On the other hand, linearizing Eq. (16) and assuming that $u^{(1)}$ depends on β only, we get the equation

$$u_{\beta\beta\beta}^{(1)} - \beta u_{\beta}^{(1)} = 0,$$

which is satisfied by Eq. (22) and consequently describes the long-time asymptotic behavior of the original system.

[1]N. J. Zabusky and M. D. Kruskal, Phys. Rev. Letters 15, 240 (1965).

[2]C. S. Gardner and G. K. Morikawa, Courant Institute of Mathematical Sciences Report No. NYO 9082, 1960 (unpublished).

[3]B. D. Fried and R. W. Gould, Phys. Fluid 4, 139 (1961).

[4]The transformation is not unique; for example, the transformation $\xi = \epsilon^{1/2}(x-t)$, $\eta = \epsilon^{3/2}t$, which is implied by the relation $kx-\omega t = (x-t)k + \frac{1}{2}tk^3$ for $k^2 \ll 1$, leads likewise to the Kortweg-deVries equation.

POTENTIOMETER FOR STUDYING THE LIQUID He II FILM*

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A simple level-sensing device is described for probing the profile of the chemical potential along the flowing liquid He II film. Measurements with these probes indicate that dissipation in the flowing film may occur either at highly localized regions or over extended regions of the flow path, depending upon experimental variations. Also it is shown that changes in transfer rate occurring during a given flow process are accompanied by a redistribution of the sites producing dissipation.

Studies of the flowing liquid He II film have a rather long history featured by numerous controversies over the observation and interpretation of "new" effects.[1] Unfortunately, a common ground for discussing these phenomena has not yet been developed. The present note describes a simple tool for investigating and analyzing the flow processes of the film.

During an informal conversation with Dr. B. D. Josephson concerning some unusual film-flow effects we had observed, he suggested the use of a "potentiometer" to probe the chemical potential μ_f along the path of the film.[2] The physical form that this probe might assume evolved in the discussion that followed.

Consider the flow of the film over the rim of a beaker with the liquid level z_i inside the beaker above the outside level z_0. The chemical potentials per atom of the two bulk liquids are, respectively, μ_i and μ_0; and their difference is given by

$$\mu_i - \mu_0 = mg(z_i - z_0) = \Delta\mu. \quad (1)$$

On the basis of recent developments in the theory of superfluidity[3] we assume the following:

(1) $\Delta\mu$ provides the driving force for film flow.

(2) The bulk He II inside the beaker communicates with the bulk liquid outside through the film; at every point along the flow path S_f connecting the two reservoirs of bulk liquid the chemical potential is defined and is related to the phase of the order parameter through the relationship

$$\langle\psi\rangle = f\exp(i\varphi) = f\exp(i\mu t/\hbar).$$

For a film-flow experiment in which the path S_f is $z_i \rightarrow \text{rim} \rightarrow z_0$ and in which the distance along S_f is s, μ_f will have a profile $\mu_f = \mu_f(s)$.

(3) Production of dissipation in the form of quantized vorticity occurs through phase slippage only in those regions where $d\mu_f/ds$ is different from zero and at a frequency

$$\omega = d(\Delta\varphi)/dt = \Delta\mu/\hbar.$$

(4) In any region along S_f in which the velocity of particle flow $\vec{v}_s$ is equal to or exceeds the critical velocity $\vec{v}_{s,c}$, μ_f varies spatially; in regions where $\vec{v}_s < \vec{v}_{s,c}$, μ_f remains constant, provided $\vec{v}_s$ remains constant.

An example of the type of probe we have devised to investigate $\mu_f(s)$ is shown in Fig. 1. The side tube extending from the inside wall of the beaker has a radius small compared with that of the beaker itself, so that adjustments of the liquid level z_t in the tube by film flow occur rapidly compared with changes of z_i and z_0. We propose that subject to the as-

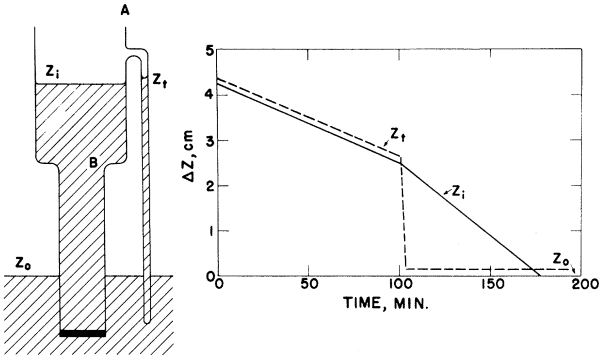

FIG. 1. Schematic drawing (left) of one of several beakers used to investigate $\mu_f(s)$, showing typical arrangement of the side tube. On the right is the time-versus-height behavior of the levels z_t and z_i for an outflow experiment.

