In the equal-mass case $\beta(s)$ is also analytic with one cut. The same theorem of Boas can be used for $\beta(s)$ if it is bounded by $\exp[|s|^{\frac{1}{2}-\epsilon}]$. This tells us that $\beta(s)$ cannot fall off faster than $\exp[-\epsilon s^{1/2}\ln^{-1}s]$. We can choose to work at fixed complex z instead of fixed t. In that case one has, instead of (5′), the condition

$$\lim_{s\to\infty} |\beta(s)P_{\alpha(s)}(z)|s^{-N}=0.$$

This gives the same result as (12a) with one power of $\ln s$ less.

[1]L. Van Hove, Phys. Letters 24B, 183 (1967).

[2]Loyal Durand, III, to be published.

[3]V. Barger and D. Cline, Phys. Rev. Letters 16, 913 (1966).

[4]N. N. Khuri, Phys. Rev. 132, 914 (1963).

[5]This argument would be more rigorous if we use instead of (1) a modified Regge representation which exhibits the correct cuts in the z plane starting at $z=1+8/(s-4)$. See, for example, N. N. Khuri, Phys. Rev. 130, 429 (1963). The part of the background term used in that paper to modify the Regge-pole contribution cannot give contributions for large s that will invalidate Eq. (5) below.

[6]J. R. Taylor, Phys. Rev. 127, 2257 (1962).

[7]A. O. Barut and D. E. Zwanziger, Phys. Rev. 127, 974 (1962).

[8]R. P. Boas, Jr., Entire Functions (Academic Press, Inc., New York, 1954), theorem 6.3.6, p. 85.

[9]A. Martin, Nuovo Cimento 37, 671 (1965).

[10]This result can also be obtained at fixed t by letting $s\to+\infty$ below the cut.

[11]In fact for large s the sequence of intervals on which (11) does not hold must get narrower as $s\to\infty$ in such a way that $\int_S ds\,(s\log s)^{-1}$ converges. Here S denotes the union of these intervals.

[12]D. Freedman and J. M. Wang, Phys. Rev. 153, 1596 (1967).

NONEXISTENCE OF SELF-CONJUGATE PARTICLES WITH HALF-INTEGRAL ISOSPIN*

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Recently it has been shown by Carruthers,[1] in the framework of local field theory, that the field operators corresponding to spinless bosons of a self-conjugate multiplet with half-integral isospin (SMHI) are nonlocal, in the sense that local commutativity between fields φ and $\varphi^\dagger$ cannot be satisfied. We shall generalize this result to particles of any spin in an SMHI. The conclusion is that the requirement of local commutativity and the group structure of SU(2) do not allow us to construct spin fields of such particles. An analogous result is proved in the analytic S-matrix framework, where the requirement of isospin invariance plus the usual crossing property entail that all scattering amplitudes involving any particles of an SMHI must vanish. Interesting physical implications of this result, and generalizations to higher internal symmetry, are also discussed.

Throughout this paper, by a self-conjugate multiplet we mean an irreducible multiplet that contains the antiparticle of each particle contained in the multiplet. Consider a self-conjugate isomultiplet of spin j and isospin I. Let $a_\alpha^\dagger(\vec{p},\sigma)$ and $a_\alpha(\vec{p},\sigma)$ be the creation and annihilation operators of the free-particle multiplet with momentum $\vec{p}$ and spin component σ, where α denotes the I_z component. Isospin symmetry is expressed by

$$U^{-1}(u)a_\alpha(\vec{p},\sigma)U(u)=\sum_{\alpha'}D_{\alpha\alpha'}^{(I)}(u)a_{\alpha'}(\vec{p},\sigma), \quad (1)$$

where $U(u)$ is the unitary operator in Hilbert space that represents the SU(2) transformation u, and $D^{(I)}(u)$ is the standard irreducible representation matrix[2] with dimension $(2I+1)$. The adjoint of (1) is

$$U^{-1}(u)a_\alpha^\dagger(\vec{p},\sigma)U(u)=\sum_{\alpha'}D_{\alpha\alpha'}^{(I)*}(u)a_{\alpha'}^\dagger(\vec{p},\sigma). \quad (2)$$

