REACTION $K^- + p \to K^- + p + \pi^- + \pi^+$ AT 4.6 AND 5.0 GeV/c; EVALUATION OF $K^-\pi^-$ SCATTERING CROSS SECTION

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The reaction $K^- + p \to K^- + p + \pi^- + \pi^+$ near 5.0 BeV/c has been examined. In events of the type $K^- + p \to K^{*0}(890) + p + \pi^-$ one-pion exchange appears to explain the interaction. Analysis of the scattering angular distribution at the $\pi^- p$ vertex indicates that the p and π^- are produced as if scattering of the virtual exchanged pion is real. Similar analysis of events of the type $K^- + p \to N^{*++}(1236) + K^- + \pi^-$, with the assumption of real scattering between the K^- and π^-, yields information on the elastic differential, total elastic, and total cross sections of $K^-\pi^-$ scattering.

Modification of the one-pion-exchange (OPE) model to include the effects of scattering of the virtual pion on the nucleon have been considered most recently by Deck[1] and by Maor and O'Halloran.[2] These authors have shown that such considerations, for example, lead to kinematic enhancements at the low-mass end of three-body mass spectra derived from four-body final states. Their calculations have led to good agreement with specific experimentally observed distributions.[1,2] Many experiments have noted angular distributions at the nucleon vertex consistent with real elastic meson-nucleon scattering.[3,4] Hence there is evidence for the hypothesis that in peripheral high-energy interactions the virtual exchanged pion scatters from the nucleon as if it were real. This report extends these ideas to the $K^-\pi^-$ vertex in the reaction $K^- + p \to N^{*++}(1236) + K^- + \pi^-$ in order to obtain information about the $K^-\pi^-$ scattering cross section.

From 60 000 pictures taken with the Brookhaven 80-inch hydrogen bubble chamber with K^- mesons incident at 4.6 and 5.0 BeV/c we have identified 1624 events of type $K^- + p \to K^- + p + \pi^+ + \pi^-$. The experimental details were described previously.[5]

The interaction is dominated by the intermediate channels

$$K^- + p \to K^{*0}(890) + \pi^- + p, \qquad (1)$$

$$K^- + p \to K^- + \pi^- + N^{*++}(1236). \qquad (2)$$

Both K^{*0} and N^{*++} are produced mainly in peripheral (low four-momentum transfer) interactions. About 20% of events forming K^{*0} showed a quasi-two-body character, $K^- + p \to K^{*0} + N^{*0}$. In contrast there is no evidence for quasi-two-body formation in channel (2).

Peripheral production of K^{*0} in channel (1) and N^{*++} in channel (2) suggest that OPE is dominant, the K^{*0} being produced via the diagram of Fig. 1(a) and the N^{*++} by that of Fig. 2(a).[6] To select events proceeding via these diagrams we adopt the following criteria: for diagrams 1(a), $860 \leq M(K^-\pi^+) \leq 920$ MeV,

$$\Delta^2(K^-_{\text{in}}(K^-\pi^+)_{\text{out}}) \leq 0.5\ (\text{BeV}/c)^2; \qquad (3)$$

for diagram 2(a), $1150 \leq M(\pi^+ p) \leq 1300$ MeV,

$$\Delta^2(p_{\text{in}}(\pi^+ p)_{\text{out}}) \leq 0.5\ (\text{BeV}/c)^2. \qquad (4)$$

These criteria minimize the nonresonant background so that (i) the Trieman-Yang azimuthal angular dependence at the corresponding pion-nucleon vertex is consistent with isotropy and (ii) the angular distribution of the decay of the resonance is consistent with $\cos^2\theta$ dependence. For $\Delta^2 > 0.5$ (BeV/c)^2 these features are less evident, indicating that OPE is not dominant.[7]

These selection criteria, (3) and (4), largely remove events produced via other mechanisms, in particular those displaying $K\pi\pi$ and

