

SPIN PRECESSION DURING INTERFEROMETRY OF FERMIONS  
AND THE PHASE FACTOR ASSOCIATED WITH ROTATIONS THROUGH  $2\pi$  RADIANS\*

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A proposed experiment which combines spin precession and two-slit interferometry of unpolarized neutrons is analyzed. The analysis elucidates the connection with a possible observation of the phase factor  $(-1)$  associated with the rotation of a half-integral spin particle through  $2\pi$  radians.

This Letter points out an interesting connection between the precession of a fermion in a magnetic field and the well-known sign reversal of the wave function of such a particle produced by the operator for rotation through  $2\pi$  radians.

To illustrate the point we consider an experiment in which a two-slit interferometer is arranged so that one of the classical paths available to the particle passes through a magnetic field (whose strength can be varied) while the other lies in a field-free region.<sup>1</sup> The Hamiltonian in the region of constant field contains a term proportional to the spin of the particle:

$$\mathcal{H}_{\text{spin}} = -\vec{\mu} \cdot \vec{H} = -(ge/2mc)\frac{\hbar}{2}\sigma_z H, \quad (1)$$

where the field  $H$  has been taken in the  $z$  direction,  $g$  is the gyromagnetic ratio,  $m$  is the mass of the particle, and  $\sigma_z$  is the diagonal spin matrix.

The results of this experiment can be most readily analyzed by treating the magnetic field as a doubly refracting medium. Fermions of one spin orientation receive an increase in momentum, while those of the opposite polarization experience a decrease. Thus there are two indices of refraction within the magnetic field. In fact, we may write for the momentum  $p$  (inside the field)

$$\frac{p_0^2}{2m} = \frac{p^2}{2m} - \frac{ge}{2mc} \frac{\hbar}{2} \sigma_z H, \quad (2)$$

where  $p_0$  is the initial momentum. The solution, for small enough  $H$ ,<sup>2</sup> may be written in the two-component form

$$p \begin{bmatrix} \Psi_+ \\ \Psi_- \end{bmatrix} = \begin{bmatrix} \left( p_0 + \frac{ge}{2mc} \frac{\hbar}{2} H \frac{m}{p_0} \right) \Psi_+ \\ \left( p_0 - \frac{ge}{2mc} \frac{\hbar}{2} H \frac{m}{p_0} \right) \Psi_- \end{bmatrix}, \quad (3)$$

where  $\Psi_+$  is the wave function of particles aligned

with the field, and  $\Psi_-$  is that of antialigned particles. Upon passing through a region of length  $L$ , the two wave functions receive different phase shifts, so that

$$\Psi(L) = \begin{bmatrix} \Psi_+(L) \\ \Psi_-(L) \end{bmatrix} = \begin{bmatrix} \exp(ip_0 L/\hbar) \exp\left\{-\frac{1}{2}i\left(\frac{geHL}{2p_0 c}\right)\right\} \\ \exp(ip_0 L/\hbar) \exp\left\{+\frac{1}{2}i\left(\frac{geHL}{2p_0 c}\right)\right\} \end{bmatrix} \\ = \begin{bmatrix} \Psi_0 \exp\left\{-\frac{i}{2}\left(\frac{geHL}{2p_0 c}\right)\right\} \\ \Psi_0 \exp\left\{+\frac{i}{2}\left(\frac{geHL}{2p_0 c}\right)\right\} \end{bmatrix}, \quad (4)$$

where  $\Psi_0 = \exp(ip_0 L/\hbar)$  is the wave function without the magnetic field. There is a value of  $H$  which produces a sign change of both components of the wave function. Thus, in the limit of infinitesimal  $H$ , the interference pattern will shift one-half fringe (corresponding to this reversal in sign of the wave function for the path through the magnetic field), when

$$geHL/2cp_0 = 2\pi \text{ or } H = 4\pi cp_0/geL. \quad (5)$$

In the case of a particle with integral spin, there is no such sign change.