sumptions listed above, z_t provides a measure of μ_f (relative to μ_i or μ_0) at the location of the tube orifice.[4]

In this way we have studied $\mu_f(s)$ in a number of different geometrical arrangements. These include placement of the orifices at several different heights, with some orifices opening to the inside and others to the outside surface of the beaker wall. Inflows ($z_i < z_0$) and outflows ($z_i > z_0$) have been measured for several beaker shapes. We illustrate the method by showing how some of the results already obtained may be used to provide new information relating to observations on films made by previous investigators. Additional details will be given in a future publication.

A rather universally accepted criterion for the transfer rate R of the film states that R should be determined by the smallest perimeter of the beaker lying above the highest liquid level.[1] Corollary to this is the notion that $\vec{v}_s$ should become critical at this point also. The beaker shown in Fig. 1 was designed to test these ideas further. It is made of glass with a copper bottom, the inside diameter of the upper section being 1.9 cm and that of the lower section being 1.0 cm. The side tube is 0.1 cm i.d. A glass cylinder with copper top and bottom surrounds the beaker and this in turn is protected by several radiation shields at the top.

The results of an out-flow experiment are shown on the right-hand section of Fig. 1, where the time history of the levels z_t and z_i are plotted relative to z_0. It is seen that when z_i is in the wide section, it is closely followed by z_t (slightly elevated by capillarity); but when z_i crosses into the narrow section, z_t rapidly falls to equilibrate with z_0. Using other beakers in which both the inside and outside walls of the beaker were probed, it was found that under these conditions of flow the level of liquid in the outside-wall probe tube always remained at z_0.

We interpret these observations as indicating that $\mu_f(s)$ is a step function with the step occurring at the smallest and highest perimeter above z_i. Thus in the first part of the experiment, $\vec{v}_{s,c}$ is reached at the rim A and elsewhere along the flow path $\vec{v}_s < \vec{v}_{s,c}$. Assuming a uniform diameter for the inside wall of the beaker, we expect that the decrease in film thickness with height would make it most likely that $\vec{v}_{s,c}$ is exceeded only at the inside edge of the rim A. Therefore, as shown schematically in Fig. 2(a), in the film on the path $z_i \rightarrow A$, μ_f is constant and equal to μ_i, and along $A \rightarrow z_0$, μ_f is also constant but equal to μ_0. As z_i drops below the knee B, $\vec{v}_{s,c}$ is exceeded here instead of at A. Therefore along $z_i \rightarrow B$, $\mu_f = \mu_i$, while along $B \rightarrow A \rightarrow z_0$, $\mu_f = \mu_0$. These conclusions imply that in the instances examined here the production of vorticity in the film may take place over only a very small region of the flow path, and that the rate-determining process occurs at the smallest perimeter above the highest liquid level as has been previously noted.[1]

Observations of the in-flow process, however, were different from those just described for outflow. With the probe tube opening into the inner wall of the beaker as shown in Fig. 1, or in a beaker with no step, during every inflow experiment for which $z_0 > B$, z_t remained at levels intermediate between z_i and z_0, indicating that for inflow $\mu_f(s)$ is no longer a well-defined step function. When both the inside and outside surfaces of a cylindrical beaker were probed during inflow, the same effect was observed in the probe tubes opening onto the inner wall, and in addition μ_f was found to be constant from z_0 to the rim, so that changes in μ_f were confined to the region between the rim and z_i. From these observations we infer the profile of μ_f in an in-flow experiment as diagramed in Fig. 2(b).