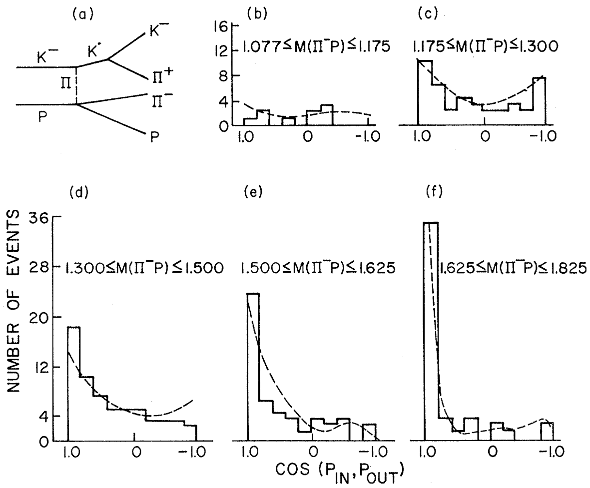

FIG. 1. (a) Diagram for OPE production of K^{*0}. (b)-(f) Distributions in the cosine of the scattering angle of the proton in the π^-p barycentric system at the lower vertex of the diagram in (a) for events satisfying condition (3). The histograms represent our data and the dashed curves are a normalized average of the real π^-p elastic scattering over the energy regions indicated.

$\pi\pi$ mass enhancements. (We find no evidence for Y^* production.) These will be reviewed extensively in another report.

Of 1624 events in the $K^-p\pi^+\pi^-$ final state, 217 are selected by condition (3) and 250 by condition (4). Both criteria are satisfied simultaneously by 29 events.

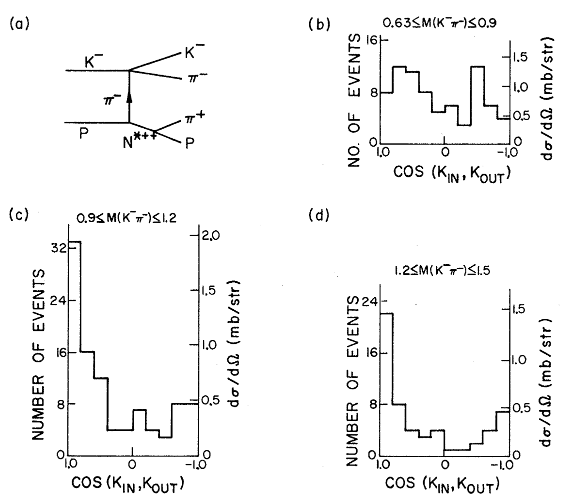

FIG. 2. (a) Diagram for OPE production of N^{*++}. (b)-(d) Distributions in the cosine of the scattering angle of the K^- in the $K^-\pi^-$ barycentric system at the upper vertex of the diagram in (a) for events satisfying condition (4).

For events selected by (3), we now examine the behavior of the π^-p vertex of diagram 1(a). The distribution in the cosine of the angle between incoming and outgoing proton in the π^-p barycentric system is shown in Figs. 1(b)-1(f) for five intervals of the invariant π^-p mass. The dashed curves show the real π^-p elastic-scattering angular distributions[8] averaged over each energy interval and normalized to the data. The similarity between the real and the derived distributions shows that the scattering at the nucleon vertex in the diagram of Fig. 1(a) is consistent with real π^-p elastic scattering.

If the exchanged meson exhibits real elastic scattering at the nucleon vertex in Fig. 1(a) then it is reasonable to assume that the exchanged meson exhibits real elastic scattering at the K^--meson vertex in Fig. 2(a). This hypothesis is supported by the result of Ref. 4 where a four-particle production is shown to be consistent with the assumption that the exchanged pion scatters elastically at both nucleon vertices of the OPE diagram for pp collisions; in fact the distribution in mass of π^+p and of π^-p resembles dramatically the energy dependence of the equivalent real π^+p and π^-p total elastic-scattering cross sections.[8]

In accordance with this hypothesis, the events satisfying (4) have been divided into three intervals of $K^-\pi^-$ mass. For these intervals the distribution of events in cosine of the $K^-\pi^-$ scattering angle in the $K^-\pi^-$ barycentric system is plotted in Figs. 2(b)-2(d). We assume that these distributions represent the relative $K^-\pi^-$ differential elastic scattering for the corresponding center-of-mass energies.[9]

To estimate approximate quantitative values for the $K^-\pi^-$ differential and total elastic-scattering cross sections we have made modified OPE calculations for diagram 2(a) to predict the rate of Reaction (2) as a function of the OPE propagator and the real on-the-mass-shell scattering of the exchanged pion at both vertices. We use the relation[10]

$$\sigma(M_{K^-\pi^-}, M_{\pi^+p}, \Omega_{K^-\pi^-}, \Omega_{\pi^+p}, \Delta^2)$$

$$=\frac{G(E, M_{K^-\pi^-}, M_{\pi^+p})}{(\Delta^2+m^2)^2}\left\{\frac{d\sigma_{\rm el}}{d\Omega}\right\}_{K^-\pi^-}\left\{\frac{d\sigma_{\rm el}}{d\Omega}\right\}_{\pi^+p}. \quad (5)$$

In each $K^-\pi^-$ mass interval $(d\sigma/d\Omega)_{\pi^+p}$ is computed using values of $(d\sigma/d\Omega)_{\rm el}$ from appropriate real π^+p elastic-scattering measurements.

