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### ISOMER SHIFTS IN THE $2^+ \rightarrow 0^+$ ROTATIONAL TRANSITION OF $\text{Yb}^{170}$

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We have carried out Mössbauer measurements of isomer shifts on the 84-keV,  $2^+ \rightarrow 0^+$  rotational transition of  $\text{Yb}^{170}$  with the objective of measuring  $\delta\langle r^{-2} \rangle$ , the difference in the mean-square charge radius between the  $2^+$  and  $0^+$  ground-state rotational levels. Such measurements are very sensitive to nonadiabatic effects in rotational nuclei and have a bearing on such interesting questions as whether or not particle-particle forces in rotating nuclei are dependent upon the angular velocity.

In a previous work,<sup>1</sup> in which an isomer shift between rotational levels was measured, the estimate of the chemical factor which appears in the theoretical expression for the isomer shift was not of sufficient accuracy to draw definite conclusions about the nuclear states. In the measurements reported here we have taken advantage of the relatively regular features of rare-earth salts and intermetallic compounds to obtain a result for  $\delta\langle r^{-2} \rangle$  of sufficient accuracy to be of theoretical interest. For the first time the sign of  $\delta\langle r^{-2} \rangle$  has definitely been established.

The measurements were made at 4.2°K with a source of irradiated  $\text{TmAl}_2$ . The absorbers consisted of Yb metal, various intermetallic compounds, and salts with Yb ions of valencies 2+ and 3+. All the absorbers used showed an unsplit absorption line narrower than four times the minimum possible width ( $2\Gamma_n$ ). Previous measurements with other  $\text{Yb}^{3+}$  salts all showed complex spectra. All the absorption lines were symmetrical, so that isomer shifts could be accurately determined. Each isomer shift determination was made from a number of spectra, some of which are shown in Fig. 1. Throughout the measurements frequent checks were made on the energy calibration. For most absorbers, spectra were also taken with the energy scale expanded by a factor of 2. All the results, which are listed in Table I, are referred to the  $\text{YbAl}_2$  absorber. The errors quoted are larger than the statistical errors and take into account possible instrumental instabilities.

The main results were these:

(1) The maximum shift between 3+ and 2+ ions occurred between  $\text{YbGa}$  garnet and  $\text{YbSO}_4$  and was  $+0.63 \pm 0.10$  mm/sec.

(2) The maximum shift between Yb metal and salts containing  $\text{Yb}^{2+}$  ( $\text{YbSO}_4$ ) was  $+0.36$

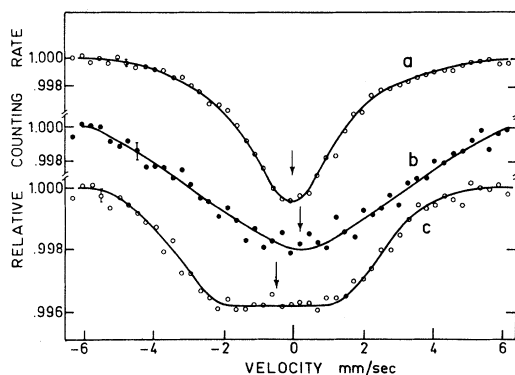


FIG. 1. The absorption of the 84-keV gamma rays of  $\text{Yb}^{170}$  at 4.2°K as a function of the relative velocity between source and absorber. (a)  $\text{YbAl}_2$  absorber. (b)  $\text{YbGa}$  garnet absorber. (c)  $\text{YbSO}_4$  absorber.

Table I. Summary of isomer-shift results.

Compound	$E$ (mm/sec)
$\text{YbCl}_2$	$-0.25 \pm 0.07$
$\text{YbSO}_4$	$-0.39 \pm 0.07$
Yb metal	$-0.03 \pm 0.07$
$\text{YbAl}_2$	0.00
$\text{YbSi}_2$	$+0.02 \pm 0.07$
$\text{YbCl}_3$ anhydrous	$+0.11 \pm 0.07$
$\text{YbGa}$ garnet	$+0.24 \pm 0.07$

$\pm 0.10$  mm/sec.

(3) The shift between all the 2+ metallic compounds was zero within the accuracy of the measurements.