In considering the origin of these results, we first thought that because of the flow geometry, bulk fluid droplets might form on the inside perimeter of the beaker and subsequent-

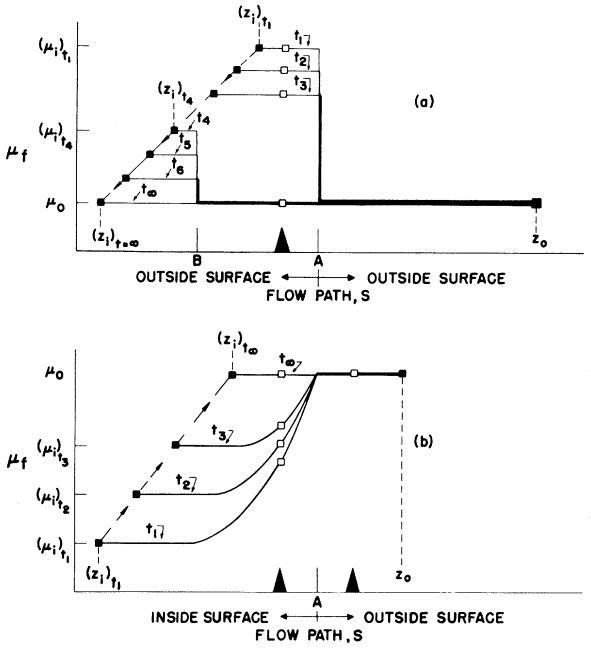

FIG. 2. Profiles of the chemical potential, μ_f, of the film, along the flow path s at several different times t_n, inferred from probes described in the text; μ_i and μ_0 are the chemical potentials of the bulk liquids inside and outside the beaker. The drawings have been idealized so that the outside level remains constant at z_0 and the inside level $(z_i)_{t_n}$ moves to equilibrate with z_0 at t_∞. Along the abscissa, s, A is the beaker rim, and the solid triangles represent the points at which μ_f is probed, with the open squares showing the probe observations and the solid squares representing the bulk-liquid levels. (a) Outflow from the beaker shown in Fig. 1, where B represents the location of the constriction in the beaker; between t_3 and t_4 (when the inner level passes the constriction B) the position of the potential drop shifts from A to B. (b) Inflow into a cylindrical beaker of constant diameter.

ly drain into the tube, giving a spuriously high value of z_t on the inside surface. To test this hypothesis, the beaker shown in Fig. 2 was constructed with three side tubes of equal radius but with differing orifices. Tube 2 has the same opening as the tube shown in Fig. 1, while tubes 1 and 3 have orifices which prevent direct drainage. During in-flow experiments with this beaker, it was found that indeed the equilibrium level was reached more rapidly in tube 2 than in the others, but that all three nevertheless settled down to the same level in a time short compared with the flow process into the beaker. Therefore, the above-mentioned effect is considered to be real and to indicate that $\vec{v}_{s,c}$ during an in-flow experiment may be exceeded and vorticity production may occur over a large portion of S_f and not just at a single point. These observations also show that the efficacy of the probe is insensitive to the geometry of the orifice.

Some outflows were also observed for the beaker shown in Fig. 3 with results typified by the schematic time-versus-height plot given in the figure. The outflow begins as expected with the three z_t's all following z_i, but then suddenly the levels in the side tubes drop to an intermediate height accompanied by an abrupt reduction in the emptying rate. The onset of the new rate sometimes occurred spontaneously, sometimes could be produced by brief (~10 sec) illumination of the region near the rim with a flashlight, and at other times could be produced only by drastic perturbations of the system. In an experiment at 1.52°K, the film flow rate R changed (via flashlight) from 9.7×10^{-5} to 8.0×10^{-5} cm^3/cm sec.

Eselson and Lasarev[5] have reported enhanced rates of the emptying of a beaker when the beaker is filled by plunging it into the bulk liquid as compared with when the beaker is allowed to fill by film flow. Recently Allen and Matheson[6] have made careful measurements of this effect and find the "normal" filling and emptying rates for glass beakers at 1.52° to be 8.0×10^{-5} and the corresponding enhanced rate to be 9.8×10^{-5} cm^3/cm sec. Since for our emptying experiments the beakers were filled by plunging, it appears likely that we are observing the same phenomena as these previous in-

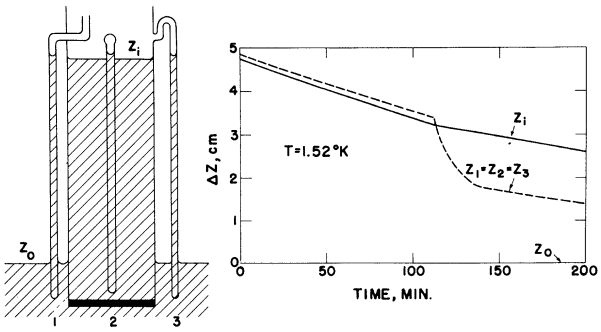

FIG. 3. Beaker (left) with three different probe orifices used for investigating drainage effects during inflow. It is shown here at the beginning of an out-flow experiment, the level history of which is given on the right; the change in rate (slope of z_i vs t) followed a 10-sec illumination of the beaker rim by a flashlight.

vestigators. Similarly, our filling experiments should correspond to the "normal" filling process.

