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SCATTERING OF MICROWAVES FROM PLASMA SPACE-CHARGE WAVES NEAR THE HARMONICS OF THE ELECTRON GYROFREQUENCY*

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An instability near the harmonics of the electron gyrofrequency has been observed by the method of combination scattering. The instability has been stimulated by a strong external microwave signal propagating along the magnetic field near the cyclotron resonance.

The analysis of plasma fluctuations and radiations near the electron-cyclotron harmonics has been undertaken by many investigators.[1] An observation of plasma waves using the method of combination[2] scattering of a weak microwave signal permits the effects which cause the fluctuation to be clearly separated from the method of detection. Using this approach, we have observed a plasma instability near the harmonics of the electron gyrofrequency which is stimulated by the presence of another, strong, microwave signal.

The experiment was performed in a very low-pressure argon gas ionized by an electron beam [Fig. 1(a)]. The plasma interacts with an external microwave field in a coaxial cylinder which is below cutoff in the circular waveguide section for all frequencies involved with the exception of frequencies very close (~100 Mc/sec) to the electron-cyclotron frequency (ω_c). The average plasma density was obtained by measuring the onset frequency of radiation coupling from the circular waveguide section to the dipole antenna as the frequency approached the cyclotron resonance.[3] The plasma profile and relative density changes were observed by a small movable double Langmuir probe. A sweep generator provided a microwave signal having frequency $\omega_x \sim \omega_c \sim 2\pi \times 3$ Gc/sec; a fraction $\sim 10^{-3}$ of this was modulated at 7 kc/sec and then coupled with the unmodulated part at the coaxial waveguide. The power level ($\lesssim$100 mW) of the unmodulated portion could be separately controlled relative to the modulated signal. A radiometer detected the signal converted by the plasma to a frequency $\omega_r = \omega_x + \omega$ (ω = space charge wave frequency, $\omega_r = 2\pi \times 9.2$ Gc/sec) which is reflected from the below-cutoff circular waveguide section; the radiometer was phase locked to the modulated part of the incident signal. To obtain the spectrum of the electrostatic waves, ω_x was swept slowly over a given range near the cyclotron frequency in order to enhance the microwave interaction with the plasma.[4] In this manner the demodulated signal provided by the radiometer is insensitive to the plasma thermal radiation and is affected by the unmodulated part of the incident wave only to the extent that

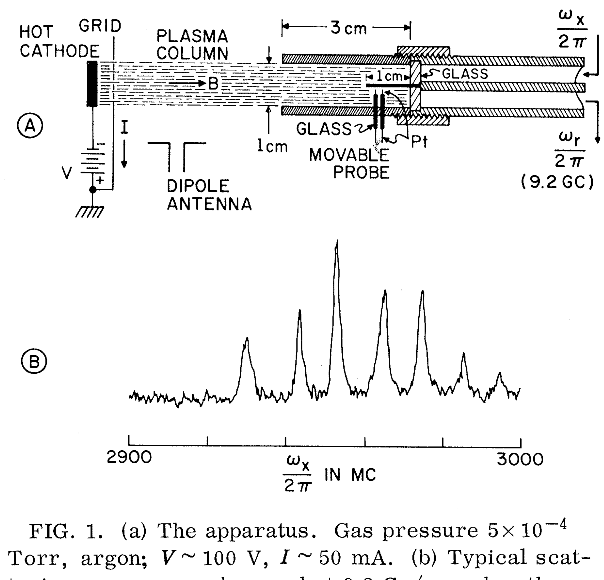

FIG. 1. (a) The apparatus. Gas pressure 5×10^{-4} Torr, argon; $V \sim 100$ V, $I \sim 50$ mA. (b) Typical scattering resonances observed at 9.2 Gc/sec when the transmitter signal is swept. Magnetic field is $\omega_c/2\pi = 3.12$ Gc/sec; space-charge wave frequencies are $>2\omega_x'$, $\omega_x = \omega_x'$.

the latter will stimulate space charge waves in the plasma.