Thus, a second scale is drawn in Figs. 2(b)-2(d) on the right side of each graph giving the calculated cross section in mb/sr. These distributions seem to indicate a predominance of S-wave scattering at low energy with a trend toward more peripheral scattering at higher energy.

Fitting the higher energy data in Fig. 2 to an exponential dependence on the square of the four momentum transfer for the $K^-\pi^-$ elastic scattering in the region $\cos(K,K) \geq 0.6$ and calculating[11] an average impact parameter $R = \langle b^2\rangle^{1/2}$ for the $K^-\pi^-$ interaction we find $R = 0.7 \pm 0.2$ F.

The average elastic-scattering cross section $\sigma_{el}(K^-\pi^-)$ for each energy range is obtained by summing over each of the distributions in Fig. 2 and is plotted in Fig. 3. Assuming the forward elastic-scattering amplitude to be purely imaginary we have estimated an upper limit for the total $K^-\pi^-$ cross section from the

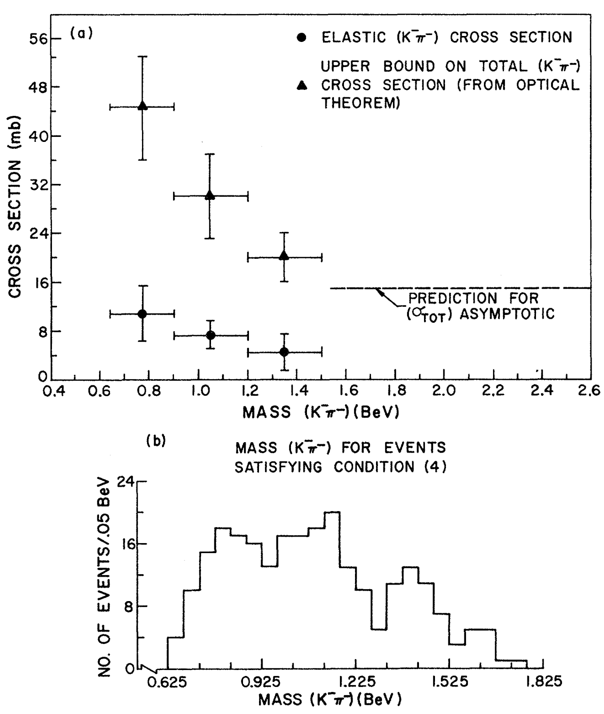

FIG. 3. (a) Cross sections for the $K^-\pi^-$ interaction as a function of the total energy in the $K^-\pi^-$ barycentric system. The total cross section is obtained from the optical theorem as explained in the text. Errors are obtained both from statistical uncertainty and from uncertainties determined by measuring the sensitivity of the cross section to cuts in the kinematical quantities. (b) Distribution in $M(K^-\pi^-)$ which demonstrates qualitative dependence of $\sigma_{el}(K^-\pi^-)$ as discussed in Ref. 9.

optical theorem

$$\sigma_{tot,\,opt} = (4\pi/k)[(d\sigma/d\Omega)_{0\ deg,\,el}]^{1/2}.$$

These values also are plotted in Fig. 3. Any real part of the forward scattering amplitude would decrease this estimate for σ_{tot} proportionally.

Finally we have compared our values for $\sigma_{tot}(K^-\pi^-)$ with the asymptotic (infinite-energy) value predicted by factorization of Regge-pole residues,[12]

$$\sigma(K^-\pi^-)_{tot} = \sigma_{tot}(K^-p)\sigma_{tot}(\pi^-p)/\sigma_{tot}(pp).$$

Using the asymptotic values $\sigma_{tot}(K^-p) = 22$ mb, $\sigma_{tot}(\pi^-p) = 25$ mb, and $\sigma_{tot}(pp) = 37$ mb, this relation gives $\sigma_{tot}(K^-\pi^-) = 15$ mb, shown as the asymptote in Fig. 3.

