J. D. Knight, and W. T. Leland, ibid. 22, 162 (1966); Bull. Am. Phys. Soc. 11, 81 (1966).

[5] R. W. Zurmühle, C. M. Fou, and L. W. Swenson, Nucl. Phys. 80, 259 (1966).

[6] R. Sherr, A. G. Blair, and D. D. Armstrong, Phys. Letters 20, 392 (1966).

[7] K. J. van Oostrum, R. Hofstadter, G. K. Nöldeke, M. R. Yearian, B. C. Clark, R. Herman, and D. G. Ravenhall, Phys. Rev. Letters 16, 528 (1966). Charge distributions for the other Ca isotopes were obtained by interpolation or extrapolation.

[8] We are indebted to E. Auerbach, whose ABACUS II computer program was used in these calculations.

[9] The calculation was similar to that carried out by K. W. Jones, J. P. Schiffer, L. L. Lee, Jr., A. Marinov, and J. L. Lerner [Phys. Rev. 145, 894 (1966)] for some of the odd-A Ca isotopes, except that a Coulomb spin-orbit term in ΔE_C of 89 keV for $1f_{7/2}$ and 27 keV for $2p_{3/2}$ was included.

[10] The method of calculation for the continuum case automatically takes into account the Thomas-Ehrman shift, which is a property of calculations involving a boundary. It is clear from Fig. 2 that, contrary to the statement of Ref. 2, the Thomas-Ehrman shift does not account for the anomaly in the $2p_{3/2}$ Coulomb energies; it is not simply a matter of a state being bound or unbound.

[11] C. Bloch and J. P. Schiffer, Phys. Letters 12, 22 (1964).

[12] F. G. Perey and J. P. Schiffer, Phys. Rev. Letters 17, 324 (1966).

[13] D. H. Wilkinson and W. D. Hay, Phys. Letters 21, 80 (1966); D. H. Wilkinson and M. E. Mafethe, Nucl. Phys. 85, 97 (1966).

AVERAGE WIDTHS FOR HIGH-ENERGY RADIATIVE TRANSITIONS*

L. M. Bollinger and G. E. Thomas
Argonne National Laboratory, Argonne, Illinois

(Received 19 April 1967)

Although the partial radiation widths of the high-energy states excited by neutron capture have been studied for many years, little is known about the average widths of transitions from many initial states to individual final states. The reason for this dearth of information is that the broad Porter-Thomas distribution[1] that governs the widths of transitions from individual initial states prevents the thermal-neutron-capture γ rays from giving useful averages, and it is not technically feasible to measure the spectra of enough individual neutron resonances to deduce average widths with the required accuracy. We have bypassed these difficulties by directly measuring the average γ-ray spectrum formed when neutrons that are distributed in a relatively broad band of energy are captured in many resonances. In this paper we describe the application of this new method to a study of the dependence of the radiation width on γ-ray energy.

Basically, the method of measurement consists of observing, with a Ge-diode γ-ray spectrometer, the capture γ rays emitted by a B^{10}-surrounded target that is placed in a high-flux region of a nuclear reactor. The boron absorber selectively removes low-energy neutrons and the $1/E$ spectrum of the incident neutron flux assures a low intensity of energetic neutrons. The combination limits the energies of the neutrons absorbed in the sample to a band that is broad enough to contain many resonances but narrow enough to preserve the excellent resolution of the Ge diode. A key concept of the measurement is that, even though there is a great variability in the magnitude of the contributions by the various resonances, a meaningful average is obtained if one has contributions from enough resonances.

The experiment outlined above is feasible only because of the high sensitivity of the internal-target facility[2] in the reactor CP-5. Here, the target is mounted in the high-flux region of a tube that passes straight through the reactor tangent to the core. Both thermal neutrons and a $1/E$ spectrum of epithermal neutrons impinge on the target from all directions. Capture γ rays from the target are viewed by a Ge-diode detector located outside the reactor, about 5 m from the target. A carefully designed collimation system ensures that the main source of radiation viewed by the detector is the target of interest.

In our measurements, the thermal flux at the target was 3×10^{13} sec^{-1} and the epithermal flux was roughly $(18/E)\times10^{11}$ eV^{-1} cm^{-2} sec^{-1}. The solid angle viewed by the detector was about 2×10^{-8} of the unit sphere. The detector had an active volume of 4 cm^3 and gave a resolution width of about 8 keV at 7 MeV.