Now we construct the $(2j+1)$-component field in the usual way[3]:

$$\varphi_\sigma^{\alpha}(x)=(2\pi)^{-\frac{3}{2}}\int\frac{d^3p}{(2p_0)^{1/2}}\sum_{\sigma'}[D_{\sigma\sigma'}^{(j)}[L(\vec{p})]a_\alpha(\vec{p},\sigma')e^{ip\cdot x}+\{D^{(j)}[L(\vec{p})]C^{-1}\}_{\sigma\sigma'}\{Ha^\dagger(\vec{p},\sigma')\}_\alpha e^{-ip\cdot x}], \quad (3)$$

where $D^{(j)}[\Lambda]$ is the $(j, 0)$ or $(0, j)$ irreducible representation of homogeneous Lorentz group, and $L(\vec{p})$ is a boost which takes a particle of mass m from rest to momentum $\vec{p}$; and C is a $(2j+1)\times(2j+1)$ matrix with the properties

$$C^*C=(-1)^{2j}, \quad C^\dagger C=I, \tag{4}$$

such that for any rotation R and boost $L(\vec{p})$ we have

$$D^{(j)*}[R]=CD^{(j)}[R]C^{-1} \tag{5}$$

and

$$D^{(j)*}[L(\vec{p})]=CD^{(j)}[L(-\vec{p})]C^{-1}; \tag{6}$$

$[Ha^\dagger(\vec{p},\sigma)]_\alpha$ stands for $\sum_{\alpha'}H_{\alpha\alpha'}a_{\alpha'}^\dagger(\vec{p},\sigma)$, where H is some matrix to be determined by the locality condition. We assume now, on the basis of their particle interpretation, that the a's and $a^\dagger$'s satisfy either the usual commutation ($[\;]_+$) or anticommutation ($[\;]_-$) rules[4]:

$$[a_\alpha(\vec{p},\sigma), a_{\alpha'}^\dagger(\vec{p}\,',\sigma')]_\pm=\delta_{\alpha\alpha'}\delta_{\sigma\sigma'}\delta(\vec{p}-\vec{p}\,'), \tag{7}$$

with all other (anti)commutators vanishing. It is then easy to work out the commutators or anticommutators for the field defined by (3):

$$\begin{aligned}
&[\varphi_\sigma^{\ \alpha}(x), \varphi_{\sigma'}^{\ \alpha'}(y)]_\pm \\
&=(2\pi)^{-3}\int\frac{d^3\vec{p}}{2p_0}\sum_\lambda\{D_{\sigma\lambda}(DC^{-1})_{\sigma'\lambda}H_{\alpha'\alpha}\exp[ip\cdot(x-y)]\mp(DC^{-1})_{\sigma\lambda}D_{\sigma'\lambda}H_{\alpha\alpha'}\exp[-ip\cdot(x-y)]\} \\
&=(2\pi)^{-3}C_{\sigma\sigma'}\int\frac{d^3\vec{p}}{2p_0}\{(-1)^{2j}H_{\alpha'\alpha}\exp[ip\cdot(x-y)]\mp H_{\alpha\alpha'}\exp[-ip\cdot(x-y)]\}.
\end{aligned} \tag{8}$$

It is well known[5] that such an integral will vanish for spacelike $(x-y)$ if, and only if, the coefficients of the two exponentials are equal and opposite, i.e.,

$$H_{\alpha\alpha'}=\pm(-1)^{2j}H_{\alpha'\alpha},$$

or in matrix form

$$H=\pm(-1)^{2j}\tilde{H}. \tag{9}$$

In deriving (8) we have used (4), (6), and $D^{(j)}[L(\vec{p})]=D^{(j)\dagger}[L(\vec{p})]$. Similarly we obtain

$$\begin{aligned}
&[\varphi_\sigma^{\ \alpha}(x), \varphi_{\sigma'}^{\ \alpha'\dagger}(y)]_\pm \\
&=(2\pi)^{-3}\int\frac{d^3\vec{p}}{2p_0}\sum_\lambda\{D_{\sigma\lambda}D_{\sigma'\lambda}^*\exp[ip\cdot(x-y)]\mp(DC^{-1})_{\sigma\lambda}(DC^{-1})_{\sigma'\lambda}^*(\sum_\beta H_{\alpha\beta}H_{\alpha'\beta}^*)\exp[-ip\cdot(x-y)]\} \\
&=(2\pi)^{-3}\int\frac{d^3\vec{p}}{2p_0}\{D^{(j)}[L(\vec{p})]\}_{\sigma\sigma'}^{\ 2}\{\exp[ip\cdot(x-y)]\mp(HH^\dagger)_{\alpha\alpha'}\exp[-ip\cdot(x-y)]\}.
\end{aligned} \tag{10}$$

