\sigma(\mathcal{P}\mathcal{P})$. A typical calculated value for the "cross section" $\sigma(\bar{\mathcal{P}}\mathcal{P})$ at 12 GeV/c laboratory momentum is 7.6 ± 6.3 mb. The large uncertainty is due to the large experimental uncertainty in the antiproton-neutron total cross-section measurements. The corresponding value for $\sigma(\mathcal{P}\mathcal{P})$ is calculated to be only 0.2 ± 0.3 mb. These values suggest that within a hadron quarks rarely engage in single collisions, except possibly for $\bar{\mathcal{P}}\mathcal{P}$ collisions. This does not imply, however, that in hadron-hadron collisions the annihilation processes, which presumably represent a substantial part of $\bar{\mathcal{P}}\mathcal{P}$ collisions, necessarily result mainly from single collisions. That part of $\sigma(\bar{\mathcal{P}}\mathcal{P})$ corresponding to absorptive processes is equal to the cross section for the absorption of a $\bar{\mathcal{P}}\mathcal{P}$ pair in hadron-hadron collisions regardless of what else happens to the remaining quarks and antiquarks during the collision. For proton-antiproton collisions, for example, it includes absorptive processes in which three quark-antiquark pairs annihilate. A relatively large value for $\sigma(\bar{\mathcal{P}}\mathcal{P})$ is therefore not incompatible with the proposal of Kokkedee and Van Hove[15] that antibaryon-baryon annihilation takes place mainly via the annihilation of three quark-antiquark pairs.

The double-scattering corrections consist of three types of terms. Those which are quadratic in $f(\bar{\mathcal{P}}\mathcal{P})$ are found to be negative at the six energies for which data exist. This is to be expected for strongly absorptive processes. For purely absorptive (opaque) interactions it may be explained in part as a "shadow" correction in a manner similar to the interpretation given by Glauber[19] for hadron-deuteron absorption cross sections, and as an additional correction for counting some absorptive processes more than once when using the additivity assumption. The remaining two types of double-scattering corrections, those quadratic in $f(\mathcal{P}\mathcal{P})$ and those bilinear in $f(\mathcal{P}\mathcal{P})$ and $f(\bar{\mathcal{P}}\mathcal{P})$, yield positive contributions.

The two types of single-scattering contributions and their sums, and the sum of the three types of double-scattering contributions, are shown in Table II for total cross sections at 12 GeV/c. The uncertainties in the values shown in the second and third columns are 86 and 153%, respectively. We note that the double-scattering contribution alone yields almost the entire pn and K^+p cross sections. This is related to the property that the $\bar{\mathcal{P}}\mathcal{P}$ (or $\bar{\mathcal{N}}\mathcal{N}$) combination appears in none of the quark collisions involved in these two cross sections. Empirically, the relative importance of double scattering appears

Table I. Tests of the sum rule $\sigma(K^-n)-\tfrac{1}{2}\sigma(K^-p)-\tfrac{1}{2}\sigma(K^+p)=\tfrac{1}{4}\sigma(\bar{p}n)-\tfrac{1}{5}\sigma(\bar{p}p)-\tfrac{1}{20}\sigma(pp)$.

Momentum (GeV/c)	$\sigma(K^-n)-\tfrac{1}{2}\sigma(K^-p)$ $-\tfrac{1}{2}\sigma(K^+p)$ (mb)	$\tfrac{1}{4}\sigma(pn)-\tfrac{1}{5}\sigma(pp)$ $-\tfrac{1}{20}\sigma(pp)$ (mb)
6	1.4 ± 0.4	1.0 ± 1.0
8	-0.8 ± 0.4	1.1 ± 1.0
12	0.8 ± 0.4	1.1 ± 0.9
14	0.7 ± 0.4	1.3 ± 0.9
16	1.2 ± 0.6	1.4 ± 0.9
18	1.3 ± 1.2	-1.0 ± 2.4

Table II. Quark single- and double-collision contributions to total cross sections at 12 GeV/c (in mb).

Cross section	$\bar{\mathcal{P}}\mathcal{P}$ single-collision contributions	$\mathcal{P}\mathcal{P}$, $\mathcal{P}\mathcal{N}$, $\bar{\mathcal{P}}\mathcal{N}$, $\lambda\mathcal{P}$ and $\bar{\lambda}\mathcal{P}$ single-collision contributions	Total single-collision contribution	Double-collision contribution	Total cross section
$\sigma(pn)$	0	1.9	1.9 ± 2.9	38.5 ± 4.6	40.4
$\sigma(K^+p)$	0	1.2	1.2 ± 1.9	16.1 ± 1.9	17.3
$\sigma(\pi^+p)$	7.6	1.0	8.6 ± 6.5	15.6 ± 1.6	24.2
$\sigma(\pi^-p)$	15.2	0.8	16.0 ± 12.7	12.9 ± 1.9	28.9 ± 12.8
$\sigma(\bar{p}n)$	30.3	1.0	31.3 ± 25.2	22.5 ± 5.8	53.8
$\sigma(\bar{p}p$	37.9	0.8	38.7 ± 31.4	13.0 ± 7.7	51.7

to decrease with the number of $\bar{\mathcal{P}}\mathcal{P}$ or $\bar{\mathcal{N}}\mathcal{N}$ combinations that can be formed from the quark constituents of the two colliding hadrons. The rather large double-scattering contributions we have calculated for some cross sections suggests that triple- or even higher-order multiple-scattering contributions may be significant. We might point out that assumptions regarding the quark amplitudes which are less restrictive than those we have considered may be used to obtain additional sum rules. These sum rules, however, have the disadvantage that they cannot be tested so readily as they involve cross sections such as $\sigma(\Lambda p)$, for example, which have not been measured with such great accuracy. Details of such calculations will be reported elsewhere.

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[16]Isospin and charge conjugation invariance are assumed throughout.

[17]For simplicity we omit those contributions to the hadron-hadron forward elastic-scattering amplitudes which arise from multiple interactions in which the identity of three or more quarks change during the collision and the final states of the two bound systems are identical to their initial states. These types of reactions have an analog in proton-deuteron collisions, for example, where an elastic double charge-exchange process may occur. In that case the incident proton charge exchanges with the neutron. The emerging particle, now a neutron, then charge-exchanges with the target proton. The final configuration of the target is a neutron-proton system which may be bound, and the final emerging particle is once again a proton. See C. Wilkin, Phys. Rev. Letters 17, 561 (1966), and Ref. 2 for more detailed discussions. A double collision which we omit for $\pi^+\pi^+$ scattering in the quark model is $\bar{\mathcal{N}}+\mathcal{P}\to\mathcal{P}+\bar{\mathcal{N}}$ followed by $\mathcal{P}+\bar{\mathcal{N}}\to\bar{\mathcal{N}}+\mathcal{P}$, with the final state of each of the two $\bar{\mathcal{N}}\mathcal{P}$ systems being the π^+ state.

[18]Our assumptions also imply $\sigma(K^+p)=\sigma(K^+n)$, $\sigma(pp)=\sigma(pn)$, $\sigma(\pi^-p)=\sigma(K^-p)$, and $\sigma(\pi^+p)=\sigma(K^-n)$. The first two relations are in agreement with experiment. In the last two relations a discrepancy of ~4 mb exists.

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