Resonances in the scattered spectrum [Fig. 1(b)] were observed in the vicinity of the second harmonic of the incident signal. The amplitude of the resonances is sensitive to the electron-cyclotron frequency ω_c, but the frequency ω of the waves is related only to ω_x. When the unmodulated incident wave is provided by a separate source (ω_x'), it was found that as ω_x' varied, the ratio ω/ω_x' remained constant. As ω_p^2 was increased, resonances both above and below $2\omega_x'$ move further from $2\omega_x'$. The amplitude of the scattered signal depended linearly upon the intensity of the modulated wave (ω_x) and approximately linearly upon the intensity of the more intense unmodulated wave (ω_x'). Similar effects were observed for scattering from waves near the first and third harmonic as well. The frequencies ω_x and ω_x' were always near ω_c.

The applied electromagnetic field propagates parallel to the dc magnetic field and causes an ordered right-hand component of the electron motion at frequency ω_x' having magnitude $v_\perp \sim eE/2m(\omega_x'-\omega_c)$. This is the only important motion for the cold plasma. It has been shown[5] that electrostatic waves will result from the interaction of the finite electron orbits with a self-consistent electrostatic field in a plasma. For the above excitation, the distribution of electron velocities assumes a δ-function form,[6] and one finds that waves will propagate at 90° across B according to the following dispersion equation:

$$0=1-2\gamma\sum_{n=1}^{\infty}\frac{n^2}{q^2-n^2}\frac{1}{\mu}\frac{d[J_n^{\,2}(\mu)]}{d\mu};$$

$$q=\omega/\omega_x', \quad \mu=k_\perp V_\perp/\omega_x', \quad \gamma=\omega_p^{\,2}/\omega_x'^{2},$$

where $k_\perp$ = wave vector of the electrostatic wave, and $V_\perp$ = transverse electron velocity. The solution of this equation predicts allowed frequency bands in the vicinity of $n\omega_x'$. If the microwave field is absent but the electron distribution is still described by a δ function, a similar equation[7] is obtained having solutions near $n\omega_c$.

In Fig. 2 the equation has been fitted to the data of the second harmonic with the aid of the adjustable parameter μ. Adjacent resonances correspond to adjacent discrete values of k, probably caused by the effects of the finite plasma diameter.[8] The fact that $|\omega-2\omega_x'|$ increases as ω_p^2 increases for resonances below and above the harmonic rules out the possibility of a Maxwellian electron-velocity distribution here.[8]

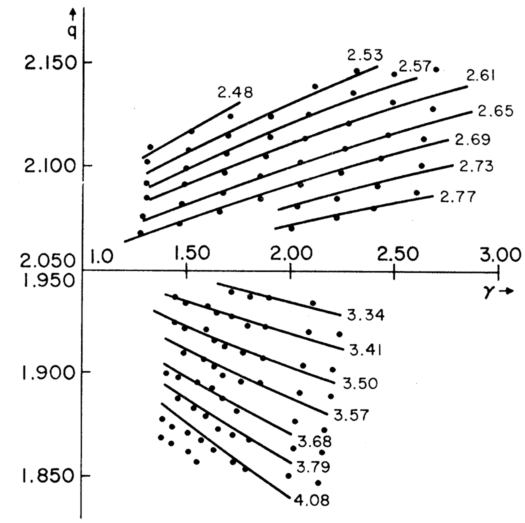

FIG. 2. Experimental data are solid points representing the peaks of the scattering resonances obtained from data such as shown in Fig. 1(b). The numerical parameter (μ) associated with each sequence of resonances is adjusted such that the solution of Eq. (1) will match the experimental data. Data for $\omega > 2\omega_x'$ and for $\omega < 2\omega_x'$ correspond to two separate runs.

It is likely that the above space-charge waves result from an instability. Harris[7,9] has shown theoretically that an instability will result for a δ-function distribution when $\mu > 2.4$, which we have observed (Fig. 2).

