from which one obtains

$$\mathrm{Re}\xi = -1.2 \pm 0.5,$$

assuming $f_\pm(q^2) = f_\pm(0)$ and $\mathrm{Im}\xi = 0$.

The previously determined values of $\mathrm{Re}\xi$ are $\mathrm{Re}\xi = +1.2 \pm 0.8$ from measurement[4] of the muon energy spectrum from $K_L^0(\mu 3)$ and $\mathrm{Re}\xi = +0.8 \pm 0.8$ from two values[5,6] of the ratio $K_L^0(\mu 3)/K_L^0(e3)$. Both of these results assume $f_\pm(q^2) = f_\pm(0)$ and $\mathrm{Im}\xi = 0$. However, the assumption $f_\pm(q^2) = f_\pm(0)$ may lead to differences in evaluating $\mathrm{Re}\xi$ from spectrum and branching-ratio data and from polarization data. [For example, it is possible to fit the muon energy spectrum from K_L^0 with $\mathrm{Re}\xi = -0.8 \pm 0.3$ as was done in Ref. 4 by assuming $f_\pm(q^2)$ to be determined by a single $J=1$ intermediate K-π state.] If this explanation is valid, a re-examination of K^+ semileptonic-decay data will be required before a meaningful comparison of $\mathrm{Re}\xi_0$ for K_L^0 and $\mathrm{Re}\xi_+$ for K^+ can be made.

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SUM RULES FOR P-WAVE PION-NUCLEON SCATTERING LENGTHS FROM CURRENT ALGEBRAS*

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From current commutation relations and the hypothesis of partial conservation of axial-vector current, sum rules are derived relating πN P-wave scattering lengths to the S-wave scattering lengths, the nucleon isovector magnetic moment, g_A, and $G_{\pi NN}$.

From current commutation relations and the hypothesis of partial conservation of axial-vector current (PCAC), we derive in this note sum rules relating the πN P-wave scattering lengths to g_A, the nucleon isovector magnetic moment $\mathfrak{M}^V$, the πNN coupling constant G, and the πN S-wave scattering lengths, and compare these sum rules with available experimental data.

Consider the matrix amplitude $M_{AA}{}^{\mu\nu}$ defined by

$$i(2\pi)^4\delta(p_f+q_2-p_i-q_1)M_{AA}{}^{\mu\nu} = \int d^4x\, d^4y\, d^4z\, d^4w \exp[i(-q_1\cdot x + q_2\cdot y - p_i\cdot w + p_f\cdot z)]$$
$$\times(-\gamma\cdot p_f + m)\langle 0|T[\mathfrak{A}_{1\alpha}{}^{\mu}(x)\mathfrak{A}_{2\beta}{}^{\nu}(y)\psi_\gamma(z)\bar{\psi}_\sigma(w)]|0\rangle(-\gamma\cdot p_i + m), \qquad (1)$$

and the amplitude M_{PP} obtained by replacing $\mathfrak{A}_{1\alpha}{}^{\mu}(x)$ and $\mathfrak{A}_{2\beta}{}^{\nu}(y)$ in (1) by $\varphi_{1\alpha}(x)$ and $\varphi_{2\beta}(y)$ and operating on the vacuum expectation value by $(\Box_x{}^2+\mu^2)(\Box_y{}^2+\mu^2)$. Here $\psi_\gamma(z)$ and $\varphi_\alpha(x)$ are nucleon and pion-field operators, $\mathfrak{A}_\alpha{}^\mu(x)$ is the axial-vector current density, μ and m are the pion and nucleon masses, and $\alpha, \beta, \gamma, \cdots$ are isospin indices. $\bar{u}(p_f)M_{PP}u(p_i)$ is the transition amplitude for πN scattering.

