approximately to an E_γ^5 dependence of the widths for transitions to the ground state when E_γ is in the neighborhood of 7 MeV.

Unfortunately, the energy dependence inferred from the giant resonance refers to ground-state transitions from a variable initial state, whereas our data consist of transitions from a fixed state to a variable final state. The (p,γ) measurements of Allas et al.[8] suggest why the data nevertheless conform to the E_γ^5 dependence. For very light nuclides such as C^{12}, these authors find that the giant resonances built on excited states are similar to the resonances built on the ground state, except that the resonance curve is shifted towards higher energy by an amount that is equal to the energy of the excited state. If this behavior is assumed to exist for heavy nuclides also, then it is easily shown that the energy dependence of radiation widths depends only on the γ-ray energy, independent of whether the energy of the initial state or the final state is varied.

It is still too early to know whether the high-energy radiative transitions in most heavy nuclides are related in a simple way to the shape of the main part of the giant resonance. However, it is clear that the average-spectrum method of measurement is capable of providing the answer.

---

*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]C. E. Porter and R. G. Thomas, Phys. Rev. 104, 483 (1956).

[2]G. E. Thomas, D. E. Blatchley, and L. M. Bollinger, to be published.

[3]H. E. Jackson, J. Julien, C. Samour, A. Bloch, C. Lopata, J. Morgenstern, H. Mann, and G. E. Thomas, Phys. Rev. Letters 17, 656 (1966).

[4]J. F. W. Jansen and H. Pauw, Nucl. Phys. A94, 235 (1967).

[5]J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics (John Wiley & Sons, Inc., New York, 1952), p. 639.

[6]D. M. Brink, thesis, Oxford University, 1955 (unpublished).

[7]P. Axel, Phys. Rev. 126, 671 (1962).

[8]R. G. Allas, S. S. Hanna, L. Meyer-Schützmeister, and R. E. Segel, Nucl. Phys. 58, 122 (1964).

DOUBLE-SCATTERING CONTRIBUTIONS TO THE PROTON-PROTON DIFFERENTIAL CROSS SECTION IN THE QUARK MODEL

David R. Harrington and Antonio Pagnamenta*
Department of Physics, Rutgers, The State University, New Brunswick, New Jersey
(Received 15 May 1967)

In a recent paper[1] Franco has pointed out that double-scattering effects in the quark model[2-8] may produce appreciable corrections to total cross sections. In this note we should like to point out that the same double scattering provides a natural explanation for the sharp break[9] in the high-energy proton-proton differential cross section at a momentum transfer square of about 0.6 $(\mathrm{GeV}/c)^2$. Provided the quark model, or any other composite model, is accepted, the possibility of such an explanation is strongly suggested by the recent work of Franco and Coleman,[10] who show that a similar structure in proton-deuteron scattering can be explained as a double-scattering effect.

In this paper we shall try to make semiquantitative estimates of the double-scattering contribution to the proton-proton differential cross section, assuming a quark-model description of the protons. We consider the elastic scattering of two high-energy protons in the center-of-mass system,

$$p(\vec{q})+p(-\vec{q})\to p(\vec{q}+\vec{\Delta})+p(-\vec{q}-\vec{\Delta}).$$

We shall assume that each proton is composed of three quarks at positions $\vec{r}_1$, $\vec{r}_2$, and $\vec{r}_3$ relative to its center of mass (thus $\vec{r}_1+\vec{r}_2+\vec{r}_3=0$), and may be described by a nonrelativistic, completely antisymmetric spatial wave function $\psi(\vec{r}_1,\vec{r}_2,\vec{r}_3)$.[11] For convenience we normalize $F(\Delta)$, the proton-proton scattering amplitude, so that

$$d\sigma/d\Delta^2=\pi|F(\Delta)|^2 \tag{1}$$

and

$$\sigma_{\mathrm{tot}}=4\pi\,\mathrm{Im}F(0). \tag{2}$$

Then, assuming that quark-quark scattering is spin and isospin independent, the usual approximations associated with the eikonal method[12] give

$$F(\Delta)=9f(\Delta)S_1^2(\Delta)+18(2i)(4\pi)^{-1}\int d^2\delta f(\tfrac{1}{2}\vec{\Delta}+\vec{\delta})f(\tfrac{1}{2}\vec{\Delta}-\vec{\delta})\times[S_1(\Delta)+S_{12}(\Delta,\vec{\delta})]S_{12}(\vec{\Delta},\vec{\delta})+\text{terms from triple- and higher-order scattering.} \tag{3}$$

