ASYMPTOTIC PREDICTIONS OF FORWARD π-N DISPERSION RELATIONS*

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The recent Brookhaven Coulomb-interference measurements of $\alpha = \mathrm{Re}f/\mathrm{Im}f$ are analyzed in terms of forward π-N dispersion relations. The results are consistent with a crossover of $\sigma^{(-)} = \frac{1}{2}(\sigma_- - \sigma_+)$ at high energies above 22 BeV. The ρ-trajectory model determined by charge-exchange experiments is compared with our analysis of the data between 8 and 20 BeV.

In earlier work one of us[1] derived from current algebra an extended version of the Pomeranchuk theorem,[2] which predicted a crossover of the antisymmetric total π-N cross section $\sigma^{(-)} = \frac{1}{2}(\sigma_- - \sigma_+)$ at high energies. The Regge-pole model and current algebras have recently been combined within a single framework and certain predictions made about the high-energy behavior of π-N cross sections[3]; in particular, a crossover in $\sigma^{(-)}$ would violate the basic assumptions of this theory. With the advent of the new, accurate Brookhaven[4] Coulomb-interference measurements of $\alpha = \mathrm{Re}f/\mathrm{Im}f$, it is possible to obtain a more critical test of the high-energy behavior of $\sigma^{(-)}$ and the forward π-N dispersion relations. Lautrup, Nielson, and Olesen[5] have developed methods for testing forward π-N dispersion relations that do not involve assumptions about the unknown cross sections above 20 BeV. However, the earlier Coulomb-interference data of Foley et al.,[6] for the real scattering amplitude, contained very large systematic errors, and therefore these investigations were inconclusive.

We shall consider the unsubtracted forward π-N dispersion relations for the real scattering amplitude

$$D^{(-)}(\omega) = \frac{2\omega f^2}{\omega^2 - (\mu^2/2M)^2} + \frac{\omega}{4\pi^2} \mathrm{P}\int_\mu^\infty \frac{(\sigma_- - \sigma_+)k'd\omega'}{\omega'^2 - \omega^2}, \quad (1)$$

where M and μ are the proton and pion masses, respectively; ω and k are the pion laboratory energy and momentum, respectively; $\sigma_\pm(\omega)$ are the total $\pi^\pm$-p cross sections; and $f^2 = 0.081$. Units $\hbar = c = 1$ are used throughout except where explicitly noted. The real antisymmetric scattering amplitude is defined by $D^{(-)}(\omega) = \frac{1}{2}[D_-(\omega) - D_+(\omega)]$. We can separate Eq. (1) into the form

$$H(\omega) = \frac{D^{(-)}(\omega)}{\omega} - \frac{2f^2}{\omega^2 - (\mu^2/2M)^2} - \frac{1}{4\pi^2} \mathrm{P}\int_\mu^{22} \frac{(\sigma_- - \sigma_+)k'd\omega'}{\omega'^2 - \omega^2}, \quad (2)$$

where $H(\omega)$ is the integral

$$H(\omega) = \frac{1}{4\pi^2} \mathrm{P}\int_{22}^\infty \frac{(\sigma_- - \sigma_+)k'd\omega'}{\omega'^2 - \omega^2}. \quad (3)$$

The integral, in Eq. (2), from μ to 6 BeV was evaluated using measured total cross-section data.[7] From 6 to 22 BeV, we used the parametric fit of Lindenbaum et al.[4] given by $\sigma_+(\omega) = 22.57 + 24.51/k^{1.02}$ mb and $\sigma_-(\omega) = 22.57 + 19.55/k^{0.664}$ mb. This parametrization is a good fit to the latest Brookhaven data, and agrees within experimental errors with the earlier data of Galbraith et al.[7] $D^{(-)}(\omega)$ was evaluated from the measured values of $\alpha_\pm = \mathrm{Re}f^\pm/\mathrm{Im}f^\pm$ given in Ref. 4 using the optical theorem and the measured total cross sections. The results of the measurements of α depend upon the relative phase shift introduced between the nuclear and Coulomb phase shifts by the long-range Coulomb interaction. This relative phase shift has been calculated by Bethe[8] to give $\delta_B = (e^2/\hbar c)\ln(1.06/pa\theta)$. Rix and Thaler[9] have recently analyzed Bethe's calculation by extending it to a relativistic treatment, and obtained agreement with Bethe's result to within 15%.[10]