Using the PCAC hypothesis, $\partial_\mu \mathfrak{A}_\alpha{}^\mu(x) = C\varphi_\alpha(x)$, and the commutation relation (c.r.)

$$[\mathfrak{A}_\alpha{}^0(x), \mathrm{A}_\beta{}^\nu(y)]_{x_0=y_0} = i\epsilon_{\alpha\beta\beta'}\mathfrak{V}_{\beta'}{}^\nu(x)\delta(\vec{x}-\vec{y}) + \text{a possible Schwinger term}, \qquad (2)$$

we obtain, using the methods of Raman and Sudarshan,[1] the equation[2]

$$q_{1\mu}q_{2\nu}M_{AA}^{\mu\nu}=(\mu^2-q_1^2)^{-1}(\mu^2-q_2^2)^{-1}C^2M_{PP}-i\epsilon_{\alpha\beta\beta'}q_{2\nu}F_{\beta'}^{\ \nu}, \tag{3}$$

where

$$F_{\beta'}^{\ \nu}=-\tfrac{1}{2}\tau_3\{f_1^V(k^2)\gamma^\nu+f_2^V(k^2)\tfrac{1}{2}[\gamma^\nu,\gamma^\lambda]k_\lambda\}. \tag{4}$$

Here, $k=p_i-p_f$, and $f_{1,2}^V$ are the isovector form factors of the nucleon, with $f_1^V(0)=1$.

Defining the variables $P=\frac{1}{2}(p_i+p_f)$, $Q=\frac{1}{2}(q_1+q_2)$, $N_\mu=\epsilon_{\mu\nu\rho\sigma}P^\nu Q^\rho k^\sigma$, $\nu=P\cdot Q/m$, $\nu_B=-q_1\cdot q_2/2m$, we make the spin decomposition

$$M_{PP}=A+(\gamma\cdot Q)B;\quad M_{AA}^{\mu\nu}=\sum_{j=1}^{16}R_j^{\mu\nu}[D_j+G_j(\gamma\cdot Q)]; \tag{5}$$

with

$$R_1^{\mu\nu}=P^\mu P^\nu,\quad R_2^{\mu\nu}=Q^\mu Q^\nu,\quad R_3^{\mu\nu}=k^\mu k^\nu,\quad R_4^{\mu\nu}=N^\mu N^\nu,$$

$$R_{5,6}^{\mu\nu}=(P^\mu Q^\nu\pm P^\nu Q^\mu);\quad R_{7,8}^{\mu\nu}=(P^\mu k^\nu\pm P^\nu k^\mu),\ \text{etc.}, \tag{6}$$

and the well-known isospin decomposition into $A^{(\pm)}$, $B^{(\pm)}$, etc. Substituting these in (3), considering nonforward scattering, equating the coefficients of the independent invariants, and then taking the "zero-mass" forward scattering limit $q_1^2=0$, $q_2^2=0$, $\nu_B=0$, we obtain[3]

$$m^2\nu^2D_1^{(+)}=\mu^{-4}C^2A^{(+)};\quad m^2\nu^2D_1^{(-)}=\mu^{-4}C^2A^{(-)}+\nu(\mathfrak{M}^V-1);$$

$$m^2\nu^2G_1^{(+)}=\mu^{-4}C^2B^{(+)};\quad m^2\nu^2G_1^{(-)}=\mu^{-4}C^2B^{(-)}-\mathfrak{M}^V; \tag{7}$$

where $\mathfrak{M}^V=1+2mf_2^V(0)$. In the limit $\nu\to 0$, these give

$$A^{(+)}(0,0,0,0)=2m\mu^4C^{-2}g_A^{\ 2};\quad A^{(-)}(0,0,0,0)=0;$$

$$B_p^{(+)}(0,0,0,0)=0;\quad B_p^{(-)}(0,0,0,0)=-\mu^4C^{-2}[g_A^{\ 2}-\mathfrak{M}^V]; \tag{8}$$

where $B_p^{(\pm)}$ are the proper (i.e., nonpole) parts of $B^{(\pm)}$, and we have used the notation $A(\nu,\nu_B,q_1^2,q_2^2)$, etc.