We assume that the maximum shift between 3+ and 2+ ions is a well-defined quantity. Strong justification for this assumption comes from measurements of magnetic susceptibility and hyperfine interaction at Yb nuclei in compounds containing  $\text{Yb}^{2+}$  and  $\text{Yb}^{3+}$ . We attribute the smaller 3+-2+ shifts relative to  $\text{YbCl}_2$  or  $\text{YbCl}_3$  absorbers to valence impurities; that is, admixtures, for example, of  $\text{YbCl}_3$  in the  $\text{YbCl}_2$  absorber. The anhydrous chloride compounds are difficult to prepare, especially the  $\text{YbCl}_2$ .

The isomer shift to good accuracy may be expressed by the formula

$$E = (2\pi/3)Ze^2[D_a - D_b]\delta\langle r^{-2} \rangle, \quad (1)$$

where  $D_a$  and  $D_b$  are the relativistic electron densities at the nucleus in the absorbers  $a$  and  $b$ . If  $\delta\langle r^{-2} \rangle$  is due only to changes in deformation characterized by  $\Delta\beta$  and constant charge density is assumed, then the following relation holds<sup>2</sup>:

$$E = Ze^2[D_a - D_b]R_0^2\beta^2(\Delta\beta/\beta). \quad (2)$$

To estimate the atomic factor  $D_a - D_b$  we use the calibration of Brix *et al.*<sup>3</sup> According to this calibration  $[D(3+) - D(2+)]_{\text{Eu}} = (1.90 \pm 0.6) \times 10^{26} \text{ cm}^{-3}$ , where  $D(3+)$  is the electron density at an Eu nucleus in an  $\text{Eu}^{3+}$  salt and  $D(2+)$  is the corresponding quantity in an  $\text{Eu}^{2+}$  salt. To extend this result to Yb salts we assume, along with Brix *et al.*,<sup>3</sup> that the difference  $D(3+) - D(2+)$  can be attributed in all rare-earth salts to the difference in the shielding of the 5s and  $5p_{1/2}$  electrons from the nuclear charge by the different 4f shells and, moreover, that this difference is constant for all rare-earth salts. The main uncertainty in the assumption that  $D(3+) - D(2+)$  is the same for Eu and Yb is in the variation of the effective charge which in going from Eu to Yb is probably greater for 4f than for 5s electrons. If we then substitute the value of Brix *et al.* for  $[D(3+) - D(2+)]_{\text{Eu}}$  into Eq. (2) we obtain (taking<sup>4</sup>  $\beta = 0.278$ )  $\Delta\beta/\beta = (2.7 \pm 1.1) \times 10^{-3}$ . The error quoted above includes the error quoted by Brix *et al.* for  $[D(3+) - D(2+)]_{\text{Eu}}$  and the error in our isomer-shift measurements. It is difficult to estimate the error in the assumption that  $[D(3+) - D(2+)]_{\text{Yb}} = [D(3+) - D(2+)]_{\text{Eu}}$  and we do not attempt to do so.

Another way of obtaining  $\Delta\beta/\beta$  is to estimate  $D(CE)$ , the contribution of the conduction electrons to  $D$  in Eu and Yb metals, where the valence is 2+. The ratio of  $D(CE)$  to  $D(3+) - D(2+)$  has been accurately measured for europium.<sup>3</sup> If we assume that in both Eu and Yb metal the effective number of 6s conduction electrons is the same (an assumption which is supported to some degree by the near equality of the respective ionic volumes and by positron annihilation rates<sup>5</sup>), we need only consider the change in the 6s electron densities at the respective Eu and Yb nuclei. According to Shirley,<sup>6</sup> the ratio of a 6s electron charge density at Yb nuclei to the corresponding density at Eu nuclei is 1.67. Taking the result of Brix *et al.*<sup>3</sup> for  $D(CE)_{\text{Eu}} [(0.9 \pm 0.4) \times 10^{26} \text{ cm}^{-3}]$  we have  $D(CE)_{\text{Yb}} = (1.5 \pm 0.8) \times 10^{26} \text{ cm}^{-3}$ . Using our experimental results for the maximum isomer shift between Yb metal and  $\text{Yb}^{2+}$  salts we have a second result for  $\Delta\beta/\beta$  which is  $(2.0 \pm 1.3) \times 10^{-3}$ . Again the quoted error includes only the errors in  $D(CE)_{\text{Eu}}$  and experimental errors in our isomer-shift measurements.

It is difficult to precisely estimate the errors involved in the assumption  $D(CE)_{\text{Yb}} = 1.67D(CE)_{\text{Eu}}$ ; however, the fact that the spread in the two values for  $\Delta\beta/\beta$  derived above is within the experimental errors indicates that the assumptions made above are not inconsistent with each other and increase the confidence in them.