It is shown in Ref. 3 that $\{D^{(j)}[L(\vec{p})]\}_{\sigma\sigma'}^{\ 2}$ is a homogeneous polynomial of order $2j$ in the momentum components p_μ, which we shall denote by $F_{\sigma\sigma'}(p_\mu)$. With this fact we can rewrite (10) as

$$[\varphi_\sigma^{\ \alpha}(x), \varphi_{\sigma'}^{\ \alpha'\dagger}(y)]_\pm=(2\pi)^{-3}F_{\sigma\sigma'}\left(-\frac{\partial}{\partial x^\mu}\right)\int\frac{d^3\vec{p}}{2p_0}\{\exp[ip\cdot(x-y)]\mp(-1)^{2j}(HH^\dagger)_{\alpha\alpha'}\exp[-ip\cdot(x-y)]\}. \tag{11}$$

In order that (11) vanish outside the light cone we must have

$$HH^\dagger = \pm(-1)^{2j} I. \quad (12)$$

Since $HH^\dagger = -I$ is absurd, (12) tells us that we must take the normal connection between spin and statistics. Hence we can express (9) and (12) together as

$$HH^* = HH^\dagger = I. \quad (13)$$

On the other hand, if we use (1) and (2) in (3) and require that $\varphi_\sigma^\alpha(x)$ transform under SU(2) irreducibly,

$$U^{-1}(u)\varphi_\sigma^\alpha(x)U(u) = \sum_{\alpha'} D_{\alpha\alpha'}^{(I)}(u)\varphi_\sigma^{\alpha'}(x), \quad (14)$$

we obtain

$$HD^{(I)}*(u)H^{-1} = D^{(I)}(u). \quad (15)$$

The most general H satisfying (15) is of the form[6]

$$H_{\alpha\alpha'} = \eta(-1)^{I+\alpha}\delta_{\alpha,-\alpha'}, \quad (16)$$

where η is an arbitrary complex number. But (16) implies

$$(-1)^{2I}HH^* = HH^\dagger = |\eta|^2, \quad (17)$$

which is incompatible with (13) if $2I$ is an odd integer. The proof for $2(2j+1)$-component fields and other general reducible (in spin space) spin fields follows in the same manner. Hence we conclude that, in the framework of the usual local field theory of particles, one cannot construct the free fields corresponding to the particles of any SMHI.

The foregoing discussion is based on the general framework of local field theory and on the usual connections between particles and fields. The question thus arises whether a physically more direct conclusion can be made in the S-matrix framework, which specifically avoids problems concerning the connection between particles and fields. We shall show next that if the S matrix possesses isospin symmetry and the usual crossing property, then particles of SMHI cannot exist in such a theory.

The crossing property of the S matrix, which can be derived[7] from the assumptions of (1) superposition principle, (2) unitarity, (3) connectedness structure, (4) Lorentz invariance, and (5) analyticity on mass shell, can be expressed as[8]

$$\langle K_1|M|K_2;p,a_\alpha\rangle = \lambda\langle K_1;-p,\bar{a}_{-\alpha}|M|K_2\rangle, \quad (18)$$

$$\langle K_1'|M|K_2';-p,\bar{a}_{-\alpha}\rangle = \lambda\langle K_1';p,a_\alpha|M|K_2'\rangle. \quad (19)$$

Here $\langle\cdots|M|\cdots\rangle$ represents a scattering amplitude, the K's denote arbitrary sets of particles which are not crossed, the symbol a_α designates a particle of type a in an isomultiplet with $I=a$ and $I_z=\alpha$, and $\bar{a}_{-\alpha}$ designates its antiparticle. The vector p $(-p)$ is the momentum of a_α $(\bar{a}_{-\alpha})$; and λ is a phase factor that is independent of p but depends on[8] a and α. The dependence of the crossing phase on spin is irrelevant here, and is suppressed. If we write $\lambda=\lambda(a,\alpha)$ in (18) and $\lambda=\lambda(\bar{a},-\alpha)$ in (19), thus exhibiting the dependence on the crossed particle in a definite way, then the equality of the factors λ in (18) and (19) is the statement

$$\lambda(a,\alpha) = \lambda(\bar{a},-\alpha). \quad (20)$$

Isospin symmetry is expressed as[9]

$$\langle a_\alpha\cdots b_\beta|M|c_\gamma\cdots d_\delta\rangle = \sum_{\alpha'\cdots\delta'}\langle a_{\alpha'}\cdots b_{\beta'}|M|c_{\gamma'}\cdots d_{\delta'}\rangle D_{\alpha'\alpha}^{(a)}*(u)\cdots D_{\beta'\beta}^{(b)}*(u)D_{\gamma'\gamma}^{(c)}(u)\cdots D_{\delta'\delta}^{(d)}(u). \quad (21)$$

It has been shown[8] that the most general form of $\lambda(a,\alpha)$ compatible with both (18) and (21) is

$$\lambda(a,\alpha) = \lambda_a(-1)^\alpha, \quad (22)$$

where λ_a is independent of α and can be selected arbitrarily for a given multiplet a.