Classically, a particle with spin  $\frac{1}{2}\hbar$  precesses in the magnetic field at the Larmor frequency,

$$\omega_L = 2\mu H/\hbar = geH/2mc, \quad (6)$$

for a total time

$$t = mL/p_0. \quad (7)$$

Thus the precession angle is  $geHL/2cp_0$ , and the field given in Eq. (5) corresponds to a precession through an angle  $\theta = 2\pi$ . Furthermore, the spatial dependence of the wave function, i.e., the plane wave  $\Psi_0$ , is invariant under a rotation through  $2\pi$ . Thus, the sign change which takes place in the case of half-integral spin particles may be regarded as associated

with the effect of the operator of rotations through  $2\pi$ , acting on the part of the wave function describing particles whose classical path is through the magnetic field.

The fact that the relation derived in this note becomes inexact for finite (instead of infinitesimal) field strength deserves some comment; it exhibits the difficulty of associating the effect of the magnetic field with the sign change of half-integer spin particles under rotations through  $2\pi$ . The point is this: Any rotation of a particle with nonzero angular momentum must be effected by applying a torque. Such a torque would be represented by a potential energy term in the Hamiltonian. It is well known that potential energy terms produce phase shifts, and there seems to be no unambiguous way of deciding which part of the phase shift is to be attributed to the rotation and which part to potential.<sup>3</sup> The sign change in the present experiment, naturally, is deducible without reference to the behavior of wave functions under rotations.

It is interesting to note that the experiment described can actually be carried out, for recent advances in neutron interferometry<sup>4</sup> allow it to be performed. The neutron's lack of charge allows us to disregard such deleterious side effects as Lorentz forces and the Bohm-Aharonov effect. Using Eqs. (6) and (7), we find for a precession of  $4\pi$  that the product of field strength  $H$  and length of region with field  $\mathcal{L}$  is given by

$$H\mathcal{L} \approx 260/\Lambda_n, \quad (8)$$

where  $H$  is measured in gauss,  $\mathcal{L}$  in centimeters, and  $\Lambda_n$  is the neutron wavelength in angstroms. The interferometer in question utilizes neutrons whose wavelength is typically  $5 \text{ \AA}$ , hence a ferromagnetic sample of between 0.1 and 1 mm thickness (for mechanical stability) would have to support a field of 5000 to 500 G.<sup>5</sup> Moreover the entire discussion above applies to unpolarized neutrons.

The problems of alignment and of sample preparation are more crucial, since the doubly refracting medium will tend to analyze the unpolarized beam into spin components, unless the beam is incident (and emergent) along a

normal to the surface of the ferromagnet. A simple calculation, however, shows that the effect of a plane sample (one whose surfaces are parallel) is to displace the spin-up slit image with respect to that for spin-down particles by an amount proportional to the ratio of wavelength to slit width. This is therefore not a significant difficulty. Similarly, the doubly refractive splitting of the beam due to nonplanarity of the sample is negligible if the faces can be made parallel to within approximately  $10^{-4}$ . Thus, there are no insurmountable difficulties.

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<sup>1</sup>Care must be taken to arrange the apparatus so no determination of which slit the particle has passed through can be made; cf. W. H. Furry and N. F. Ramsey, Phys. Rev. **118**, 623 (1960), for similar remarks concerning the Bohm-Aharonov effect. In the present experiment, this may be accomplished by complicating the apparatus so that a relative rotation between the two beams is effected, rather than simply inserting the field in one beam.

<sup>2</sup>In this context, small enough  $H$  means  $ge\hbar H/4cp_0^2 \ll 1$ , i.e.,  $\mu H \ll p_0^2/2m$ .

<sup>3</sup>The author is indebted to N. Kroll for discussion of this point.

<sup>4</sup>H. Maier-Leibnitz and T. Springer, Z. Physik **167**, 386 (1962).

<sup>5</sup>We note that the inequality of footnote 3 is quite well satisfied for these values of the experimental parameters. After completion of this work, a paper discussing the possible observability of the phase factor of  $2\pi$  rotations for fermions has come to my attention [Y. Aharonov and L. Susskind, Phys. Rev. (to be published)].