We now wish to compare the experimentally determined values of  $\Delta\beta/\beta$  with the value predicted by the centrifugal stretching model of Davydov and Chaban<sup>7</sup> as treated by Diamond, Stephens, and Swiatecki<sup>8</sup> and Ref. 1. Using the accurately measured energies for the 2+ and 4+ levels of  $\text{Yb}^{170}$  as 84.262 and 277.7 keV,<sup>9</sup> we can fit the spectra to the phenomenological expression  $E = AI(I+1) + BI^2(I+1)^2$ , by taking  $A = 14.112 \pm 0.002$  keV and  $B = -11.3 \pm 0.1$  eV.

The centrifugal stretching theory then predicts

$$\Delta\beta/\beta = -6B/A = (4.80 \pm 0.17) \times 10^{-3}.$$

We assume here that the nucleus is axially symmetric so that a nonzero  $B$  is not due to axial asymmetry. For the case of  $\text{Yb}^{170}$ , sufficient data to determine a value for  $\gamma$ , the axial asymmetry parameter, are (as far as we know) not available.

From the arguments presented previously, we thus conclude as follows:

(1)  $\delta\langle r^2 \rangle$  or  $\Delta\beta/\beta$  has for the first time been established to be greater than 0.

(2) The results tend to show that  $\Delta\beta/\beta$  is smaller than the value predicted by the theory of Davydov and Chaban, as treated by Diamond, Stephens, and Swiatecki.

With regard to conclusion (2), we note that the indicated discrepancy is not evidence for nonadiabatic effects or a need to depart from a purely rotational model. In the centrifugal stretching model, as originally introduced by Davydov and Chaban,<sup>7</sup> the authors intend  $\gamma$  to be associated with a rotational degree of freedom of an axially asymmetric nucleus. If this is done it might be possible to calculate a smaller value of  $\Delta\beta/\beta$  using purely rotational degrees of freedom.

Another calculation for  $\Delta\beta/\beta$  has been given by Faessler and Greiner.<sup>10</sup> According to them  $\delta\langle r^2 \rangle$  is very sensitive to  $E_\beta$ , the energy of the second  $0^+$  level sometimes observed in even-even nuclei. In order to bring their calculation into agreement with our result it is required that  $E_\beta \sim 6$  MeV whereas nuclear systematics and theoretical considerations suggest that  $E_\beta \sim 1$  MeV.

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## POLARIZATION MEASUREMENTS NEAR ISOBARIC ANALOGS OF STATES IN $N = 83$ ISOTONES\*

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Measurements of the polarization of protons scattered from barium and cerium have been made to determine the total angular momenta,  $J$ , of isobaric analogs of states in <sup>138</sup>Ba and in <sup>141</sup>Ce. The second  $p$ -wave state in each case has been found to have angular momentum  $\frac{1}{2}$  in contrast to previous assignments of  $J = \frac{3}{2}$  to this level.

The existence of isobaric analog states in proton elastic and inelastic scattering from <sup>138</sup>Ba and <sup>140</sup>Ce has recently been established by von Brentano et al.<sup>1</sup> Since these targets have a closed neutron shell, analog states in the compound nuclei can provide information about the configurations consisting of one nucleon outside a closed shell. We have measured the polarization in proton elastic scattering between 9 and 12 MeV for each target in order to determine the total angular momenta of five analog states in each compound nucleus. The levels occur at energies comparable with the Coulomb barrier. Therefore the polarization away from the resonances is relatively large, and it was necessary to study the polarization between the resonances as well.<sup>2</sup>

Measurements were made at 80 and 110° for each target and in addition at 145° for barium. The polarization of the scattered protons was deduced from the left-right asymmetry when the polarized proton beam of the Wisconsin tandem accelerator was scattered from thin foils of the target metals. The results are shown in Figs. 1 and 2. The error bars indicate the statistical uncertainties. The curves were calculated with a scattering matrix  $S$ , defined by<sup>3,4</sup>

$$S_l^\pm = \exp[2i(\omega_l + \lambda_l^\pm)] \left\{ \exp(-2\mu_l^\pm) + \exp(2i\phi_p) \left[ \frac{i\Gamma_p}{E_0 - E - \frac{1}{2}i\Gamma} \right] \right\},$$