To extrapolate these relations to the zero-energy forward scattering of massive pions, we assume that $B_p^{(+)}(0,0,0,0)\approx B_p^{(+)}(0,0,\mu^2,\mu^2)/K^2(0)$, etc. [where $K(k^2)$ is the $NN\pi$ form factor, with $K(\mu^2)=1$], neglect the change caused by varying ν_B from 0 to $-\mu^2/2m$, as $\mu^2/2m$ is small, and estimate the differences $\Delta A^{(+)}=A^{(+)}(\mu,-\mu^2/2m)-A^{(+)}(0,-\mu^2/2m)$ and $\Delta B_p^{(-)}=B_p^{(-)}(\mu,-\mu^2/2m)-B_p^{(-)}(0,-\mu^2/2m)$ by writing a dispersion relation[4] in ν for $A^{(+)}$ and $B^{(-)}$ and using the experimental πN phase shifts[5] to evaluate the dispersion integrals. The amplitudes $A^{(+)}$ and $B^{(-)}$ at the physical threshold are obtained by adding $\Delta A^{(+)}$ and $\Delta B_p^{(-)}$ and the exact pole term in $B^{(-)}$ to the amplitudes at $\nu=0$, $\nu_B=0$. As the dispersion integrals for $\Delta A^{(-)}$ and $\Delta B^{(+)}$ do not converge sufficiently rapidly, these differences are evaluated by using $\Delta A^{(+)}$, $\Delta B_p^{(-)}$, the exact pole terms, the relations (8), and the experimental S-wave scattering lengths[6] $a_0^{(I)}$ in the relation $a_0^{(I)}=[m/4\pi(m+\mu)][A^{(I)}+\mu B^{(I)}]$ for $\nu=\mu$, $\nu_B=-\mu^2/2m$. The results are as follows:

$$\Delta A^{(+)}\approx 6.67;\quad \Delta A^{(-)}\approx 4.26;\quad \Delta B_p^{(+)}\approx -6.55;\quad \Delta B_p^{(-)}\approx -1.39, \tag{9}$$

in the units $\hbar=c=\mu_\pi=1$. Equations (8) and (9) give $A^{(\pm)}$ and $B^{(\pm)}$ at the physical threshold.[7]

On the other hand, from the partial-wave expansions of $A^{(I)}$ and $B^{(I)}$, one obtains (e.g., see Ref. 6)

$$a_{1-}^{(I)}-a_{1+}^{(I)}=-\frac{a_0^{(I)}}{4m^2}+\frac{B^{(I)}(\mu,-\mu^2/2m)}{8\pi m}=\frac{(m+\frac{1}{2}\mu)}{2\mu m^2}a_0^{(I)}-\frac{A^{(I)}(\mu,-\mu^2/2m)}{8\pi\mu m}, \tag{10}$$

where $a_{1\pm}^{(I)}$ are the P-wave scattering lengths. The two equations here are equivalent when one uses the relation $a_0^{(I)}=m[A^{(I)}+\mu B^{(I)}]/4\pi(m+\mu)$ at threshold. Equations (8)-(10) then give the sum rules

$$a_{11}-a_{13}=-\frac{a_0^{(1/2)}}{4m^2}-\frac{G^2}{8\pi m^3}[1-g_A^{-2}\mathfrak{M}^V]-\frac{G^2(1-\mu/m)}{8\pi\mu m^2(1-\mu^2/4m^2)}+\frac{\Delta B_p^{(1/2)}}{8\pi m}, \tag{11}$$

$$a_{31}-a_{33}=-\frac{a_0^{(3/2)}}{4m^2}+\frac{G^2}{16\pi m^3}[1-g_A^{-2}\mathfrak{M}^V]-\frac{G^2}{8\pi\mu m^2(1-\mu/2m)}+\frac{\Delta B_p^{(3/2)}}{8\pi m}, \tag{12}$$

where we have denoted $a_{1+}^{(I)}=a_{2I,\,2J}$ and have expressed the PCAC constant C as $C=\sqrt{2}g_A m_N\mu_\pi^2 \times G^{-1}K^{-1}(0)$. The S-wave scattering lengths here may further be expressed in terms of g_A and G, using the PCAC hypothesis.[8]