Allen and Matheson[6] have interpreted their results by assuming that there are two types of films, "thick" films formed by the plunging procedure and "normal" films. The present experiments suggest that such an assumption may not be necessary, but rather that changes in the profile $\mu_f(s)$ and a redistribution of the vorticity production areas in the film are responsible for switching the rates. This viewpoint is consistent with recent observations made by us on the isothermal flow of He II through narrow slits.[7] When the liquid was driven by a gravitational head such that throughout the experiment $\vec{v}_S > \vec{v}_{S,c}$, the pressure difference ΔP, and hence $\Delta\mu$ also, was shown to be equally distributed (statistically) over the entire length of the slit. In this case the reproducible flow rates obtained were consistently lower than when the fluid was driven with a plunger beginning with $\vec{v}_S < \vec{v}_{S,c}$. In the forced-flow experiments, it appeared as if vorticity was preferentially produced at certain sites not uniformly distributed along the channel.

The probings reported here serve to illuminate some of the gross features of film flow. Further studies are in progress with multiple probes in order to define $\mu_f(s)$ more completely in various geometries and, in particular, to investigate to what additional extent changes in R are accompanied by changes in the chemical potential profile.

The authors are indebted, of course, to Dr. B. D. Josephson for his original comments, and as well to Dr. E. Huggins for stimulating discussions, and to Mr. A. P. Roensch for his painstaking and ingenious glass blowing.

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[1]See, for example, K. R. Atkins, Liquid Helium (Cambridge University Press, Cambridge, England, 1959), Chap. 7.

[2]The ingenious double beaker experiment [J. Daunt and K. Mendelssohn, Phys. Rev. 69, 126 (1946); B. S. Chandrasekhar and K. Mendelssohn, Proc. Phys. Soc. (London) A68, 857 (1955)], represents a type of potentiometer which has shown that in the film, mass motion may take place under zero potential. Our objective in the present paper is quite different, since we wish to study nonzero potentials as well. Whereas this has not previously been accomplished for the film, R. Bowers and K. Mendelssohn [Proc. Phys. Soc. (London) A63, 178 (1950)] and R. Bowers, B. S. Chandrasekhar, and K. Mendelssohn [Phys. Rev. 80, 856 (1950)] have reported pressure measurements along the flow path of He II through narrow channels.

[3]See, for example, P. W. Anderson, Rev. Mod. Phys. 38, 298 (1966).

[4]The chemical potential $\mu(\vec{v})$ of the moving film has been shown [J. Tilley, Proc. Phys. Soc. (London) 84, 77 (1963)] to be given by $\mu(\vec{v}) = \mu(0) - (\rho_n/2\rho)(\vec{v}_n - \vec{v}_s)^2$, where $\mu(0)$ refers to the chemical potential of the film at rest and $\vec{v}_n$ is the normal fluid velocity assumed to be zero. The probes described here are not influenced by the velocity term but measure only $\mu(0)$.

[5]B. N. Eselson and B. G. Lazarev, Dokl. Akad. Nauk SSSR 81, 537 (1951); Zh. Eksperim. Teor. Fiz. 23, 552 (1952).

[6]J. F. Allen and C. C. Matheson, Proc. Roy. Soc. (London) A290, 1 (1966).

[7]W. E. Keller and E. F. Hammel, Physics 2, 221 (1966).

HYPERFINE FIELD SPECTRA IN Fe-Mn AND Fe-V ALLOYS

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The hyperfine field distributions at both the host and the impurity nuclei in iron-rich Fe-Mn and Fe-V alloys have been examined by nuclear resonance techniques.

In this Letter we report the observation of corresponding satellites on the host and impurity nuclear resonance lines in Fe-rich alloys containing small concentrations of either Mn or V. The most striking result of this study is the observation of almost identical hyperfine spectra in the Mn and Fe nuclear resonances in the Fe-Mn alloys. This is interpreted as evidence that the changes in the hyperfine fields at nuclei near a Mn impurity arise predominantly from induced changes in the conduction-electron spin polarization (cesp). In the Fe-V alloys no simple relationship between the hyperfine field distributions is found for the Fe and V spectra. Here it appears that, in addition to perturbing the cesp, a V impurity