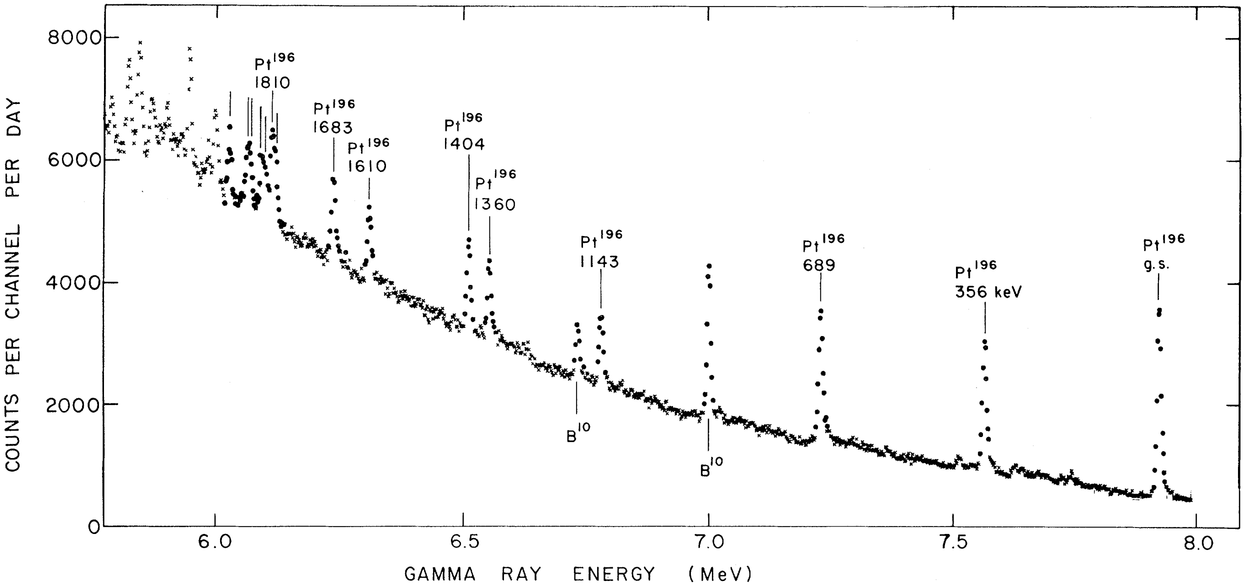

FIG. 1. Average spectrum for the reaction $Pt^{195}(n,\gamma)Pt^{196}$. The energies associated with the peaks are the energies of final states fed by the high-energy transitions in Pt^{196}.

The final collimator allowed about 0.6 cm^2 of the detector to be illuminated by the source.

The validity of the method outlined above was checked by measuring the average spectrum of $Pt^{195}(n,\gamma)Pt^{196}$ and comparing the resulting average widths for high-energy transitions with the values obtained by Jackson et al.[3] from an observation of 22 individual resonances of Pt^{195}. In this comparison the B^{10} absorber was made relatively thin (~0.30 g/cm^2) so that the resonances that contribute most to the average spectrum are the same as the ones studied by Jackson et al.[3] The platinum target was a disk 1 in. in diameter and $\frac{1}{8}$ in. thick.

The average spectrum of $Pt^{195}(n,\gamma)Pt^{196}$ for the thin B^{10} absorber is given in Fig. 1. Here we see γ-ray lines of roughly equal intensity for transitions to all the low-energy states in Pt^{196} that have been observed in the study of individual resonances. In addition, there are prominent lines from capture in B^{10}. At γ-ray energies less than 6.1 MeV the spectrum is too complex to be interpreted quantitatively.

In addition to the excellent quality of the data, two features of the spectrum in Fig. 1 are noteworthy. First, the plantinum lines have a distinct tail on the high-energy side, an effect that is not present in the boron lines. This asymmetry is even more pronounced for the platinum lines obtained with a thicker B^{10} absorber. Presumably the tail results from capture of neutrons in the keV region and its presence is definite evidence that the lines result from primary transitions.

The second interesting feature is the presence of a strong line at 6560 keV. Although this line was not seen in the previously reported measurements of resonance capture, it appears to be associated with a transition to a state in Pt^{196} at 1360 keV. The existence of such a state has recently been reported by Jansen and Pauw.[4]

The average widths obtained in our measurement are compared with those of Jackson et al.[3] in Table I. Except for the problem with the 6560-keV line, the agreement is good in view of the fact that the contributions from the various resonances are weighted in entirely different ways in the two experiments. On the strength of this agreement and the fact that everything about our data is consistent with what is expected, we conclude that our method is basically sound.