To compare the sum rules (11) and (12) with experiment, we substitute in them the S-wave scattering lengths of Hamilton and Woolcock,[6] and the values of g_A, G, and $\mathfrak{M}^V$, and use (9), obtaining

$$a_{31}-a_{33}\approx-0.234;\quad a_{11}-a_{13}\approx-0.137. \tag{13}$$

Using the estimates of $a_0^{(I)}$ by other workers[5,9] does not alter (13) significantly.

The experimental estimates of a_{31} and a_{13} of different workers[5,6] agree well, giving $a_{31}\approx-0.036$, $a_{13}\approx-0.029$. For a_{33}, we take the best estimate as that of Ref. 6, giving $a_{33}\approx0.215$, so that $a_{31}-a_{33}\approx-0.251$. This agrees well with the first relation in (13) and supports the validity of (11). The different estimates of a_{11} do not agree; thus the 0- to 100-MeV solution of Roper, Wright, and Feld[5] give $a_{11}\approx-0.04$, $a_{11}-a_{13}\approx-0.01$, while Hamilton and Woolcock[6] give $a_{11}=-0.101\pm0.007$, $a_{11}-a_{33}=-0.072+0.012$. The present data then cannot test (12) adequately. On the other hand, assuming the validity of (12) may provide a useful guide for preferring one experimental estimate to another. Thus it suggests preferring Hamilton and Woolcock's solution for a_{11} to the 0- to 100-MeV solutions of Roper, Wright, and Feld.

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[2]In (3), we have omitted the contribution of the Schwinger term in (2), as it vanishes in the limits to be used here. We have also omitted a term arising from the commutator $[\mathfrak{A}_\beta^0(x),\varphi_\alpha(y)]$, as this term is found to be proportional to the combination $[a_0^{(1/2)}+2a_0^{(3/2)}]$ of the S-wave scattering lengths, which is very small. {This term was evaluated using the c.r. $[\mathfrak{A}_\beta^0(x),\varphi_\alpha(y)]\delta(x_0-y_0)=id_{\beta\alpha\alpha'}\zeta_{\alpha'}(y)\delta(x-y)$ suggested by a quark model, where $\zeta_{\alpha'}(y)$ is a scalar operator.}

[3]Taking the forward scattering limit before separating Eqs. (7) gives only the combinations of these equations involving $A^{(\pm)}+\nu B^{(\pm)}$, which give the Adler-Weisberger relation and the S-wave scattering length relations. Equations (7) contain more information.

[4]This follows the method of S. L. Adler, Phys. Rev. 137, B1022 (1965), who obtained $A^{(+)}(\mu,0)-A^{(+)}(0,0)\approx7.4$ for $q_1^2=0$, $q_2^2=\mu^2$.

[5]For 0 to 700 MeV, we have used solution (14) of L. D. Roper, R. M. Wright, and B. T. Feld, Phys. Rev. 138, B190 (1965), and for 700 to 1050 MeV, the results of B. H. Bransden, P. J. O'Donnell, and R. G. Moorhouse, Phys. Letters 19, 420 (1965). The contribution of energies above 1 GeV to the $\Delta A^{(+)}$ and $\Delta B^{(-)}$ dispersion integrals was neglected. In obtaining scattering lengths from Roper, Wright, and Feld's analysis, their 0- to 100-MeV solutions A to D were used.

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[7]We are indebted to Dr. C. Callan for pointing out that the change in the amplitudes in going to the physical threshold cannot, in general, be ignored.

[8]See Ref. 1 and the following papers: Y. Tomozawa, to be published; A. P. Balachandran, M. Gundzik, and F. Nicodemi, to be published; B. Hamprecht, to be published; S. Weinberg, to be published.

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