In this formula $f(\Delta)$ is the quark-quark scattering amplitude at the appropriate center-of-mass momentum,[13] while $S_1(\Delta)$ and $S_{12}(\Delta,\delta)$ are one- and two-body form factors:

$$S_1(\Delta)=\int d\tau\,|\psi|^2\exp(-i\vec{\Delta}\cdot\vec{r}_1), \tag{4}$$

$$S_{12}(\Delta,\delta)=\int d\tau\,|\psi|\exp[-\tfrac{1}{2}i\vec{\Delta}\cdot(\vec{r}_1+\vec{r}_2)]\exp[-i\vec{\delta}\cdot(\vec{r}_1-\vec{r}_2)]. \tag{5}$$

In general, the double-scattering term will be smaller than the single-scattering term in the forward direction but, because it decreases more slowly with increasing Δ, will dominate at larger angles. This gives a qualitative explanation of the sudden break in the differential cross section; for a more quantitative discussion we assume the following forms for $f(\Delta)$ and ψ:

$$f(\Delta)=(\alpha+i)f_I\exp(-\tfrac{1}{2}b^2\Delta^2), \tag{6}$$

$$\psi\propto(r_1^2-r_2^2)(r_2^2-r_3^2)(r_3^2-r_1^2)\times\exp[-(r_1^2+r_2^2+r_3^2)/2a]. \tag{7}$$

(The real parameters α, f_I, and b may, of course, depend upon q.) The wave function we have chosen is identical to one mentioned by Thirring[14]; it is completely antisymmetric, yet simple enough to give analytic (but lengthy) expressions for the form factors. We find

$$S_1(\Delta)=e^{-X}[1-2X+(17/10)X^2-(62/105)X^3+(27/280)X^4-(1/140)X^5], \tag{8}$$

where

$$X=\tfrac{1}{6}\Delta^2a^2. \tag{9}$$

The exact expression for $S_{12}(\Delta,\delta)$ is even more complex. Fortunately, when the arguments are not too large, the following simple approximate form should be quite accurate:

$$S_{12}(\Delta,\delta)\approx\exp[-3Y-3Z], \tag{10}$$

where

$$Y=\Delta^2a^2/24 \tag{11}$$

and

$$Z=\tfrac{1}{2}\delta^2a^2. \tag{12}$$

If there were no double-scattering term the differential cross section would be

$$d\sigma/d\Delta^2=81\pi|f(\Delta)|^2S_1^4(\Delta). \tag{13}$$

Because of the antisymmetrization requirement on the wave function $S_1(\Delta)$ has considerable structure; in particular, after it has decreased smoothly for almost five orders of magnitude $S_1^4(\Delta)$ suddenly develops a broad shoulder. As shown in Fig. 1, however, this is important for our purposes only in the rather unrealistic case $a\gg b$. In general, the single-scattering contribution to the differential cross section decreases smoothly through the region

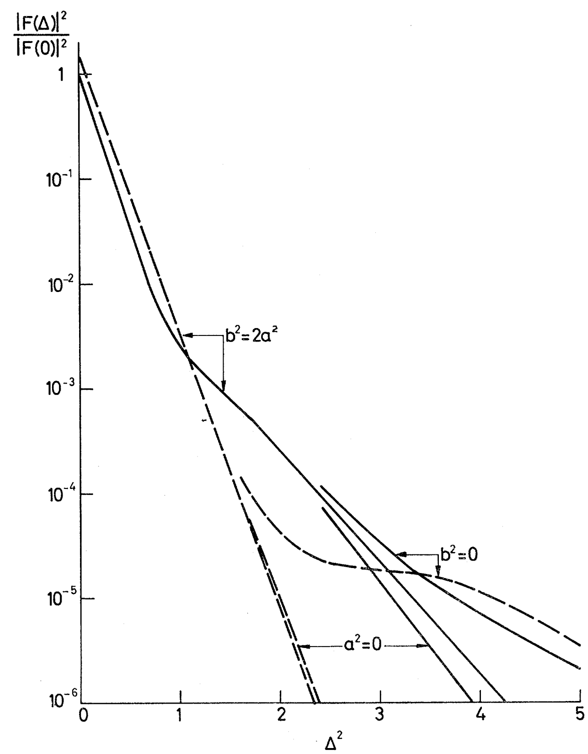

FIG. 1. The proton-proton cross section in the quark model for $|\alpha|=1$. The solid curves include double scattering and have been drawn with the horizontal scales adjusted to give common initial slopes. The corresponding curves without double scattering are drawn with dashed lines. The horizontal scale is in arbitrary units.