The emphasis in the initial applications of the new method of measurement has been on the study of the widths of high-energy radiative transitions as a function of the γ-ray energy. The first reaction studied has been $Pt^{195}(n,\gamma)Pt^{196}$. The measurement to determine

Table I. Average intensities measured with a thin (0.30 g/cm^2) B^{10} absorber. The data are normalized at the ground-state transition. Most of the energies listed are those reported by Jackson et al.[3]

γ-ray energy (MeV)	Final-state energy (keV)	Relative intensity Jackson et al.	Relative intensity This experiment[a]
7.920	0	2.0	2.0
7.564	356	1.2	1.25
7.231	689	1.4	1.24
6.777	1143	0.7	0.75
6.560	1360	•••	0.70
6.516	1404	1.1	0.79
6.310	1610	•••	0.66
6.237	1683	•••	0.77

[a] All values ±20%.

the energy dependence is like the one that led to the spectrum in Fig. 1, except that it is made in such a way as to allow a greater number of resonances to contribute signicantly to the average spectrum. This is done by surrounding the platinum sample with a thicker B^{10} absorber, which has the effect of diminishing the contribution of low-energy resonances and hence expanding the effective energy range in proportion to the square of the absorber thickness.

The widths obtained from a measurement with a thick (0.60 g/cm^2) B^{10} absorber are given in Table II and are presented in Fig. 2 as a plot of $\Gamma_{\gamma j}/E_\gamma^3$ vs E_γ, where $\Gamma_{\gamma j}$ is the average radiation width for transitions to a final state j and E_γ is the γ-ray energy. One sees that the data appear to be inconsistent with the E_γ^3 energy dependence that is expected from the single-particle model[5]; rather, the widths seem to vary approximately as E_γ^5.

Table II. Average intensities measured with a thick (0.60 g/cm^2) B^{10} absorber. The quantities Γ, Γ/E_γ^3, and Γ/E_γ^5 are in arbitrary units and the uncertainty in each quantity is about ±11%.

Final state (keV)	Γ	Γ/E_γ^3	Γ/E_γ^5
0	1000	1000	1000
356	707	811	890
689	696	915	1099
1143	393	629	860
1360	378	665	972
1404	354	635	937
1610	331	654	1029
1683	326	668	1072

Before the E_γ^5 dependence of the widths can be accepted as significant, the uncertainties in the widths must be evaluated. Statistical errors associated with the number of counts are only a few percent and may be neglected. The uncertainty in the energy dependence of the detector efficiency is also small, since the relative efficiency was accurately determined from the relative intensities of well-understood γ-ray cascades in Be^{10}, B^{11}, C^{13}, and N^{15}, as described in detail in Ref. 2. The main error results from Porter-Thomas fluctuations in the radiation and neutron widths of individual resonances. In principle, the magnitude of this uncertainty in the average widths can be calculated from the known conditions of the experiment, but we consider this calculation to be too complicated to be reliable. Therefore, we have used an approach in which the key information is obtained from the average spectrum itself. This information is the pulse-height spread or width w (full width at half-maximum in energy units) of the observed platinum lines (which are broadened by neutron capture over a range of energy), as compared to the width w_γ of the unbroadened line from capture in boron. Then the width w_n of the spectrum of captured neutrons is given by $w_n^2 \approx w^2 - w_\gamma^2$. Moreover, approximate calculations show that the rms uncertainty in the intensity I_γ of a line from resonance capture is given

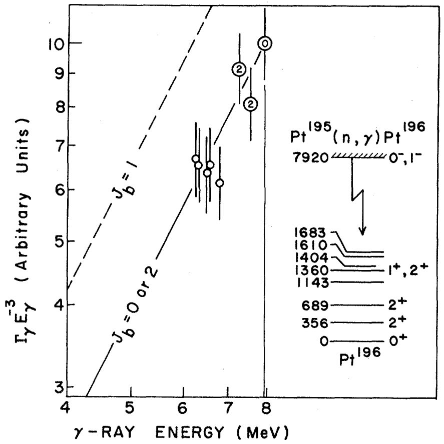

FIG. 2. Energy dependence of widths of high-energy transitions in Pt^{196}. The decay scheme on the right summarizes the published information about low-energy states in Pt^{196} that are fed by strong neutron-capture γ rays—i.e., states with $J = 0$, 1, or 2.

by

$$\frac{\Delta I_\gamma}{I_\gamma} \approx \left(\frac{2+\mu}{w_n/D}\right)^{1/2}, \tag{1}$$

where D is the spacing of resonances that form the average spectrum. The quantity μ, which satisfies the relation $0<\mu<2$, is associated with the fluctuation of neutron widths. Numerical calculations show that the accuracy of Eq. (1) is quite insensitive to the shape of the spectrum of captured neutrons.