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[6]We have examined the possibility that $K^*(890)$ production goes by K^{*0} exchange where the K^- dissociates into K^{*0} and π^- and the K^{*0} scatters elastically with the proton. The azimuthal scattering angular dis-

tribution of the data at the hypothetical K^*-p vertex fits the shape predicted for this angle assuming instead that we are observing OPE as in Fig. 1(a). The fit gives $\chi^2 = 14$ for 10 degrees of freedom. There is then no evidence for any mechanism other than OPE for events in this sample.

[7]Our results for the limits on Δ^2 for OPE dominance agree with those found by G. Goldhaber, in Proceedings of the Athens Topical Conference on Recently Discovered Resonant Particles, Ohio University, Athens Ohio, 1963, edited by B. A. Munir and L. J. Gallahan (Ohio University Physics Department, Miami, Ohio, 1963) p. 86, and Ref. 3, in K^+p interactions at 1.96 and 4.6 BeV/c to produce $K^* + N^*$.

[8]See compilation of elastic-scattering $\pi^{\pm}p$ data by E. Urvater and D. Alvarado, University of Colorado Report No. UA-3, 1967 (unpublished).

[9]Our $K^-\pi^-$ mass distribution is shown in Fig. 3(b). According to our hypothesis this represents a rough indication of the energy dependence of the $K^-\pi^-$ elastic-scattering cross section except for necessary phase-space modifications. The energy dependence corrected for phase space, according to the model discussed in the text, is illustrated in Fig. 3(a).

[10]E. Ferrari and F. Selleri, Nuovo Cimento 21, 1028 (1961); Nuovo Cimento Suppl. 24, 453 (1962); 27, 1450 (1963); we use

$$G(E, M_1, M_2) = \frac{M_1 M_2}{(2\pi)^3 E^2}\left[\frac{\lambda(E^2, M_1^2, M_2^2)}{\lambda(E^2, M^2, M_K^2)} \times \lambda(M_1^2, M_K^2, m^2)\lambda(M_2^2, M^2, m^2)\right]^{1/2},$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$, M is the proton mass, M_K is the K-meson mass, and m is the pion mass. This is equivalent to the calculations of Ferrari and Selleri for the limiting case $\Delta^2 \to -m^2$.

[11]See, for example, Stephen Gasiorowicz, Elementary Particles (John Wiley & Sons, Inc., New York, 1966), p. 482.

[12]M. Gell-Mann, Phys. Rev. Letters 8, 263 (1962); V. N. Gribov and I. Ya. Pomeranchuk, Phys. Rev. Letters 8, 343 (1962).

POSSIBLE NONADDITIVITY OF QUARK AMPLITUDES IN HIGH-ENERGY CROSS SECTIONS*

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The possible nonadditive character of quark amplitudes and the resulting additional contributions to total cross sections are investigated. A sum rule is derived and it is suggested that the nonadditive contributions to some cross sections may be dominant.

High-energy hadron-nucleus cross sections exhibit the effects of multiple scattering to varying degrees, depending in part upon the reactions measured and the momentum transfers involved. For example, proton-deuteron elastic-scattering angular distributions exhibit a secondary maximum or shoulder and a backward peak which appear to be attributable to double scattering.[1-3] Proton-He^4 elastic scattering exhibits secondary and tertiary maxima which may be attributable to double and triple scattering, respectively.[4] Double-scattering effects in high-energy hadron-deuteron total cross sections are not quite so dramatic, but generally do amount to 5-20% of the corresponding hadron-nucleon cross sections.[5-7] The analyses of multiple scattering at high energies are usually made by means of the Glauber approximation.[8] In quark models, where hadrons may be thought of as bound states of three quarks (or antiquarks), or of a quark and an antiquark, one might expect multiple-scattering corrections to hadron-hadron total cross sections to be significant. We investigate the possible effects of "multiple scattering," i.e., nonadditivity of quark amplitudes, by applying the Glauber approximation to the quark model.

Most of the total cross-section relations obtained from the quark model with the assumption of additivity of the quark-quark and quark-antiquark amplitudes[9-15] appear to be in rather good or very good agreement with the measurements. This agreement need not, however, imply that the quark amplitudes are additive. The treatment of nonadditivity corrections in the quark model may be illustrated for $\pi^+\pi^+$ collisions for which the Glauber approximation leads naturally to a consideration of single, double, triple, and quadruple scattering. We use the notation $\mathcal{P}\mathcal{N}\lambda$ for the three quarks and we consider the π^+ meson to be a bound state of the antiquark-quark pair $\bar{\mathcal{N}}\mathcal{P}$. We assume that the internal velocities of the quarks may be neglected in comparison to the velocity of the incident π^+ meson. We also assume that the π^+ meson has an internal wave function and