Up to this point Eqs. (18) through (22) are completely general; now if we let a be an SMHI, then the multiplet labels a and $\bar{a}$ are identical in these equations, and in particular we can write (20) as

$$\lambda(a,\alpha) = \lambda(a,-\alpha). \quad (23)$$

But (23) is incompatible with (22) since the latter implies

$$\lambda(a,\alpha)=\lambda_a(-1)^{\alpha}=(-1)^{2\alpha}\lambda_a(-1)^{-\alpha}$$
$$=-\lambda(a,-\alpha). \qquad (24)$$

Thus we shall have a contradiction unless all the scattering amplitudes involving particles of SMHI vanish identically. This result is, however, physically equivalent to the statement that particles of SMHI do not exist in the theory since we can never establish a set of noninteracting particles as an isomultiplet.

In conclusion we make the following remarks:

(a) Although our S-matrix proof is based on exact symmetry, it can be used as an explanation why self-conjugate mesons with half-integral isospin[10] are absent from the family of the strongly interacting particles. The usual belief is that strongly interacting particles will obey the exact SU(2) symmetry if the electromagnetic interaction is "turned off." Now by our result above, mesons of any SMHI do not interact through pure strong interaction at all but only through electromagnetic interaction[11]; since the latter does not obey isospin symmetry even in the approximate sense, we would never recognize these mesons as members in an isomultiplet.

(b) The method presented here can be applied to higher internal symmetry as well. Whether an irreducible self-conjugate multiplet is allowed or not depends on the existence of a matrix H satisfying (13) and

$$\sum_{\beta,\gamma} H_{\alpha\beta}D_{\beta\gamma}^{(\nu)*}(g)H_{\gamma\delta}^{-1}=D_{\alpha\delta}^{(\nu)}(g), \qquad (25)$$

where $D^{(\nu)}(g)$ is the irreducible representation characterized by ν and g is any member of the symmetry group in question; now α is a set of quantum numbers which specifies a state. For example, in the case of SU(3), it is easy to show that whenever D and D^* are equivalent irreducible representations, there always exists a conjugation matrix that satisfies[12] both (13) and (25). In contrast to the case of SU(2), here all the possible irreducible self-conjugate multiplets (i.e., those with a regular hexagonal weight diagram) are allowed.

(c) Note that in the proofs presented above, parity does not enter. In a theory where parity is conserved, we have only self-conjugate bosons to consider; fermion and antifermion must have opposite parity, hence they cannot belong to the same irreducible multiplet.

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[1]P. Carruthers, Phys. Rev. Letters 18, 353 (1967).

[2]A. R. Edmonds, Angular Momentum in Quantum Mechanics (Princeton University Press, Princeton, New Jersey, 1960).

[3]See, e.g., S. Weinberg, Phys. Rev. 133, B1318 (1964).

[4]We have not assumed any connection between spin and statistics.

[5]W. Pauli, Phys. Rev. 58, 716 (1940).

[6]That H takes this form is because we use the standard phase convention. In general $H=\eta D^{(I)}\exp[\frac{1}{2}i\pi\sigma_2]$, but the relation $HH^\dagger=(-1)^{2I}HH^*=|\eta|^2$ is independent of convention (see Ref. 2).

[7]See, e.g., H. P. Stapp, University of California Lawrence Radiation Laboratory Report No. UCRL-16816, 1966 (to be published). Reference to earlier works on this subject can be found in this paper.

[8]For the derivation of these assertions, we refer to J. R. Taylor, J. Math. Phys. 7, 181 (1966).

[9]Although it is only for convenience that we require all the indices in the bra (ket) transform according to D^* (D) representation, this is possible only when D and D^* are equivalent, which is true for SU(2).

[10]By charge conservation, these mesons would have to carry half-integral charges if we assume they could interact strongly with the existing hadrons.

[11]It is tacitly assumed that the "pure" strong-interaction S-matrix possesses the crossing properties (18) and (19).

[12]J. J. De Swart, Rev. Mod. Phys. 35, 916 (1963).