For the platinum data $\mu \approx 1$, $D \approx 25$ eV, and $w_n \approx 6.5$ keV. Hence the rms uncertainty in the measured intensity is about ±11%. This value is believed to be quite reliable and is in satisfactory agreement with the ±9% rms deviation of the data points from the solid line in Fig. 2.

A second major source of uncertainty about the significance of the data is our ignorance about the spins and parities of some of the final states. What is known about the states from other measurements is summarized in Fig. 2. The γ-ray spectrum surely is formed almost entirely by s-wave neutron capture. Thus, since the spins and parities J^π of the first three states of Pt^{196} are known to be 0^+, 2^+, and 2^+, the three transitions of highest energy are surely $E1$ in character. Moreover, in view of the smooth E_γ dependence of the average width the other transitions must also be $E1$—since other multipolarities would be expected to be an order of magnitude weaker. This expectation implies that all of the low-energy states involved must have positive parity and $J=0$, 1, or 2. The results of Jackson et al.[3] suggest strongly but do not prove absolutely that none of the states have $J=1$.

The data of Fig. 2 may also be used to show that none of the final states have $J=1$. In spite of multiple scattering of neutrons in the sample, it can be shown that the intensity I_{ab} of transitions from initial state a with spin J_a to a final state b is given approximately by the relationship

$$I_{ab} \propto \frac{\overline{\Gamma}_{ab}}{D_a}(\overline{\sigma}_0)^m \propto \frac{\overline{\Gamma}_{ab}}{D_a}\left(g_a \frac{\overline{\Gamma}_n}{\overline{\Gamma}}\right)^m, \tag{2}$$

where σ_0 is the peak cross section of a resonance, Γ_n is the neutron width, and Γ is the total width. The statistical factor g_a is $\frac{1}{2}(2J_a+1)(2I+1)^{-1}$, where I is the spin of the target nucleus. The exponent m satisfies the relationship $\frac{1}{2}<m<1$ and its value is roughly 0.6 for this experiment. Thus, on the assumption that $\overline{\Gamma}_\gamma/D_a$, $\overline{\Gamma}_n/D_a$, and the total radiation width are all independent of J_a, one sees from Eq. (2) that the transitions $1^- \to 0^+$, $1^- \to 1^+$, and $1^- \to 2^+$ are of equal strength, and that the transition $0^- \to 1^+$ is roughly 0.65 ± 0.2 times as strong. As a result, since a 1^+ final state can be populated by both the 0^- and 1^- initial states that are formed by s-wave capture in Pt^{195}, in the average spectrum the intensity of transitions to the 1^+ final state is expected to be about 1.65 times the intensity of transitions to 0^+ or 2^+ final states, and the data points would be expected to fall along the two lines drawn in Fig. 2. Unfortunately, the platinum data do not provide a good test of this expectation, since all of the states known to have $J=1$ are at too high an energy to be resolved. Nevertheless, since all of the points cluster along one line and since the reduced width of the poorly resolved transition to the 1810-keV state with $J=1$ appears to lie well above the line for $J=0$ or 2, the data provide fairly convincing evidence that none of the final states of interest have $J=1$.

The arguments outlined above show that the transitions listed in Table II are almost certainly $E1$ transitions from 1^- initial states to either 0^+ or 2^+ final states and that the width of each of these transitions is uncertain by about 11%. If the widths are assumed to vary as $E_\gamma{}^\alpha$, where α is a constant, then a least-squares fit to the data gives the value $\alpha = 4.9 \pm 0.5$ where the uncertainty in α results from the 11% error in the individual widths. Thus, the data are almost surely inconsistent with the $E_\gamma{}^3$ energy dependence predicted by the single-particle model.