Table I. Comparison of calculated parameters describing the shape of the differential cross section with the corresponding experimentally determined parameters. The calculated values are determined by fitting each of the solid curves in Fig. 1 (by eye) with two straight lines, then reading off the ratio of slopes and the ordinate at the point of intersection. The experimental values were determined in the same way from Fig. 9 of Ref. 9.

	Calculated with $\|\alpha\|=1$			
	$b^2=0$	$b^2=2a^2$	$a^2=0$	Experiment
Slope ratio	0.31	0.36	0.38	0.35
Intersection ordinate	2.8×10^{-3}	3.6×10^{-3}	3.5×10^{-3}	3×10^{-3}

where the break is observed experimentally.

If we include the double-scattering term, then our calculated results depend upon only two free parameters, α and the ratio of a to b, since the experimental values for the total cross section and $B\equiv-d(\ln|F|^2)/d\Delta^2|_{\Delta^2=0}$ determine the other two. [We take $\sigma_{\rm tot}=40$ mb and $B=10$ (GeV/c)$^{-2}$.] The shape of the calculated differential cross section over a range including the region of the break is rather insensitive to the ratio of a to b; the critical parameter is α. As in proton-deuteron scattering,[10] if α is small the destructive interference of double- and single-scattering terms produces a deep minimum where their magnitudes are equal. Since no such dip is observed experimentally we are forced to take α comparable to one; Fig. 1 is drawn for case $|\alpha|=1$. An α this large is in disagreement with recent Coulomb-interference experiments,[15] but this is not too disturbing since we have probably oversimplified by assuming α independent of Δ.

From Fig. 1 and Table I we see that the calculated ratios of secondary slope to initial slope, and cross section at the break to forward cross section, agree quite well with experiment. Our main point is in any case clear from a glance at Fig. 1: Even though we have chosen the simplest possible forms for $f(\Delta)$ and ψ, we automatically obtain a sudden change in the slope of the differential cross section because of double scattering. These results, of course, do not prove that the quark model is correct. There are many other possible explanations for the structure in the proton-proton cross section[16] (but perhaps none which fall out so easily, with so little fitting of parameters). When taken together with its other successes,[17] however, they do perhaps suggest that, in spite of its obvious defects, the quark model is worthy of further consideration.

If the quark model is accepted, it is of course natural to ask whether triple scattering might be responsible for the second break observed at higher momentum transfers. Also, preliminary calculations indicate that, provided the phase of the quark-antiquark amplitude is chosen properly, and allowed to vary somewhat with energy, it may be possible to account for the structure in the pion-proton differential cross section[18] as a double-scattering effect. We hope to return to these points in a future publication.

We should like to thank Professor A. D. Krisch for a preprint of Ref. 9 and a helpful discussion of the experimental situation. One of the authors (D.H.) wishes to thank Dr. V. Franco for suggesting that the eikonal method might be useful in discussing quark models, and for other helpful comments.

---

*Present address: CERN, Geneva, Switzerland.

[1]V. Franco, to be published.

[2]E. M. Levin and L. L. Frankfurt, Zh. Eksperim. i Theor. Fiz.—Pis'ma Redakt. 2, 105 (1965) [translation: JETP Letters 2, 65 (1965)].

[3]H. J. Lipkin and F. Scheck, Phys. Rev. Letters 16, 71 (1966).

[4]H. J. Lipkin, Phys. Rev. Letters 16, 1015 (1966).

[5]J. J. Kokkedee and L. Van Hove, Nuovo Cimento 42A, 711 (1966).

[6]J. J. Kokkedee, Phys. Letters 22, 88 (1966).

[7]G. Alexander, H. J. Lipkin, and F. Scheck, Phys. Rev. Letters 17, 412 (1966).

[8]J. J. Kokkedee and L. Van Hove, Nucl. Phys. B1, 169 (1967).

[9]C. W. Akerlof, R. H. Hieber, A. D. Krisch, K. W. Edwards, L. G. Ratner, and K. Ruddick, to be published.

[10]V. Franco and E. Coleman, Phys. Rev. Letters 17, 827 (1966).