Our results for $H(\omega)$ are shown in Fig. 1 for Bethe's relative phase shift. The errors due to the integral from μ to 22 BeV, in Eq. (2), are neglected as the main errors arise from the calculated $D^{(-)}(\omega)$. Höhler, Baacke, and Strauss[11] have calculated the experimental values of $D^{(-)}(\omega)$ from the formula

$$D^{(-)}(\omega) = \pm\left[\frac{1}{2}\frac{d\sigma(0)}{d\Omega_{\mathrm{lab}}} - \left(\frac{k}{4\pi}\sigma^{(-)}\right)^2\right]^{1/2} \quad (4)$$

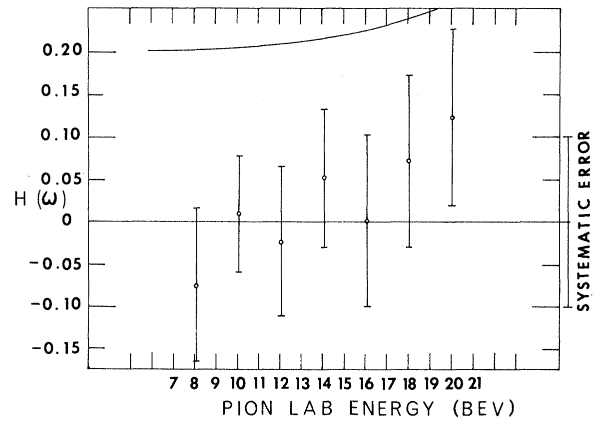

FIG. 1. The function

$$H(\omega)=\frac{1}{4\pi^2}\int_{\omega_0}^{\infty} d\omega'\, k' \frac{[\sigma_-(\omega')-\sigma_+(\omega')]}{\omega'^2-\omega^2}$$

(in units BeV^{-2}), based on Bethe's value of δ, is plotted versus the pion laboratory energy. $\omega_0=22.14$ BeV. The systematic error from $\alpha_\pm=\mathrm{Re}f/\mathrm{Im}f$, as determined by the Brookhaven experiment (Ref. 4), is also shown. The solid curve represents the prediction of $H(\omega)$ according to the Regge ρ trajectory with $\alpha_\rho(0)$ and $c^{(-)}$ determined by charge-exchange data in Ref. 11.

using experimental data for $d\sigma/d\Omega$ and $\sigma^{(-)}$. Branch I of Eq. (4) does not agree as well as branch II with the results of Fig. 1 within the statistical and systematic errors.

Höhler, Baacke, and Strauss[11] have calculated $H(\omega)$ on the basis of the ρ-trajectory Regge-pole fit to the charge-exchange data. They used

$$\sigma^{(-)}(k)=c^{(-)}k^{\alpha_\rho-1}$$

with $\alpha_\rho=0.56$ and $c^{(-)}=0.315$ (in units $\hbar=c=m_\pi=1$). This gives, for the lower limit of integration $\omega_0=22.14$ BeV,

$$H_\rho(\omega)=[3.91+0.704(\omega/\omega_0)^2+0.387(\omega/\omega_0)^4+\cdots]10^{-3}. \quad (5)$$

The solid curve in Fig. 1 shows the comparison of the results of Eq. (5) with the $H(\omega)$ obtained from our analysis of the data.

The results of Fig. 1 indicate the possibility of a negative $H(\omega)$ with $H(\omega)$ passing through 0 at ≈ 12 BeV. This would suggest a crossover of $\sigma^{(-)}(\omega)$ above 22 BeV, which would agree with the predictions of the high-energy theorem in Ref. 1. This theorem predicts that $\sigma_+-\sigma_-=R/k$, where R is a positive constant. If the crossover occurs at about 50 BeV, then "asymptopia" is at much higher energies than are presently being considered. It would be interesting to check this prediction by means of total cross-section experiments on the 70-BeV machine.

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