SUM RULES FOR PION-HYPERON SCATTERING

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The forward sum rule for $\pi\Sigma$ scattering is shown to be inconsistent with the current-algebra scattering lengths. A further class of dispersion-theory sum rule shows that the dynamics of the $\frac{3}{2}^+$ resonances cannot be dominated by baryon exchange.

Many authors have shown that the current commutation relations,[1] together with partially conserved axial-vector currents (PCAC),[2] lead to predictions of pion scattering lengths, and in particular Weinberg[3] has derived a formula for the s-wave scattering lengths of pions on any target (except pions). The predictions for the pion-nucleon system are in excellent agreement with experiment, and the result for a_1-a_3, together with the dispersion-relation sum rule on the antisymmetric forward amplitude

$$F^{(-)}(1)=\frac{m+1}{3m}(a_1-a_3)=\frac{2f^2}{1-1/4m^2}+\frac{2}{\pi}\int_1^\infty\frac{d\nu\,\mathrm{Im}F^{(-)}(\nu)}{\nu^2-1}, \tag{1}$$

is equivalent to the Adler-Weisberger relations.[5] We shall first evaluate this relation on the basis of assumptions which can easily be used in πY scattering, to illustrate the accuracy of these assumptions in this case.

We assume that the integrals are dominated by the resonances, together with an s-wave background, which could be important in the low-energy region because of the factor $(\nu^2-1)^{-1}$. The latter is evaluated in a scattering-length approximation using the current-algebra scattering lengths. The higher resonances are evaluated in the narrow-resonance approximation, but the $\Delta(1236)$, which is large and quite near threshold, is treated more carefully using the phase shift. Evaluation of the right-hand side of (1) gives

$$\begin{aligned}&0.165\pm0.004\,[N(940)]+0.014\,(s\text{ waves})-0.093\pm0.003\,[\Delta(1236)]+0.016\pm0.005\,[N(1400)]\\&+0.009\,[N(1525)]+0.002\,[N(1570)]+0.006\,[N(1670)]-0.003\,[\Delta(1670)]\\&+0.000\,[N(1700)]+0.007\,[N(1688)]-0.007\,[\Delta(1930)]=0.116\pm0.007,\end{aligned}$$

where the errors are estimated solely from the experimental widths.[6] The result is in excellent agreement with the current algebra + PCAC prediction

$$[(m+1)/3m](a_1-a_3)=0.115. \tag{2}$$

Here we have merely reproduced the well-known result obtained using the experimental total cross sections. However, PCAC may well work less well when coupling to particles other than nucleons is involved, and we now test these in the same way. A relation analogous to (1) exists for the antisymmetric $\pi\Sigma$ forward scattering amplitude ($I=1$ exchange):

$$F^{(1)}(\nu)=\tfrac{1}{3}F_0(\nu)+\tfrac{1}{2}F_1(\nu)-\tfrac{5}{6}F_2(\nu), \tag{3}$$

where, in $F_I(\nu)$, I is the isospin in the $\pi\Sigma$ channel. The Born terms[7] are estimated using SU(3) for a wide range of f ($0<f<0.5, f+d=1$). The isospin-1 and -2 s waves are estimated using the Weinberg scattering lengths

$$a_0=0.39,\quad a_1=0.20,\quad a_2=-0.20,$$

and the $Y_0^*(1405)$ is calculated using a reduced width

$$\gamma=0.43/(1.26-0.26\nu)$$

which gives the correct width and scattering length. The $Y_1^*(1385)$ is approximated by a two-channel Breit-Wigner formula, and the other resonances are treated in a narrow width approximation. We obtain

$$0.123\pm0.018\ [\Lambda(1115)+\Sigma(1190)]\ -0.018\ (I=1,2\ s\ \text{waves})\ +0.044\pm0.015\ [\Sigma(1385)]\ +0.076\ [\Lambda(1405)\ +I=0\ s\ \text{wave}]\ +0.007\pm0.001\ [\Lambda(1520)]\ +0.006\pm0.006\ [\Sigma(1660)]\ +0.001\ [\Lambda(1820)]=0.223\pm0.024,$$