[11]G. Morpurgo, Physics 2, 95 (1965).

[12]See Ref. 10; V. Franco and R. J. Glauber, Phys. Rev. 142, 1195 (1966), and reference given there.

[13]There may be some ambiguity as to how this appropriate momentum is determined. Fortunately this question is of no importance for the purposes of this paper.

[14]W. Thirring, in Elementary Particle Theories, edi-

ted by Paul Urban (Springer-Verlag, Berlin, Germany, 1966).

[15]K. J. Foley, R. S. Gilmore, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. H. Willen, R. Yamada, and L. C. L. Yuan, Phys. Rev. Letters 14, 74, 862 (1965); K. J. Foley, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. Platner, C. A. Quarles, and E. H. Willen, Bull. Am. Phys. Soc. 12, 103 (1967).

[16]See, for example, Ref. 9 and further references given there.

[17]Certain of these have recently been criticized by V. Barger and L. Durand, III, Phys. Rev. 156, 1525 (1967), but the discrepancies these authors point out may in some cases be explained by nonadditive effects. See Ref. 1 and Ref. 8 in this connection.

[18]C. T. Coffin, N. Dikmen, L. Ettlinger, D. Meyer, A. Saulys, K. Terwilliger and D. Williams, in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, California, 1966 (University of California Press, Berkeley, 1967).

SATURATION OF SUPERCONVERGENCE RELATIONS AND CURRENT-ALGEBRA SUM RULES FOR FORWARD AMPLITUDES*

Frederick J. Gilman
California Institute of Technology, Pasadena, California

and

Haim Harari†
Stanford Linear Accelerator Center, Stanford University, Stanford, California

(Received 3 April 1967)

We use the algebra of charges and their time derivatives, partially conserved axial-vector currents, and Regge high-energy behavior to derive sum rules for strong-interaction forward amplitudes. The saturation of all sum rules by a finite number of states is self-consistent and leads to relations among coupling constants and masses. Saturating all sum rules for π-ρ scattering by π, ω, and A_1, we predict $m_{A_1}=1100$ MeV, $m_\omega=m_\rho$, $\Gamma_{A_1}=120$ MeV, $g_{\omega\rho\pi}=21\ \mathrm{BeV}^{-1}$, in good agreement with experiment.

A new set of strong-interaction sum rules has recently been proposed by De Alfaro, Fubini, Rossetti, and Furlan[1] who noticed that the high-energy behavior of certain amplitudes may lead to superconvergent dispersion relations of the form[2]

$$\int \mathrm{Im}A(s,t)ds=0. \tag{1}$$

Other sum rules for strong amplitudes have been previously derived by writing unsubtracted dispersion relations for amplitudes satisfying low-energy theorems based on the algebra of currents and partially conserved axial-vector currents (PCAC).[3] The complete set of all sum rules obtained in this way for a given scattering process represents a significant amount of new dynamical information. In particular, if the sum rules are approximately saturated by the contributions of a small number of s-channel resonances, they lead to sets of equations in the masses and coupling constants of the involved particles.[4]

In this paper we demonstrate the use of such a set of equations for the particular case of π-ρ scattering and show that the complete set of $t=0$ π-ρ sum rules leads to a determination of the masses and coupling constants of the ω and A_1 mesons in good agreement with experiment.

We use the following set of assumptions:

(1) The vector and axial-vector charges Q^i and $Q_5^{\,i}$ $(i=1,2,3)$ obey the equal-time commutation relations of the chiral $\mathrm{SU}(2)\otimes\mathrm{SU}(2)$ algebra.[5]

(2) The time derivatives of $Q_5^{\,i}(t)$ satisfy

$$[D^i(t),Q_5^{\,j}(t)]=\delta_{ij}S(t), \tag{2}$$

where $D^i(t)=(d/dt)Q_5^{\,i}(t)=-i[Q_5^{\,i},H]$. $S(t)$ does not include an $I=2$ piece and is therefore a pure isoscalar.[6]

(3) The matrix elements of the divergence of the axial-vector current are dominated by the pion pole (PCAC).

(4) The high-energy behavior for all isospin and helicity amplitudes for π-x scattering at $t=0$ is given by the Regge-theory expression $s^{\alpha_I(0)-|\Delta h|}$, where $\alpha_I(0)$ is the $t=0$ intercept of the leading meson trajectory with isospin I and Δh is the difference between the t-chan-