There is no adequate, theoretically based explanation of the $E_\gamma{}^5$ dependence of the experimental widths, although an idea introduced by Brink[6] and developed by Axel[7] may be relevant. These authors postulate that the high-energy transitions following neutron capture result from the same physical processes as are responsible for the dipole giant resonance and that the energy dependence of the transitions may be inferred by extrapolating to low energies the shape of the dipole resonance. Axel has considered this idea in detail and shown that the classical Lorentzian shape (which fits the central part of the giant resonance) leads approximately to an E_γ^5 dependence of the widths for transitions to the ground state when E_γ is in the neighborhood of 7 MeV.

Unfortunately, the energy dependence inferred from the giant resonance refers to ground-state transitions from a variable initial state, whereas our data consist of transitions from a fixed state to a variable final state. The (p,γ) measurements of Allas et al.[8] suggest why the data nevertheless conform to the E_γ^5 dependence. For very light nuclides such as C^{12}, these authors find that the giant resonances built on excited states are similar to the resonances built on the ground state, except that the resonance curve is shifted towards higher energy by an amount that is equal to the energy of the excited state. If this behavior is assumed to exist for heavy nuclides also, then it is easily shown that the energy dependence of radiation widths depends only on the γ-ray energy, independent of whether the energy of the initial state or the final state is varied.

It is still too early to know whether the high-energy radiative transitions in most heavy nuclides are related in a simple way to the shape of the main part of the giant resonance. However, it is clear that the average-spectrum method of measurement is capable of providing the answer.

---

*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] C. E. Porter and R. G. Thomas, Phys. Rev. 104, 483 (1956).

[2] G. E. Thomas, D. E. Blatchley, and L. M. Bollinger, to be published.

[3] H. E. Jackson, J. Julien, C. Samour, A. Bloch, C. Lopata, J. Morgenstern, H. Mann, and G. E. Thomas, Phys. Rev. Letters 17, 656 (1966).

[4] J. F. W. Jansen and H. Pauw, Nucl. Phys. A94, 235 (1967).

[5] J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics (John Wiley & Sons, Inc., New York, 1952), p. 639.

[6] D. M. Brink, thesis, Oxford University, 1955 (unpublished).

[7] P. Axel, Phys. Rev. 126, 671 (1962).

[8] R. G. Allas, S. S. Hanna, L. Meyer-Schützmeister, and R. E. Segel, Nucl. Phys. 58, 122 (1964).

DOUBLE-SCATTERING CONTRIBUTIONS TO THE PROTON-PROTON DIFFERENTIAL CROSS SECTION IN THE QUARK MODEL

David R. Harrington and Antonio Pagnamenta*
Department of Physics, Rutgers, The State University, New Brunswick, New Jersey

(Received 15 May 1967)

In a recent paper[1] Franco has pointed out that double-scattering effects in the quark model[2-8] may produce appreciable corrections to total cross sections. In this note we should like to point out that the same double scattering provides a natural explanation for the sharp break[9] in the high-energy proton-proton differential cross section at a momentum transfer square of about 0.6 $(\mathrm{GeV}/c)^2$. Provided the quark model, or any other composite model, is accepted, the possibility of such an explanation is strongly suggested by the recent work of Franco and Coleman,[10] who show that a similar structure in proton-deuteron scattering can be explained as a double-scattering effect.

In this paper we shall try to make semiquantitative estimates of the double-scattering contribution to the proton-proton differential cross section, assuming a quark-model description of the protons. We consider the elastic scattering of two high-energy protons in the center-of-mass system,

$$p(\vec{q})+p(-\vec{q})\to p(\vec{q}+\vec{\Delta})+p(-\vec{q}-\vec{\Delta}).$$

We shall assume that each proton is composed of three quarks at positions $\vec{r}_1$, $\vec{r}_2$, and $\vec{r}_3$ relative to its center of mass (thus $\vec{r}_1+\vec{r}_2+\vec{r}_3=0$), and may be described by a nonrelativistic, completely antisymmetric spatial wave function $\psi(\vec{r}_1,\vec{r}_2,\vec{r}_3)$.[11] For convenience we normalize $F(\Delta)$, the proton-proton scattering amplitude, so that

$$d\sigma/d\Delta^2=\pi\,|F(\Delta)|^2 \tag{1}$$

and

$$\sigma_{\mathrm{tot}}=4\pi\,\mathrm{Im}F(0). \tag{2}$$

Then, assuming that quark-quark scattering is spin and isospin independent, the usual approximations associated with the eikonal meth-