where the details of the evaluation of the resonances are relatively unimportant, since all the large terms add (cf. pion-nucleon case). The Weinberg formula gives 0.44. We note that any contribution from $I=2$ makes the agreement worse, as does any rescattering correction to the scattering lengths [e.g., from $\Lambda(1405)$]. An unobserved resonance, to account for the discrepancy, would have to be at low energies, and if at, say, 1400 MeV, give a contribution to the $\Sigma\pi$ total cross section 3 to 4 times that of the $\Lambda(1405)$. Finally, the correction from the high-energy region can be estimated,[8] and is much smaller than the discrepancy. We thus conclude that there is probably an error of about 100% in the predicted $I=1$ exchange scattering length, corresponding to an error of about 50% in the Goldberger-Treiman relation for this case. In this context, we remark that Bugg[9] has shown the importance of the $I=1$ 3π cut in $N-N$ scattering. If this coupled equally to Σ, the suppression of $g_{\Sigma\Sigma\pi}$ relative to $g_{\pi NN}$ would already increase the error in the above relation in this case.

There is also a sum rule for $\pi\Xi$ scattering. With $f=0.25$ for the pion-baryon couplings, we obtain

$$F^{(1)}(1)=0.010\pm0.020\ [\Xi(1318)]+0.013\ (s\ \text{waves})+0.032\pm0.007\ [\Xi(1530)]=0.055\pm0.021,$$

whereas the current-algebra result is

$$F^{(1)}(1)=0.11.$$

Here the situation is not so clear, but again a $T=\frac{3}{2}$ contribution would have the wrong sign, and again considerable further resonance structure would be required to bring the result into agreement.

We now use a set of dispersion-theory sum rules to show that s-wave $(\pi\pi)^0$ exchange is probably much more important in πY scattering than in πN scattering. Such a result would of course affect the current-algebra results for $T=0$ exchange (which are 0).

Lyth[10] has pointed out that a current-algebra result obtained by Furlan, Jango, and Remiddi[11] could be obtained from the unitary sum rule[12] for the p_{33} pion-nucleon partial wave

$$\frac{1}{\pi}\left\{\int_L+\int_{s_0}^{\infty}\right\}\frac{ds\ \mathrm{Im}f_{1+}^{\ 3}(s)}{q^2}=0 \qquad (4)$$

by retaining the long-range nucleon-exchange term only on the left-hand cut. That this simple dynamical situation cannot extend to the other $\frac{3}{2}^+$ resonances is clear. In the $\Xi(1530)$ case, the Born term is necessarily repulsive, so that the Ξ and Ξ^* terms add. For $Y_1^*(1385)$ in the $\pi\Lambda$ channel, the relation would require $g_{\Sigma\Lambda\pi}^{\ 2}\sim19$, which is much larger than present estimates. We note that the inclusion of coupled channels cannot solve these problems, without the addition of further forces in the left-hand cut, since they can only increase the integral over the right-hand cut. Thus at least one other term must be important, and since in the latter case rho exchange is forbidden, the obvious candidate is s-wave $(\pi\pi)^0$ exchange. The importance of this term is also suggested by the fact that the sum rule analogous to (4) for the $Y_1^*(1385)$ in $\pi+\Lambda\to\pi+\Sigma$, in which $(\pi\pi)_0$ exchange is forbidden, is well satisfied with

$$g_{\Sigma\Lambda\pi}g_{\Sigma\Sigma\pi}\sim8\pm2\ (f\sim0.37)$$

in agreement with SU(3). (The latter has been used to give the relative sign of the $Y_1^*\Lambda\pi, Y_1^*\Sigma\pi$ couplings, and the narrow-width approximation used.)