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[6]Under the above experimental conditions, if the distribution function for the transverse velocity assumes the form $\delta(\vec{V}-\vec{V}_\perp)$, the electron's driven gyrofrequency must be ω_x'. Collisions will randomize this distribution on a time scale >1 μsec in this experiment; however, the confinement time of the plasma is itself <10 μsec. From the mode spacing of Fig. 2. we deduce that $\frac{1}{2}mV_\perp^2 \sim 10$ eV which is about four times the unperturbed mean electron thermal energy (determined by a Langmuir probe). Thus the assumption of the cold plasma model is not unreasonable.

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REFLECTION OF A PLASMA WAVE AT AN ELECTRON SHEATH*

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An electrostatic wave propagating into a region of decreasing electron density is generally taken to be absorbed with small reflection,[1] as a consequence of the onset of strong Landau damping. The knowledge of the reflection coefficient, however small, is important in the attempt to suppress convectively unstable waves by limiting the plasma size. For example, there have been a number of calculations of the critical length of a mirror machine subject to a loss-cone instability.[2-5] These calculations have yielded small reflection coefficients. However, they have omitted certain effects of electron reflection at the sheath which we will show below may, for certain density distributions, lead to a coefficient of order one.

As a basis for calculation, we will adopt the model used by Berk, Rosenbluth, and Sudan (BRS).[6] This is a one-dimensional plasma with electron density uniform for $-\infty < x < 0$ but going to 0 for $x \to +\infty$, and a plasma wave impinging from the left whose frequency is slightly above ω_p, the value of the plasma frequency for $x<0$. In the region where the plasma wave is only weakly damped, we will assume that the static potential, $-m\Phi(x)/e$, is slowly varying on the scale of the plasma wavelength. Further to the right, where the wave is heavily damped, the potential may have arbitrary variation. In this latter sense, we generalize the model of BRS in which the potential was taken to be slowly varying everywhere.

When all quantities vary only in the x direction, BRS obtain the following equation for the perturbed electric field $\mathcal{E}(x)$:

$$\mathcal{E}(x) = -\frac{\omega_p^2}{i\omega}\int_{\Phi(x)}^{\infty} dE\,\frac{\partial F}{\partial E}\left[\int_{-\infty}^{x} dx'\,\mathcal{E}(x')\exp\left(i\omega\int_{x'}^{x}\frac{dx''}{v(x'')}\right) + \int_{x}^{x_0} dx'\,\mathcal{E}(x')\exp\left(i\omega\int_{x}^{x'}\frac{dx''}{v(x'')}\right)\right.$$
$$\left. -\exp\left(2i\omega\int_{x}^{x_0}\frac{dx''}{v(x'')}\right)\int_{-\infty}^{x_0} dx'\,\mathcal{E}(x')\exp\left(i\omega\int_{x'}^{x}\frac{dx''}{v(x'')}\right)\right] \qquad (1)$$

In Eq. (1) we have set $E=\frac{1}{2}v^2+\Phi(x)$ and $v(x)=\sqrt{2}[E-\Phi(x)]^{1/2}$ and defined x_0 to be the turning point, $v(x_0)=0$. $F(E)$ is the Maxwell-Boltzmann distribution. All time-dependent quantities have been taken to vary as $\exp(-i\omega t)$ with $\mathrm{Im}\omega>0$. Whenever appropriate, it is understood that the limit $\mathrm{Im}\omega \to 0^+$ is to be taken.

Because of the assumed properties of $\Phi(x)$, we look for solutions of Eq. (1) of the form

$$\mathcal{E}(x) = \mathcal{E}_+(x)\exp(i\int_0^x k\,dx') + \mathcal{E}_-(x)\exp(-i\int_0^x k\,dx'), \qquad (2)$$

where

$$\frac{1}{k\mathcal{E}_\pm}\frac{d\mathcal{E}_\pm}{dx} \sim \frac{1}{k^2}\frac{dk}{dx} \sim \frac{1}{k\Phi}\frac{d\Phi}{dx} \equiv \delta \ll 1$$

within the region of weak damping. The local wave number $k(x)$ will be taken to be real, and such