We finally note that, for $m_\Lambda=m_\Sigma$, Weinberg's argument gives the s-wave scattering length for $\pi+\Lambda\to\pi+\Sigma$ to be 0. Again, putting this in the forward sum rule, we obtain, using the above coupling and the experimental reduced widths,

$$0\sim0.084\pm0.021\ (\Sigma)\ -0.107\pm0.027\ [\Sigma(1385)]\sim-0.023\pm0.034,$$

so that the Adler-Weisberger relation is in this case roughly satisfied.

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DECAY RATES OF THE DECUPLET RESONANCES FROM O(3, 1)⊗SU(3) DYNAMICS. DEFINITE SU(3) SYMMETRY BREAKING*

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The simplest dynamical group including SU(3), O(3, 1)⊗SU(3), is used to describe the decay widths of the $j^P = \frac{3}{2}^+$ decuplet resonances. SU(3) breaking comes in naturally (and uniquely) through the mass differences in the multiplet. The only two parameters of the theory have been fixed in an independent fit of the $I = \frac{3}{2}$ baryon decay rates for different spins. The agreement with experiment is excellent.

Recently, the idea of a dynamical group governing baryon form factors has passed an impressive test. The simplest possible dynamical group, O(3, 1), has been able to describe the pionic decay rates of many resonances extremely well using only two parameters, a coupling constant G and a Casimir operator ν specifying the representation.[1]

We recall that after extension by parity and assuming the existence of an electromagnetic current operator on the representation space, the fermion spectrum consists of a tower of particles with spins $\frac{1}{2}^+, \frac{3}{2}^-, \frac{5}{2}^+, \cdots$ or $\frac{1}{2}^-, \frac{3}{2}^+, \frac{5}{2}^-, \cdots$ and their antiparticles. Neglecting isospin, the nucleon and the isospin-$\frac{1}{2}$ resonances $N^*(1525)$, $N^*(1688)$, $N^*(2190)$, $N^*(2650)$, and $N^*(3030)$ are assigned to the first representation and the isospin-$\frac{3}{2}$ resonances $\Delta(1236)$, $\Delta(1920)$, $\Delta(2420)$, $\Delta(2850)$, and $\Delta(3230)$ to the second one, both with the same Casimir operator ν. The process of a baryon decaying into a lower state under emission of a pion is then described in the following way: One uses the special frame where the initial baryon $|a', j'\rangle$ is at rest and the final one $|a, j\rangle$ moves with rapidity ζ [$\zeta = \tanh^{-1}(v/c)$] into the z direction and interprets the matrix element

$$A_m^{j'j}(\zeta) = (G/\sqrt{2})\langle a'j'm|Pe^{i\vec{M}\cdot\vec{\zeta}}|ajm\rangle \quad (1)$$

as the transition amplitude. G is a coupling constant for the whole multiplet, $\vec{M}$ denotes the Lorentz generators of O(3, 1), and P is the only pseudoscalar in the representation space, $P = \vec{L}\cdot\vec{M}/|\vec{L}\cdot\vec{M}|$. This interpretation is suggested by the observation that hydrogenic electric form factors can be written in the same form.[2-4] $A_m^{j'j}(\zeta)$ can be given in terms of global representations of O(3, 1) as

$$A_m^{j'j}(\zeta) = (G/\sqrt{2})B_m^{\pm j'j}(\zeta[\tfrac{1}{2}, \nu]) \quad (2)$$

with

$$B_m^{\pm j'j}(\zeta[\tfrac{1}{2}, \nu]) = \tfrac{1}{2}\{B_m^{\pm j'j}(\zeta[\tfrac{1}{2}, \nu]) \pm B_m^{\pm j'j}(\zeta[\tfrac{1}{2}, \nu])\}, \quad (3)$$

where ν is the only undetermined Casimir operator of O(3, 1) and + or − have to be used according to whether the parities of the ground states of the initial and final baryon multiplets are opposite or